Title:
Benefitting Pension Contributors
– An Analysis of the Danish Pension Fund Sector

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Executive Summary

The purpose of this thesis is to investigate, whether it is possible to enhance the performance of the Danish pension fund sector. It will be investigated whether the pension contributors could achieve a larger benefit if the pension funds’ investment strategy was changed.

Offering the defined benefit products, the Danish pension fund sector is responsible of carrying out the investments of retirement savings and holds the risk by setting a guaranteed return. To ensure a proper conduct with regards to portfolio management in the pension contributors’ interest, legislation, both Danish and European, highly restrict the investment strategy. The day-to-day solvency requirement and the risk-based focus on investments themselves force the pension funds to act rather short term in their investments.

According to theory equity outperforms risk-free assets over time. Further, volatility decreases with time since annual returns on assets are mean-reverting. This combined with the fact, that the Danish pension funds are long term investors facing an infinite investment horizon supports that holding risky assets on a long term basis increases probability in achieving higher returns, benefiting the pension contributors to a higher extent.

From a comparison of the derived theoretical optimal portfolios and the estimated actual portfolio of the Danish pension fund sector, it is concluded possible to achieve a higher performance. The day-to-day solvency requirement and the risk focus on investments are not implemented in the theoretical optimised portfolios, and these legal requirements are assessed part of the divergent performances of the compared portfolios. The time perspective shows important both with respect to setting the investment strategy and executing it. A relaxation of the solvency requirement enables the Danish pension fund sector to incorporate a long term focus in its investments, enlarge equity ratio in the portfolio and thereby opening up for achieving higher returns. Pension funds should include both the short term and long term perspective in the investment strategy. Executing the investments should be aligned with the revised strategy, why it is important that incentive based targets and remuneration packages are changed to reflect both the short term and long term focus in the investment strategy.
Contents

1 Chapter 1 - The Framework ...........................................................................................................1
  1.1 Introduction .......................................................................................................................1
  1.2 Motivation .........................................................................................................................2
  1.3 Research Objective ............................................................................................................3
  1.4 Structure of Thesis .............................................................................................................5
  1.5 Methodology ......................................................................................................................6
    1.5.1 Data Collection ...........................................................................................................6
    1.5.2 Data Review ...............................................................................................................7
    1.5.3 Overall Conditions of the Thesis ...........................................................................10
    1.5.4 Specific Comments to the Theoretical Optimisation Model ....................................11
    1.5.5 Delimitations ...........................................................................................................13
    1.5.6 Structure of Analysis ................................................................................................14
2 Chapter 2 - The Danish Pension Fund Sector .............................................................................17
  2.1 Description of the Danish Fund Pension Sector ....................................................................17
    2.1.1 The Structure of the Danish Pension Fund Sector .......................................................19
      2.1.1.1 Commercial and Labour Market Pension Funds ..............................................19
    2.1.2 Products offered by the Pension Fund Sector ..............................................................20
      2.1.2.1 The Defined Benefit Products ...........................................................................21
      2.1.2.2 The Defined Contribution Products ..................................................................23
    2.2 Regulations within the Danish Pension Fund Sector .......................................................24
      2.2.1 The Danish Legislation – Financial Business Act ..................................................25
      2.2.2 EU Legislation – The Solvency II Directive ...........................................................27
    2.3 The Actual Portfolio of the Danish Pension Fund Sector ..................................................30
      2.3.1 Data on the Actual Portfolio ....................................................................................31
      2.3.2 Asset Allocation of the Pension Sector’s Portfolio ...................................................33
      2.3.3 The Average Actual Portfolio ..................................................................................34
    2.4 Conclusion on Chapter 2 .................................................................................................36
# Chapter 3 - Theoretical Review

3.1 Pension Funds as Long Term Investors

3.1.1 The Equity Premium
3.1.2 Time Diversification
3.1.3 The Challenge of the Danish Pension Fund Sector
3.1.4 The Theoretical Foundation of Long Term Investments

3.2 Modern Portfolio Theory

3.2.1 The Standard Mean-Variance Model
3.2.1.1 The Efficient Frontier
3.2.2 CAPM

3.3 The Black-Litterman Optimisation Model

3.3.1 The Intuition of the Black-Litterman Optimisation Model
3.3.2 The Black-Litterman Model

3.4 Performance Measures

3.4.1 Sharpe Ratio

3.5 Investing Internationally

3.5.1 ICAPM
3.5.2 Exchange Rate Hedging
3.5.2.1 What is Exchange Rate Hedging?
3.5.2.2 Hedging Strategies and Derivative Instruments
3.5.2.3 The Chosen Hedging Approach in the Data Set

3.6 Conclusion on Chapter 3

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1.1 The Equity Premium</td>
<td>39</td>
</tr>
<tr>
<td>3.1.2 Time Diversification</td>
<td>41</td>
</tr>
<tr>
<td>3.1.3 The Challenge of the Danish Pension Fund Sector</td>
<td>43</td>
</tr>
<tr>
<td>3.1.4 The Theoretical Foundation of Long Term Investments</td>
<td>44</td>
</tr>
<tr>
<td>3.2.1 The Standard Mean-Variance Model</td>
<td>45</td>
</tr>
<tr>
<td>3.2.1.1 The Efficient Frontier</td>
<td>49</td>
</tr>
<tr>
<td>3.2.2 CAPM</td>
<td>51</td>
</tr>
<tr>
<td>3.3.1 The Intuition of the Black-Litterman Optimisation Model</td>
<td>54</td>
</tr>
<tr>
<td>3.3.2 The Black-Litterman Model</td>
<td>55</td>
</tr>
<tr>
<td>3.4.1 Sharpe Ratio</td>
<td>58</td>
</tr>
<tr>
<td>3.5.1 ICAPM</td>
<td>59</td>
</tr>
<tr>
<td>3.5.2 Exchange Rate Hedging</td>
<td>61</td>
</tr>
<tr>
<td>3.5.2.1 What is Exchange Rate Hedging?</td>
<td>61</td>
</tr>
<tr>
<td>3.5.2.2 Hedging Strategies and Derivative Instruments</td>
<td>61</td>
</tr>
<tr>
<td>3.5.2.3 The Chosen Hedging Approach in the Data Set</td>
<td>62</td>
</tr>
<tr>
<td>3.6 Conclusion on Chapter 3</td>
<td>63</td>
</tr>
</tbody>
</table>

# Chapter 4 - The Theoretical Optimised Portfolio

4.1 Data for the Theoretical Portfolio Optimisations

4.1.1 Presentation of Data for the Theoretical Portfolio Optimisations
4.1.2 Information on Data for the Theoretical Optimisations

4.2 Statistical Analysis of Data for the Theoretical Optimisations
4.2.1 Normality Test ......................................................................................................... 69
4.2.2 Test for Autocorrelation ........................................................................................ 71
4.3 The Individual Asset Markets .................................................................................. 72
4.4 Challenging Historical Data .................................................................................... 75
4.5 Theoretical Portfolio Optimisation & Comparison to the Actual Portfolio ............. 78
  4.5.1 First Part of the Optimisation Processes .............................................................. 80
  4.5.2 Portfolio One – The Minimum Variance Portfolio ............................................. 81
  4.5.3 Portfolio two - Maximise Return ....................................................................... 86
4.6 Addressing the Actual vs. the Theoretical Optimised Portfolio ................................. 93
4.7 Conclusion on Chapter 4 .......................................................................................... 97

5 Chapter 5 – The Time Perspective in Portfolio Management ..................................... 100
  5.1 Time Perspective – Central Aspects ....................................................................... 101
    5.1.1 Managing Long Term and Short Term Interests ................................................ 102
      5.1.1.1 Long versus Short Term Horizon ............................................................... 102
      5.1.1.2 Is a Short Term Investment Strategy the Most Beneficial to Pension
         Contributors? ............................................................................................................ 103
    5.1.2 Managing Long Term Investments on a Short Term Basis ................................. 106
  5.2 Bringing Suggestions Forward Addressing the Time Perspective .......................... 108
    5.2.1 Incorporation of the Long Term Perspective ................................................... 108
      5.2.1.1 Risk in a Long Term Perspective ............................................................... 108
      5.2.1.2 The Suggested Model ............................................................................. 110
    5.2.2 Alignment of Strategy and Execution ............................................................... 116
  5.3 Conclusion on Chapter 5 ........................................................................................ 117

6 Chapter 6 - Conclusion ............................................................................................... 120
7 Chapter 7 - Bringing the Analysis into Perspective ....................................................... 123
8 Bibliography ............................................................................................................... 126
9 Appendices ................................................................................................................ 126
   Appendix A – List of Expressions .............................................................................. 126
Appendix B – The Commercial Pension Fund Sector  
Appendix C – The Theoretical Optimised Portfolios  
Appendix D – The Actual Portfolio  
Appendix E – Description of Data for the Theoretical Optimisation Modelling  
Appendix F – Frequency Distributions of Returns  
Appendix G – SAS Outputs  
Appendix H – Sharpio-Wilk W Test & Skewness and Kurtosis  
Appendix I – Standard Mean-Variance Model  
Appendix J – Correlation Matrix  
Appendix K – Explorative Interviews  
Appendix L – Returns in DKK  
Appendix M – Practical Approach to Exchange Rate Changes in the Data

A list of the most applied expressions in this thesis is provided in appendix A, including a brief description of them.
1 Chapter 1 - The Framework

1.1 Introduction

In standard economics and in reality, wealth is transferred to later stages in life by saving up in earlier stages of life. This allows consumption to be smoothed over a lifetime, so when the work related income is stopped at retirement, a certain standard of living for the individual can be maintained. The balancing of income together with wealth accumulation is typically done through retirement savings in pension funds. The main interest of the pension contributors is to maximize utility of their savings. At the same time they are bound by their risk profile.

To maximise the utility of the savings, the pension contributors need to consider; how much they should invest in risky assets given their level of risk aversion. How is the trade-off between value maximization and risk profile dealt with in the most beneficial way? And what influence, if any, does the time perspective in relation to the investment strategy play with regards to the wealth transfer throughout working life?

Most people don’t have the interest or the knowledge to manage their retirement savings properly. Therefore, pension contributors hand over the responsibility of accumulating and managing their wealth opportunities to institutional investors, in this case pension funds. It is assessed that higher performance of the investments and thus utility maximisation is very possible since pension funds are specialised in evaluating investment opportunities and the associated risks. This is done by incorporating their professional knowledge of the capital markets to carry out the most beneficial investments. Furthermore, it is argued that the economics of scales\(^1\), of the total wealth accumulation in the funds, also contributes to the achievement of more beneficial outcomes than most individual pension contributors are able to themselves. The pension fund service is not free and administrative costs are incurred, however, the overall return from making pension funds capitalise retirement savings are assessed outweighing these.

\(^1\) These include processing more information, possible lower transaction costs etc.
Chapter 1 - The Framework

The primary function of a pension fund is to invest the accumulated pension contributions as appropriately and optimally as possible whilst adhering to the legal regulations. The Danish pension fund sector manages a large amount of capital in terms of pension contributions which corresponded to 43.20% of the Danish GDP\(^2\) in 2009 (OECD Statistics, 2011). Furthermore, The Danish Economic Council estimates that by 2045, 50% of all pension payouts will come from private retirement savings (Andersen, J. G., 2004)\(^3\). Thus, the future welfare and consumption opportunities as retiree of the Danish pension contributors is contingent upon the Danish pension fund sector’s ability to achieve high performing investment portfolios.

The pension fund sector is responsible for managing the pension contributors’ wealth. This is a great responsibility and it might not always be handled in an appropriate way seen from the pension contributors’ point of view. Over time, this has led to a tightening of the legislation restricting the freedom of movement of the pension funds with regards to their investment strategies. Such legislation is put in place to protect the pension contributors.

A pension fund’s primary function is to invest contributions as optimally as possible, given selected investment strategies and the legal restrictions to portfolio management. Portfolio management is defined by investments in asset markets, both domestic and foreign, where the objective is to provide adequate returns on investments at an acceptable risk level in order to finance the consumption needs of pension contributors in retirement. This thesis will focus on the portfolio management of the Danish pension fund sector.

1.2 Motivation

The motivation for the selected research objective is based on the below observations and findings about the Danish pension fund sector:

- The Danish pension fund sector is an important sector to the overall Danish economy.

\(^2\) Gross Domestic Product, annually.

\(^3\) These contributions include the contributions paid by pension contributors themselves and from employers.
- The pension contributors hand over the responsibility to pension funds to manage their wealth in the belief that the pension funds can generate investments with higher performance than the individual pension contributors are able to.
- According to modern portfolio theory, the portfolio allocation decision should be based on the risk and the expected return. In the Danish pension fund sector, there is an increased focus on risk.
- Theory suggests that equity will gain a higher expected return over time compared to risk-free investments. This combined with the fact that pension funds are assessed to have an infinite investment horizon should enable the sector to exploit this.
- The pension funds have the responsibility to manage the pension contributions in an appropriate way so the pension contributors gain the highest possible benefit at retirement. The question is whether this is done from a theoretical point of view?
- To ensure that the Danish pension fund sector conducts appropriate investments the sector is highly regulated. The strict legislation forces the Danish pension fund sector to manage investment portfolios with a high focus on solvency and risk on a very short term basis, why possible gains from more risky long term investments are not achievable. Therefore, it can be questioned whether the current restrictive environment is hampering the possibility to benefit the pension contributors to the highest possible extent.

1.3 Research Objective

It is presumed that the Danish pension fund sector might not benefit the pension contributors to the highest extent possible. The purpose of this thesis is to investigate whether it is possible for the Danish pension fund sector to optimise its portfolio management and through that benefit the pension contributors to a higher extent than it does

From what has been stated above, the below research question is investigated and answered in this thesis by addressing the subsequent sub questions.
Chapter 1 - The Framework

Research Question

Is it possible for the Danish pension fund sector to enhance the performance of its investments by changing its investment strategy and benefit pension contributors to a higher extent than it does?

Sub Questions

- How is the Danish pension fund sector’s portfolio comprised and how does it perform?

- By use of the Black-Litterman optimisation model, does the Danish pension fund sector hold the optimal portfolio from a theoretical point of view and how does this optimal portfolio perform?

- How does legislation affect the portfolio management and does it hamper the Danish pension fund sector in achieving higher performance? And does it hamper a long term perspective in the sector’s investment strategy?

- Can the possible divergence in the performance of the Danish pension fund sector’s actual portfolio vs. the theoretical optimal portfolio partially be explained from theory and practical factors? And do these factors affect the basis for comparison of the two portfolios?

- Can another investment strategy incorporating the time perspective be applied to benefit the pension contributors to a higher extent?
1.4 Structure of Thesis

Figure 1.4.1 Structure of Thesis

<table>
<thead>
<tr>
<th>The Outset</th>
<th>Page 1-16</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Chapter 1 – <em>The Framework</em></td>
<td></td>
</tr>
<tr>
<td>• Introduction</td>
<td></td>
</tr>
<tr>
<td>• Motivation</td>
<td></td>
</tr>
<tr>
<td>• Research Objective</td>
<td></td>
</tr>
<tr>
<td>• Methodology</td>
<td></td>
</tr>
<tr>
<td>• Chapter 2 – <em>The Danish Pension Fund Sector</em></td>
<td>Page 17-36</td>
</tr>
<tr>
<td>• Description of the Danish Pension Fund Sector</td>
<td></td>
</tr>
<tr>
<td>• Regulations within the Danish Pension Fund Sector</td>
<td></td>
</tr>
<tr>
<td>• The Actual Portfolio of the Danish Pension Fund Sector</td>
<td></td>
</tr>
<tr>
<td>• Chapter 3 – <em>Theoretical Review</em></td>
<td>Page 37-64</td>
</tr>
<tr>
<td>• Pension Funds as Long Term Investors</td>
<td></td>
</tr>
<tr>
<td>• Modern Portfolio Theory</td>
<td></td>
</tr>
<tr>
<td>• The Black-Litterman Optimisation Model</td>
<td></td>
</tr>
<tr>
<td>• Performance Measures</td>
<td></td>
</tr>
<tr>
<td>• Investing Internationally</td>
<td></td>
</tr>
<tr>
<td>• Chapter 4 – <em>The Theoretical Optimised Portfolio</em></td>
<td>Page 65-99</td>
</tr>
<tr>
<td>• Data for the Theoretical Portfolio Optimisations</td>
<td></td>
</tr>
<tr>
<td>• Statistical Analysis of Data for The Theoretical Optimisations</td>
<td></td>
</tr>
<tr>
<td>• The Individual Asset Markets</td>
<td></td>
</tr>
<tr>
<td>• Challenging Historical Data</td>
<td></td>
</tr>
<tr>
<td>• Theoretical Portfolio Optimisation &amp; Comparison to the Actual Portfolio</td>
<td></td>
</tr>
<tr>
<td>• Addressing the Actual vs. The Theoretical Optimised Portfolio</td>
<td></td>
</tr>
<tr>
<td>• Chapter 5 – <em>Time Perspective in Portfolio Management</em></td>
<td>Page 100-119</td>
</tr>
<tr>
<td>• Time Perspective – Central Aspects</td>
<td></td>
</tr>
<tr>
<td>• Bringing Suggestions Forward Addressing the Time Perspective</td>
<td></td>
</tr>
<tr>
<td>• Chapter 6 – <em>Conclusion</em></td>
<td>Page 120-122</td>
</tr>
<tr>
<td>• Chapter 7 – <em>Bringing Things into Perspective</em></td>
<td>Page 123-125</td>
</tr>
</tbody>
</table>

Source: Own contribution
The structure of the thesis is now presented. In the next section, the methodological outset for the thesis is conducted. This includes a number of different factors that are found important to include in order to provide the outset of the thesis.

1.5 Methodology

This section contains the methodologies applied to answer the research question. The choice of methods refers to the techniques and procedures used to collect and analyse data (Saunders, Lewis, & Thornhill, 2007).

First, the techniques for collecting data are described. Secondly, a data review containing a critique on the validity and reliability of the data, analysis and conclusions in this thesis. Thirdly, focus will be on the procedure used to analyse data to answer the research question. Some overall conditions regarding the thesis are presented and fourthly, general aspects specifically related to the theoretical model are found appropriate to mention. Fifth, the delimitations of the thesis as a whole follow this section. And finally, the structure of analysis is presented in order to provide the reader with the most appropriate outset for reading the thesis.

1.5.1 Data Collection

This thesis is based on both quantitative and qualitative data. The quantitative data applied mainly consist of; (1) official data from the Bloomberg database and the Central Bank of Denmark, (2) annual reports from the pension funds and (3) data from OECD, Statistics Denmark, The Danish Financial Services Authority⁴ and the sector association of insurance and pension, The Danish Insurance Association⁵ are, among others, applied.

The qualitative data partly stems from two explorative interviews; one with an employee working in one of the pension funds in the sector and the second with a more impartial informant; a consultancy firm which is specialised within institutional investors, among these pension funds (Kirstein Finansrådgivning A/S, 2011). Furthermore, different official homepages, body of laws,

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⁴ From this point in the thesis referred to as the Danish FSA.
⁵ In Danish: Forsikring & Pension
journal articles, academic papers, reports, market surveys, market reports and further theoretical literature are applied.

The type of explorative interviews used in this thesis, is a semi-structured interview (Andersen, I., 2006). The interview guide mostly consists of short questions. The reason for the choice of this interview form is that the interviewer approach to information is inductive. Rather than strictly controlling the interview to deduct specific information for the focus is to discover new information by letting the interviewee choose the direction when answering (Andersen, I., 2006). Since these interviews mainly have served as introductions to the sector and are applied to underpin specific facts related to the sector, they will be referred to as explorative interviews without stating the specific reference throughout the thesis. This is found reasonable since most information is supported by both interviewees. In appendix K – Explorative Interviews, main points from the interviews are listed.

This section was about how data is collected for use in the analysis. In the next section a more thorough review of the data is made. This contains comments on the validity and reliability of the data as well as the conclusions based upon this. It will include both the quantitative and qualitative data.

1.5.2 Data Review

The data applied in the analysis influences the conclusion, why it is found relevant to make a note on the overall reliability and validity. In this section, the primarily focus is the quantitative data, however, the qualitative data will be also be touched upon. Also the consequences with respect to validity and reliability of the conclusion are mentioned when it is found appropriate.

1.5.2.1 Validity

When doing analyses, it is important to ensure that the findings are actually about what they appear to be about. Furthermore, one needs to ensure that the methods applied can be used to assess the data as intended, thereby generalising it to reality – this is what defines validity (Saunders et al., 2007).
Chapter 1 - The Framework

The validity in comparing the theoretical optimised portfolios and the actual portfolio of the Danish pension fund sector can be questioned. This is partly due to the fact, that the data set is based on historical data. It is a known fact that historical data is a poor predictor of the future, which affects the validity (This will be further elaborated in section 4.4 Challenging Historical Data).

Data extracted, for use in the theoretical optimisations is primarily based on indices. It is acknowledged that this limits the validity. It is accepted that an active approach to carry out investments is evident in the real world. Using indices in the theoretical optimisations makes this active approach impossible to picture in the modelling and therefore the outcomes are affected by this. The basis for comparison is also affected. This will be elaborated upon in section 4.6 Addressing the Actual vs. the Theoretical Optimised Portfolio.

Is it possible to generalise findings based on the theoretical analysis to reality and afterwards compare these findings with the actual portfolio held by the Danish pension fund sector? It is argued that main conclusions can be extracted from the analysis, but the specific numerical outcomes are uncertain. However, for the purpose of this analysis, conclusions will necessarily be drawn, also based on the numbers. Furthermore, it has been necessary to make some assumptions in order to compare the actual and the theoretical model.

In general, this analysis is conducted under certain assumptions specified by the authors throughout the thesis. This controlled research setup is naturally affected by exogenous factors, which either are excluded from the setup or might be more subordinate, hence, not possible to specify. Many things affect the outcome and not all is possible to include in the setup, why the validity is questioned. With this acknowledgement, the analysis is nonetheless found possible to draw general conclusions from.

1.5.2.2 Reliability

Reliability refers to whether data, collection techniques and analysis procedures bring consistent findings (Saunders et al., 2007, p. 149). Reliability is an essential prerequisite for validity.

For the purpose of this thesis the individual companies’ information has been united to construct an overall depiction of the sector. This has caused some challenges during the process. These
challenges are addressed continuously throughout this thesis, when it is found relevant. It is recognised that the reliability is affected by the constructed depiction of the Danish pension fund sector. This constructed united depiction, the actual portfolio, serves as the point of comparison for the theoretical optimal portfolio in lack of available data on sector level.

The estimated actual portfolio is based on five year data; 2006-2010. It is a small sample and this affects the estimation precision negatively. Optimally, the data should cover the period of interest of this thesis; 2001-2010. However, due to changes in corporate structure of the pension funds over time, it has not been possible to include previous years. Appendix B - *The Commercial Pension Fund Sector* contains a brief description of the modifications in corporate structures of relevant companies. Both the reliability and the validity are affected by the small sample.

The theoretical optimisation model is based on data from 2001 to 2010. Obviously, the more observations included the more accurate the estimations. This speaks in favour of a wider time spectrum. 10 years data is assessed reasonable for this type of research. Data from a specific period is sensitive to the development in the different markets in this period. This affects the reliability of the results based on the data (this will be elaborated upon in section 4.1.2 *Information on Data for the Theoretical Optimisations*).

The difference in the data sets of the actual portfolio and the theoretical optimised portfolio and how this influences the comparability is addressed in section 4.6 *Addressing the Actual vs. the Theoretical Optimised Portfolio*. Overall, it is acknowledged that the outcomes of this part of the analysis are sensitive to the period over which data is obtained. Testing robustness of the conclusions by investigating outcomes in sub periods, e.g. 2003-2005 would make it possible to either support or question the conclusions drawn from the period, 2001-2010. This is delimitated from in this thesis.

The data for use in the optimisation process is assessed to be reliable as it has been obtained from recognised and popular sources such, as Bloomberg.

Having addressed the quantitative data, the qualitative data is briefly touched upon. The interviews conducted can, to some extent, be biased by the information provided by these contacts. This possibly affects the reliability. Though, it has been prioritised to get the most
accurate understanding of the sector possible, why both the sector (Alm. Brand Liv & Pension A/S) and the impartial informant (Kirstein) are represented. This is done in order to provide the most reliable basis for the analysis with respect to the qualitative information on the sector.

With respect to data, including quantitative as well as qualitative data obtained from the internet, it should be noted that a critical approach has been applied. By this, only sources that in general are assessed reliable are used (see bibliography).

Having conducted a review of the data for use in this thesis, general conditions of the thesis are addressed in the next section. This will include; definitions, assumptions and choices.

1.5.3 Overall Conditions of the Thesis

In this section the overall conditions of the thesis are stated in the form of assumptions and choices made by the authors. These are outlined separately and are stated in any order.

- The commercial part of the Danish pension fund sector is chosen as the point of reference for the analysis. A definition of the sector will follow in chapter 2 - The Danish Pension Fund Sector.

- The sector is composed by a number of pension funds, which operate individually and do not take collective decisions. However, this analysis is carried out at a sector level which therefore does not directly reflect the investment decisions of the individual pension funds. This is imposed in order to be able to draw general conclusions about the Danish pension fund sector as a whole - although it is acknowledged that the sector does not behave as a single entity in reality. Interchangeably, the sector and the pension funds will be referred to, meaning the same group of investors.

- The Danish pension fund sector is defined as long term investors facing an infinite investment horizon. On a continuous basis, pension contributors enter the labour market and start their contributions to their retirement savings in order to secure their future wealth. At the same point in time, others enter the life as retirees requiring the capital they have saved up during their working life. It is assumed that this cycle will continue in infinite time.
- The defined benefit products are the ones of interest with respect to answering the research question of this thesis. A definition of these products will follow in chapter 2 - *The Danish Pension Fund Sector*.

- All return and volatility measures are stated on a yearly basis unless else is stated. Outcomes are based on monthly input. These are converted to yearly numbers for the purpose of disclosure of the outcomes.

- The investment opportunities used as data input in the theoretical optimisation model contain inflation, hence they are nominated in nominal values, why the returns do not measure the actual purchasing power.

- It is acknowledged that the market value based portfolio composition extracted from the Danish pension fund’s annual reports only to some extent depicts the investment strategies of the pension funds. In this thesis, these portfolios are approximated to depict the investment strategies.

- In this thesis, the returns of interest with respect to the actual portfolio are the returns before retirement saving return tax on investments from only pension contributions, \( \text{N1}^F_6 \).

- It is acknowledged that the return on the actual portfolio contains returns on investments not included in the optimised portfolios. For example, the \( \text{N1}^F \) returns include returns on interest rate swaps. Furthermore, returns on the investments in property, investment in emerging market bonds etc. are included in this return.

- Since this thesis is financially and economically founded, the legal aspects of the matter are applied and interpreted to the best of the authors’ ability in order to answer the research question.

The general conditions of the thesis are now addressed. Additional conditions specific to the theoretical optimisation model are found appropriate to isolate.

### 1.5.4 Specific Comments to the Theoretical Optimisation Model

In this section, specific comments to the selected theoretical optimisation model, which are found important conditions to state before entering the analysis, are made clear.

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6 In Danish: afkast før pensionsafkastskat på kundemidler.
The theoretical optimisation model used is the Black-Litterman model (Black & Litterman, 1992). This model takes its outset in modern portfolio theory; it employs the moments of expected return and the standard deviation. These are arguably not the only important factors when determining an optimal portfolio. Taking the complexity of models (incorporating more of these factors) and the constraints into account, this model is found best suitable for the purpose of this thesis.

The Black-Litterman model includes a measure of confidence in the opinions of the investor (see section 3.3 The Black-Litterman Optimisation Model). This confidence is not part of this thesis. This is done in order to ensure simplicity in the model.

Identifying the optimal portfolio requires a relevant level of risk aversion. In this thesis a fixed risk aversion level is chosen for the sector when optimising the portfolios.

The regulations\(^7\) described in the section 2.2 Regulations within the Danish Pension Fund Sector, are only to some extent possible to model. Therefore, it is acknowledged that the legal environment only to a limited extent is possible to incorporate. The legal restrictions that have been possible to include in the model are two allocation restrictions on investments in equity (70%) and in mortgage bonds (40%). In chapter 4 - The Theoretical Optimised Portfolio and onwards these are referred to as the two allocation restrictions.

In the model, foreign money markets could have been included to illustrate the hedging activity. In this thesis the exchange rates effects on foreign investments are corrected for in the foreign investments instead. More on this will follow in section 3.5.2 Exchange Rate Hedging.

In theory, investors have the opportunity to include the domestic money market as a part of the investment strategy either in terms of borrowing or lending money at the risk-free rate of return (see section 3.2.1.1 The Efficient Frontier). From explorative interviews it has been confirmed, that the pension fund sector does not take on leveraged investments. Therefore, a restriction on short sales is applied to the model. Lending money as a part of the investment portfolio is neither included in the model.

Several optimisations have been conducted with different setups with different restrictions. Only the ones found relevant in answering the research questions are included in the thesis.

\(^7\) Throughout the thesis, the word regulations will apply to legislation. Possible regulations defined by the overall pension fund sector will not be addressed.
All the models that have been developed including the ones not included in the analysis are found in appendix C - *The Theoretical Optimised Portfolios*.

Important conditions with regards to the theoretical model have been presented to provide the reader with some overall conditions which are found important state before the analysis is carried out. In the following section, the factors which have been delimitated from in this thesis are addressed.

### 1.5.5 Delimitations

In this section the delimitations of this thesis are presented. This is done in order to state aspects which this thesis does not include in the analysis of the Danish pension fund sector.

- It is known that the Danish pension funds invest in various investment funds. It is assessed that investment funds are only used for specific investment niches rather than in general terms, why investments in investment funds are delimitated from for the purpose of this thesis.

- Since this thesis focus on the overall investments, this means that when talking about e.g. investments in risky assets, an investment in an individual asset within the asset class is not considered. The analysis is conducted on total portfolio level.

- This thesis does not include the contributing principle\(^8\). This is assessed reasonable since the focus of the thesis is on the pension funds’ total investments and how the portfolio composition benefits the pension contributors in total. It does not address the division of returns between the pension contributors, hence, the individual wealth opportunities.

- The consequences of not obeying Danish or European legislation are not found relevant to include in answering the research question.

- The thesis delimits from the insurances often included in pension schemes. Further, the fees that are incurred by holding retirement savings in a pension fund to cover the operating and administrative costs are not included. It is nevertheless acknowledged that these are of high importance with respect to competition within the sector. Transaction costs from investments

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\(^8\) Departmental order (in Danish: Bekendtgørelse) no 358 by 6 April 2010 on the contribution principle (The Danish Financial Services Authority (FSA), 2010).
and tax are not included in the model. The returns obtained from the sector are excluding taxes to make the comparison in a “tax free world”.

- Volatility in the form of the standard deviation is used as a measure of risk in this thesis. This contains the total risk meaning both systematic and non-systematic risk. Alternatively, beta, which measures the systematic risk, among other, could be applied.

Now the overall outset of the thesis is presented. This includes some methodical aspects and conditions important to state with respect to the overall thesis, more specific comments related to the theoretical optimisation model and what has been chosen not to be included in the analysis. In the next section, the structure of the analysis is brought forward.

1.5.6 Structure of Analysis

Having set the outset for the thesis, this section outlines the structure found applicable in analysis conducted in order to answer the research question. A foundation serves as the background for the in-depth analysis, and at last the conclusion will sum up. The structure of the analysis is illustrated in figure 1.5.6.1.
From figure 1.5.6.1, it appears that the overall analysis is built from different blocks, which are linked to each other directly or indirectly.

This structure of analysis must not be confused with the structure of the thesis, illustrated by figure 1.4.1. The structure of analysis focuses on elements drawn from the chapters and hence it does not aim at displaying either the entire content of - or the chronological order of the sections included in the thesis.

1. A description of the Danish pension fund sector is brought forward to sketch out the context of interest.
2. An introduction to the legal environment, in which the pension fund sector operates, is included, partly as a contribution to the description of the sector. It is isolated in this context in order to stress the importance of the legislation to the Danish pension fund sector in its portfolio management and possibilities in carrying out more beneficial
investments. Legislation affects both the actual portfolio and is incorporated in the theoretical optimisations to the highest extent possible. Further the legislation is important for the time perspective, which is a highly relevant factor in the analysis too.

3. A theoretical review is presented. It serves as the theoretical frame of reference for the in-depth analysis illustrated in the sixth block.

4. How the Danish pension fund sector manages its portfolio is highly relevant in order to conclude on its possible outset for benefitting the pension contributors to a higher extent than it does. Therefore, an estimation of the actual portfolio, representing the investment strategy on sector level, is conducted.

5. The theoretical optimised model is conducted and a number of optimal portfolios are the outcomes of this optimisation process. Different “environments” are depicted in the theoretical optimised portfolios by imposing restrictions in the optimisation model. These include, among other, the legal regulation.

6. The five blocks described above serve as the foundation of the subsequent in-depth analysis. This contains a comparison of the theoretical optimised portfolios and the actual portfolio held by the pension sector. From this quantitative part, it is possible to draw conclusions on the performance of the investments carried out by the sector, compared to what theory predicts possible. Moreover, the time perspective is incorporated in the more conceptual part of the analysis investigating the research objective, bearing in mind that pension funds are defined long term investors facing an infinite investment horizon.

7. Every block individually and combined constitutes the overall analysis needed to investigate the research objective. This will be concluded upon in the seventh block.

Throughout the analysis, main points are highlighted in boxes.

The entire outset for the thesis is brought forward in chapter 1. In chapter 2 - *The Danish Pension Fund Sector*, the Danish pension fund sector will be investigated.
2 Chapter 2 - The Danish Pension Fund Sector

The purpose of this chapter is to give the reader a basic knowledge about the pension fund sector in Denmark and create a conceptual framework with regards to specific expressions within the pension sector. Firstly, in section 2.1 a description of the Danish pension fund sector is provided. Secondly, the structure of the sector is described in section 2.1.1, introducing the labour market pension funds and the commercial pension funds. Thirdly, in section 2.1.2 the two main products available to the pension contributors are introduced. Subsequently, the legal environment in which the Danish pension fund sector operates - both touching upon Danish and European legislation is presented in section 2.2. Finally, the actual portfolio held by the Danish pension fund sector is estimated with regards to both composition and performance in section 2.3.

Throughout the sections, the field of interest with respect to this thesis will be stated with the aim of focusing on what is relevant information to answer the research question.

2.1 Description of the Danish Fund Pension Sector

The Danish pension fund sector is capital intensive. As illustrated in figure 2.1.1\(^9\), the size of the pension sector, measured as a percentage of GDP, was 43,26% in 2009 (OECD Statistics, 2011). The number indicates that the sector of private pensions relative to the size of the economy is important.

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\(^9\) The figures and tables are numbered according to the section in which they appear. E.g. figure 2.1.1 is the first figure appearing in section 2.1.
As seen from the graph, the importance of the sector to the overall economy has increased over the last 10 years. Further, in 2009, 116,84 million DKK were contributed to pension schemes corresponding to 7,06 % of the total GDP (Statistics Denmark, 2011). The total amount of pension contributions has increased over the last 10 years (The Danish Insurance Association, 2011).

The Danish pension fund sector is capital intensive and its relative size to the economy is increasing. This combined with the increase in pension contributions support the importance of how pension funds manage the pension contributions.

With respect to the state pension in Denmark, a part of the needed payouts are financed by a “pay-as-you-go” solution whereas the other part is financed through individual savings based initiatives paid through taxes (Andersen, J. G., 2004). These initiatives include ATP, SP and LD\(^\text{10}\). The latter part is hence what differentiates the Danish pension system from most European countries. The Danish Economic Council estimates that by 2045, 70% of the pension payouts will come from savings based pensions (including the savings based initiatives described above with 20% and the private pensions with 50%) and only the 30% will be financed by the state.

\(^{10}\) ATP: Arbejdsmarkeds Tillægspension, SP: Den Særlige Pensionsopsparing, LD: Lønmodtagernes Dyrtidsfond.
Andersen, J. G., 2004). This indicates that the Danish system is and will become increasingly independent of the state contribution. Therefore, to a higher degree, pension contributors will be dependent on their private retirement savings and how these contributions are managed.

2.1.1 The Structure of the Danish Pension Fund Sector

The pension schemes in Denmark are multi-pronged and divided into three different pillars. The first pillar contains the state pension also including the savings based initiatives— all pension payouts financed through taxes. The second pillar contains the work related pensions, for people who are engaged in active employment. Both the employer and the employee pay a contribution each month, and the total contribution is invested and paid out in retirement. Membership in some labour unions forces the pension contributors into mandatory pension funds. This will be elaborated upon in the next section. The third pillar contains all schemes where individuals voluntarily pay a contribution independent of employment or membership of a labour union (Ministry for Economics and Business Affairs, 2003). This thesis focuses on the pension schemes in the second and third pillar, and is referred to as private retirement savings.

This thesis focuses on the pension schemes in the second and third pillar.

2.1.1.1 Commercial and Labour Market Pension Funds

All pension contributions are managed by two types of pension funds; labour market pension funds and the commercial pension funds. These two constitute the overall pension fund sector. A study conducted by the Danish sector association of insurance and pension, The Danish Insurance Association in 2009, shows that the two types of funds have close to the same market share distributed on gross contribution and studying data for the period 2001-2008, the distribution is similar (The Danish Insurance Association, 2011).

As stated in section 2.1.1 The Structure of the Danish Pension Fund Sector, the labour market pension funds are affiliated with specific sectors through collective agreements, examples could be the Lærernes Pensionskasse or Arkitekternes Pensionskasse. Contributions are automatically transferred to the specific pension fund and the pension contributors are bound to this specific
pension fund through their membership in a certain labour union (The Danish Insurance Association, 2011). Obviously, the competitiveness in this part of the overall pension sector is non-existing since the pension contributors cannot chose what pension fund to place their retirement savings.

The commercial pension funds administer contributions not specified through collective agreements. Many companies have an agreement with a specific pension fund. However, the pension contributors are free to move their retirement savings to another pension fund.

The range of products offered in the commercial part of the pension fund sector is wider than it is in the labour market pension funds (explorative interviews). The ability to move ones retirement savings between the different funds and the wide product range make the commercial pension funds compelled to be competitive to maintain their customers (The Danish Insurance Association, 2011)\(^{11}\).

Due to this competitive aspect this thesis will evolve around the commercial pension funds. Eight companies are defined in this subsector - they are; AP Pension Livsforsikringsselskab, PFA Pension, Nordea Liv & Pension, livsforsikringsselskab A/S, Alm. Brand Liv & Pension A/S, TopDanmark Liv Holding A/S (TopDanmark Livsforsikring koncernen), Danica Pension, SEB Pensionsforsikring A/S and Skandia\(^{12}\).

This thesis focuses on the commercial part of the overall pension fund sector.

2.1.2 Products offered by the Pension Fund Sector

There are a number of different products, pension contributors can choose from when investing their retirement savings. The product market is divided into two main groups; the defined benefit products\(^{13}\) and the defined contribution products\(^{14}\). The next two sections will describe these.

\(^{11}\) It should be noted, that since the sector does not seem transparent to the customers, activity of moving from one pension fund to another might be lower than it is assessed to be if the sector was more transparent. New regulations are enforced by 1 July 2011 regarding transparency, why the mobility activity might increase (The Danish Insurance Association, 2011).

\(^{12}\) The number of companies included in the sector of commercial pension funds differs, depending on the source. The eight companies used in this thesis are based on a report by Kirstein Finans (Kirstein Finansrådgivning A/S, 2010).

\(^{13}\) In Danish: Gennemsnitsrenteprodukt.

\(^{14}\) In Danish: Markedsrenteprodukt.
2.1.2.1 The Defined Benefit Products

The defined benefit products guaranty a specified yearly return, which makes it possible to calculate the minimum wealth available at retirement\(^{15}\). The pension funds issue a rate of interest\(^{16}\) each year, which is the expected rate of return received on the pension contributions. It can be revised on monthly basis as a consequence of changing markets and hence expectations to the return possible to achieve. This rate cannot be below the guaranteed return. To make an example, observing Nordea Liv & Pension, livsforsikringsselskab A/S, they offer a guaranteed yearly return of 0,50 % infinitely and a rate of interest of 3,75 % for 2011\(^{17}\) (Nordea Liv & Pension, 2011).

The pension funds are responsible for investing the money in an appropriate and responsible way to ensure the guaranteed return. To ensure the guaranteed return, the pension funds withhold a part of the profit when achieving high returns, a so-called collective bonus potential. This amount of capital is used to pay out the guaranteed return in years with low returns on investments. Moreover, the collective bonus potential pool contains the “real” bonus potential. There are specified limits on how large the collective bonus potential is permitted to be. Exceeding this level, capital contained in the collective bonus potential must be paid out to the contributors as a bonus (The Danish Insurance Association, 2011). This implies that the investment strategy of the pension funds effects how much bonus the pension contributors are able to receive.

Overall, when choosing the defined benefit product, the individual pension contributor does not take any risk, but at the same time he denounces the right to have a say in the investment strategy (The Danish Insurance Association, 2011). This type of product is suitable for risk averse investors, who do not wish to play an active role in the investment strategy. The entire group of pension contributors is risk averse. Though, it is assessed that the dispersed age composition

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\(^{15}\) In Danish: Garantirente.

\(^{16}\) In Danish: Kontorente.

\(^{17}\) This rate of interest is assessed as a competition parameter for the pension funds with regards to this product. Through this, the pension funds signal how well they invest pension contributors’ future wealth. It is acknowledged that the competing pension funds also compete on costs and other parameters. However, for the purpose of this thesis, only the rate of interest is included.
within this group implies different views on how important retirement savings are to pension contributors at specific point in time – underpinning the importance of incorporating the time perspective in the analysis. Time to retirement is relevant with respect to how investments should be carried out. This will be elaborated further upon throughout the thesis.

The defined benefit products serve as a kind of a call option for the pension contributors. The guaranteed return serves as an insurance eliminating the downside risk for the individual pension contributor. It provides an opportunity for exploiting the upside potential. This possible upside is allocated to the collective bonus potential pool. It is recognised that this upside potential is limited.

Due to the fact that the pension funds themselves decide the investment strategy, legislation is enforced to protect the pension contributors’ interest. Thus, the legislation helps to ensure that the pension funds are solvent and capable of meeting their guaranteed returns and pension payments in form of the future liabilities. The pension funds manage risk of pension contributors and to ensure this is done in an appropriate way both Danish and European legislation is applied. This legislation is very restrictive and is presently being tightened further. The specific legislation regarding the defined benefit products is further investigated in section 2.2 Regulations within the Danish Pension Fund Sector.

Over the years, the level of the guaranteed return has dropped remarkably. The Danish sector association of insurance and pension, The Danish Insurance Association (2011) states that the overall Danish pension fund sector, including both labour market pension funds and the commercial part of the sector has guaranteed 1.5 million pension contributors a return above 4% over the years. This corresponds to approximately 19.80% of all pension schemes in 2009. As stated earlier, Nordea Liv & Pension, livsforsikringsselskab A/S now offers a guaranteed return of only 0.5% to new pension contributors (Nordea Liv & Pension, 2011). The drop in the guaranteed returns is caused by decline in the different markets, which the pension contributions are invested in. Consequently, it is difficult for the pension funds to achieve high returns on their portfolios (Nymark, 2000). Also the solvency requirement in Danish legislation and the coming European Solvency II directive are contributing factors challenging the opportunity to oblige to
the guaranteed returns. By this directive, the pension funds are enforced to hold portfolios with low risk, hence, low return assets. The legislation will be described in section 2.2 Regulations within the Danish Pension Fund Sector.

For the purpose of this thesis the defined benefit products are the products of interest. The argumentation for this choice will be elaborated upon after having presented the second group of products available in the Danish pension fund sector in the next section.

The defined benefit products are the products of interest in this thesis.

2.1.2.2 The Defined Contribution Products
The defined contribution product is defined as a value based pension scheme, where pension contributions earn the market interest rate. There are no guarantees of a specific yearly return meaning, in years where the market returns are high so will the interest rate on the pension contribution. On the contrary, the return on pension contributions will be low or even negative in years where markets face hard times (The Danish Insurance Association, 2011).

The defined contribution products can be divided into three groups; unit link, profile- and generation products. Common to all three products is that the pension contributor holds the risk of investing the pension contributions, why the pension funds’ are not obliged to be solvent to all future payouts at all points in time, as it is the case for the defined benefit products. Defined contribution products are also not subject to the same legislative regulations as the defined benefit products. This thesis will not elaborate further on the defined contribution product, including the difference in the three sub products.

In this thesis the research objective pivots on portfolio performance. Since this thesis focus on the portfolio management of the Danish pension fund sector, and it is only responsible for the defined benefit products, these products are of interest in this thesis. As described above, pension contributors holding defined contribution products, hold the entire risk of the investments themselves, why these products are out of scope of this thesis.
A discussion on the future development of the two product groups will follow in Chapter 7 - *Bringing the Analysis into Perspective*.

Summing up, the sector has been presented including how it is structured, what products are offered to the pension contributors, among other aspects and narrowing the field of interest with respect to this thesis. With regards to the defined benefit products, the sector struggles to fulfil the guaranteed returns due to the developments in the capital markets over the last years. Moreover, the existing and upcoming legislation constitutes the boundaries which the sector operates within and it might be a contributing factor to, why the Danish pension fund sector may not achieve the needed returns.

The next section will take its outset in the legislation which partly describes the environment, which the pension fund sector operates in and further states the boundaries which the investment strategies are limited to be executed within. As mentioned in section 1.5.6 *Structure of Analysis* the next section is important to the analysis. Besides being relevant to the description of the pension fund sector, it serves as input provider for the theoretical optimisation model as well as the further analysis of the Danish pension fund sector.

### 2.2 Regulations within the Danish Pension Fund Sector

As brought forward in section 2.1 *Description of the Danish Fund Pension Sector*, the Danish pension fund sector struggles to fulfil the guaranteed returns promised the pension contributors. It was further assessed that a part of this challenge might be due to the legislation. This section will describe the current Danish legislation, the EU legislation - the Solvency II directive.

The various legislations affect the investments which the pension funds are responsible of. Therefore, legislation affects the defined benefit products, referring to section 2.1.2 *Products offered by the Pension Fund Sector*.
2.2.1 The Danish Legislation – The Financial Business Act

The Danish legislation; The Financial Business Act\textsuperscript{18} applies to financial undertakings thereby including the Danish pension fund sector\textsuperscript{19}. Narrowing it down to the pension funds, the purpose of the legislation is to protect the interest of the pension contributors by limiting the pension funds in their investment strategies so the pension contributors can be certain of the pension payouts in retirement. This is ensured by limiting the pension fund sector’s exposure to risk. This is for example achieved by enforcing specific restrictions on the allocations of investments within given asset classes. Articles §158-169 are regulations regarding liquidity and solvency as well as placement of capital specific for insurance companies and pension funds. Investments are divided into two types: (1) gilt-edged assets, which among others are government bonds, mortgage bonds, specific property, and (2) non-gilt-edged assets including equity, mutual funds, investment fund and loans (Industriens Pension, 2011).

In table 2.2.1.1, the relevant articles in The Financial Business Act, for the purpose of this thesis, are identified and briefly stated. Investigating the legislation, which the pension funds sector is subject to, allows for identification of how pension funds are limited in their investment strategy. A more thorough description of the articles will follow below the table.

Table 2.2.1.1 Extract of The Financial Business Act, 2011

<table>
<thead>
<tr>
<th># Article</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>§158</td>
<td>Capital must be invested in an appropriate and advantageous manner for the pension contributors, so that the pension fund at all times can meet its obligations.</td>
</tr>
<tr>
<td>§159</td>
<td>The pension fund is required to hold a group of assets which value at all times correspond to the company’s future liabilities (1\textsuperscript{20}).</td>
</tr>
<tr>
<td>§163</td>
<td>Limitations on investments for Pension funds (regulation on group of assets)</td>
</tr>
<tr>
<td></td>
<td>• A maximum of 70% shares in non-gilt-edged securities e.g. equity, mutual funds and investment funds (1-1\textsuperscript{21})</td>
</tr>
</tbody>
</table>

\textsuperscript{18} In Danish; \textit{Lov om Finansiell Virksomhed}.

\textsuperscript{19} E.g. banks, mortgage credit institutions, investment companies and insurance companies (\textit{The Financial Business Act} article § 5 (1), 2011).

\textsuperscript{20} States that it is article § 159, subsection 1. This term will be used throughout this thesis.
In Article § 158 it is stated that pension funds are obliged to manage pension contributions in an appropriate manner. It is argued, that the article restricts the pension funds not to invest in excess risk to achieve higher return.

At all points in time, pension funds are obliged to hold a group of assets comprising their portfolios that cover all liabilities in the future according to article § 159(1)\(^{22,23}\). This capital requirement with respect to liquidity and solvency constrains the pension fund sector to invest very conservatively so it can meet these liabilities even when the capital or other markets experience shock. By this article, the proportion of risky assets in a portfolio is restricted indirectly since the returns on these assets are more volatile than e.g. bonds. The fact that pension funds must be solvent at all points in time, necessarily limits the level of uncertainty possible to hold in their investments. Uncertainty increases with time. This implies a short term focused investment strategy and hampers a long term view on investments. This article will be included in the analysis throughout this thesis.

By article § 163 and § 164 specific limitations on asset allocation in portfolios are enforced. A pension fund must not invest more than 70% in non-gilt-edged assets overall. In this thesis this is approximated to 70% in equity. Maximum 2% of the capital must be invested in an individual equity – hence e.g. in an individual company. This limit is 3% if the equity capital, the asset relates to, exceeds DKK 250 million (The Financial Business Act, 2011). Arguably, the purpose

\(^{21}\) States that it is article § 163, subsection 1, item 1. This term will be used throughout this thesis.

\(^{22}\) Since the wording of the regulations enforced by law is very similar in English, it is found relevant to state the difference in use of term in this thesis. When referring to the solvency requirement in this thesis, it concerns article § 159(1) in The Financial Business Act. It will be stated clearly, when the mentioned legislation refers to the European Solvency directive.

\(^{23}\) In this thesis, this requirement of solvency at all points in time will be referenced to as a day-to-day restriction since in practice this solvency ratio is made up on daily basis (Explorative interviews, 2011).
of the article is to ensure diversification within investments in stocks. This implies elimination of the unsystematic risk in the portfolio, referring to section 3.2.1 The Standard Mean-Variance Model. Not only investments in equity are regulated, also investments in bonds are subject to the legislation. Minimum 30% of the portfolio must consist of gilt-edged assets and pension funds are not allowed to invest more than 40% in mortgage bonds. This enforces a certain level of diversification in the portfolio.

Assets covered by article § 159(1), meaning all assets contained in the pension funds’ portfolios are further restricted. At least 80% of the assets must be held in congruent currency. The liabilities of the Danish pension funds are nominated in DKK. Half of the 80% is allowed to be held in EUR nominated assets.

From explorative interviews, it is known that in practice, the 80% is reached partly through hedging. Foreign investments are hedged to cover the exchange rate risk that is involved in such investments. This is done in order to oblige to the legislation and naturally also to limit the exchange rate risk exposure in the investments in general.

Apart from the Danish legislation, the Danish membership of the European Union, Danish operators are obliged to respect the European legislation which generally, except in some cases, overrules national law. This will be elaborated upon in the following section.

2.2.2 EU Legislation – The Solvency II Directive

The Solvency II directive will replace the Solvency I directive which is the directive enforced presently. Both of the solvency directives cover insurers and reinsurers excluding the smallest companies (European Union, 2007). Only the Solvency II directive will be elaborated upon in this thesis. In Denmark, insurance companies and pension funds are regulated within the same legislation, therefore the Danish pension fund sector is affected by the Solvency I and II directives.

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24 This thesis delimit from going further into details on this legal set up.
The main objective of the coming Solvency II, in relation to the pension fund sector, is to improve the protection of the interests of the pension contributors to a higher degree. This is in line with the aim of the Danish legislation. The new solvency directive creates a link between risk profile and capital requirements thereby securing rational business conduct of the pension funds and stabilise the financial markets as a whole (The Danish Insurance Association, 2011).

The solvency II is an economic risk-based solvency requirement system whereas the Solvency I is based solely on the capital requirements. Solvency II is risk-sensitive since specific individual solvency requirements will be imposed on each type of asset. The solvency requirement will be high on investments in risky assets, e.g. equity, and low on low risk asset investments e.g. in government bonds. If the pension fund sector holds portfolios with a high ratio of risky assets, the attached solvency requirement consequently will be high compared to a solvency requirement of a portfolio including mainly bonds. It is assessed that this focus on risk in investments, will limit the investment strategy of the Danish pension fund sector (European Commission, 2009). The coming solvency requirement will, ceteris paribus, imply a short term focus in pension funds’ risk management. The enhanced focus on more proactive risk management for the pension funds requires an increased need to devote resources to identify and measure risk.

The Solvency II directive is still a draft, the commencement of the directive is expected January 1 2013 (The Danish Insurance Association, 2011). As a part of the preparation process of solvency II, a number of quantitative impact studies (QIS) have been conducted to evaluate the quantitative consequences of the directive. Recently the QIS5 was conducted.

Over the years, the Danish FSA has implemented risk-focused regulations in the Danish pension fund sector. One of these is the traffic light scenario testing, which was implemented in 2001 (The Danish Financial Services Authority (FSA), 2008). This implies that the Danish pension fund sector already focus on risk in their portfolio composition.

The Danish sector association of insurance and pension, The Danish Insurance Association (2011) assess, that the coming changes in the solvency rules will not affect the Danish sector to the same extend as many of the other EU countries due to the existing focus on risk in investments.
The Danish FSA supports the argument of the Danish readiness, following the QIS5, as 107 of 121 Danish pension funds already even before the rules are set into force satisfy the quantitative conditions (The Danish Financial Services Authority (FSA), 2011b). The sector is prepared for the changes and has incorporated it into its investment strategy already. This being said, the sector does to some degree still prepare for the coming tightening of solvency I since the current draft of the Solvency II directive is preliminary and is still undergoing changes (The Danish Insurance Association, 2011).

Some of the new regulations in the tightening of the solvency directive will though affect the Danish pension fund sector to a higher degree. Particularly with regards to specific changes applying to mortgage bonds, the Danish pension fund sector holds its breath. The current draft of the solvency II directive requires that a maximum of 15% of mortgage bonds held in the portfolio must originate from one issuer. This regulation shows challenging for the Danish pension funds, since there is a limited number of issuers in the Danish mortgage bond market, why high exposure to one issuer will occur (Ministry of Economic and Business Affairs, 2011). If a pension fund wishes to invest 40% in mortgage bonds, which it is allowed to according to Danish legislation and only few issuers are available, the limit of 15 % is assessed to be reached rapidly. Mortgage bonds are preferred investments of the Danish pension funds and the mortgage bond market is an important market for the Danish economy (Ministry of Economic and Business Affairs, 2011).

Rounding off, the legislation affecting the defined benefit products is extensive. The Danish pension fund sector is highly restricted in its investment decisions both by domestic and international legislation. The combined focus on solvency and risk management is assessed to limit the sector since it restricts the liberty of the pension fund sector to exploit more risky, uncertain and possibly high return investment opportunities. It forces the sector to act very short termed in its investments.

The Danish pension fund sector is highly restricted in its investment strategy by legislation - both by Danish and European law.
Having outlined the legal environment in which the Danish pension fund sector operates, the purpose of the next section is to look into how the pension fund sector invests. Hence, how the total investment is allocated across the different asset classes in the portfolio. The section includes an estimation of how the Danish pension fund sector performs.

2.3 The Actual Portfolio of the Danish Pension Fund Sector

In order to assess whether the portfolio management in the Danish pension fund sector is done in the most beneficial way, a point of comparison to the theoretical optimised portfolios is necessary. Estimating the actual portfolio of the Danish pension fund sector is therefore essential for the analysis. This point of comparison will be based on data from the eight commercial pension funds. The comparison is necessary in order to conclude on the performance of the sector as a whole compared to what is optimal from a theoretical point of view.

The actual portfolio held by the Danish pension fund sector serves as the point of comparison to the theoretical optimised model.

This section pivots on the sector’s actual portfolio, hence the portfolio of comparison. It starts by presenting the data from which the actual portfolio is estimated. This contains an explanation of how the portfolios held in the sector are divided into different asset classes and a description of these different groups. Subsequently, the focus will be drawn to the asset allocation over time and this will act as a basis for how the average allocation is estimated. Finally, the actual portfolio on sector level is presented. This is the weighted average of the eight funds operating in the commercial pension fund sector. This portfolio will be used to compare performance of the theoretical optimal portfolio which will be addressed in chapter 4 - The Theoretical Optimised Portfolio.
2.3.1 Data on the Actual Portfolio

As mentioned in section 1.5.3 *Overall Conditions of the Thesis*, data from the annual reports of the eight pension funds of interest is applied for the analysis of the actual investment strategy of the Danish pension fund sector and the split among the assets held in the portfolio. An overview of the portfolio allocation on sector level is not available, why it is constructed from data of the individual companies. The data is available for a five year period; 2006-2010. This is commented on in section 1.5.2 *Data Review*.

In a statutory instrument on financial reporting for pension funds\textsuperscript{25}, the Danish FSA requires some standards for the disclosure of different key figures (Announcement Regarding Financial Reporting for Insurance and Pension Funds, 2011). In the specification of assets and their return measured in market values, a classification is made including seven subgroups\textsuperscript{26}. From this, a slightly different split has been made for this purpose. This is done since several of the eight companies, additionally to the above mentioned specification, present their portfolio in this more simplified way to the stakeholders. Furthermore, this current split is appropriate with respect to data availability in modelling the optimal portfolio.

The sector invests approximately 8% of its total capital in property. Unfortunately, a proper index for property return developments in market prices is not available. Therefore, it is not included, either in the actual or the theoretical optimal portfolio. It is acknowledged that this is not satisfactory with regards to validity of the analysis. Investments in affiliated companies have been excluded however, these investments count for almost nothing (2% in SEB Pension and 0% in all others).

To estimate the actual portfolio on sector level, data from Skandia is aligned to make all eight pension funds comparable and enabling construction of a united depiction of the sector. A description of the alignment, general information and further elaboration on the data set is accessible in appendix B - *The Commercial Pension Fund Sector*. In general, data is based on a number of assumptions, e.g. in relation to market shares in the sector, to reach a joint data set

\textsuperscript{25} This statutory instrument is in force towards insurance companies and pension funds (de tværgående pensionsselskaber) (Announcement Regarding Financial Reporting for Insurance and Pension Funds, 2011).

\textsuperscript{26} The seven groups are divided by investments: Property, affiliated companies, total other investments, total bonds, secured loans, other financial investments, derivative financial investments to hedge (Announcement Regarding Financial Reporting for Insurance and Pension Funds, 2011).
which is as true and fair as possible. This is done to get the most accurate actual portfolio. See appendix D - *The Actual Portfolio* sheet *Skandia*.

The actual portfolio is divided into several groups and the structure is as follows:

- **Danish equity** includes direct investments in listed as well as unlisted individual Danish companies. This group also contains investments in mutual funds, equity funds etc.

- **Foreign equity** includes direct investments in listed as well as unlisted individual foreign companies. Investing in foreign equity markets secures portfolio diversification. This group also contains investments in mutual funds, equity funds etc.

- **Government bonds** are not necessarily Danish, however no further split is specified in the annual reports of the pension funds.

- **Mortgage bonds** are, like government bonds not solely domestic investments opportunities. In this thesis the category also includes a group named “other bonds”\(^{27}\).

- **Inflation-linked bonds** are bonds in which the return is adjusted for inflationary tendencies.

- **Corporate bonds** are issued by companies to raise capital. This group contains investment grade as well as non investment grade bonds. The Danish pension fund both invests in these bonds domestically and abroad. Investments in emerging market bonds are further included in this group.

- **Others** contain other financial investment assets, investments in derivative financial instruments for hedging purposes, and secured loans.

The above grouping of assets in the portfolio is the outset for the optimal portfolio to the extent possible. Further elaboration on this will be made in section 4.1 *Data for the Theoretical Portfolio Optimisations*, where the optimal portfolio will be presented.

| The asset grouping in the actual portfolio held by the Danish pension fund sector is the outset for the opportunity set in the Theoretical portfolio optimisation. |

\(^{27}\) This one is not defined or further elaborated upon. Neither, it is included in the optimal portfolio.
2.3.2 Asset Allocation of the Pension Sector’s Portfolio

Figure 2.3.2.1 illustrates that the allocation among the different assets in the portfolio on sector level has not changed to a large extent over the period of interest, 2006-2010.

![Asset Allocation in the Pension Fund Sector, 2006-2010 and Average](image)

Source: Own contribution, based on data from annual reports of the eight pension funds, 2006-2010.

From figure 2.3.2.1 it is seen, that the main portion of the investments are allocated to bonds, primarily mortgage bonds. The Danish equity market has accounted for a very small portion of the investments, especially declining after the strike of the financial crisis. The same tendency is seen in the foreign equity market. Investments in equity have ranged from constituting approximately 12% of the total investment in 2008 to around 27% in 2007. By implication, approximately 70% was invested in bonds in 2007 whereas at the other extreme, about 81% of the portfolio in 2008 and 2009 was held in bonds. These changes, in the asset allocation were expected from the economic outlook the different years.

As stated above, the actual portfolio is estimated from market values as of 2006-2010. This is only an approximation of the portfolio allocation hence investment strategy referring to section
1.5.3 Overall Conditions of the Thesis. The use of market values in estimating the actual portfolio implies that the proportions invested in the different assets vary with the specific conditions in the different markets. Therefore, the actual portfolio, estimated for this purpose, does not perfectly reflect the investment strategy of the sector. However, in lack of more accurate information and based on explorative interviews of how this problem is dealt with in other analyses on this sector, this way is found the best possible.

2.3.3 The Average Actual Portfolio

As previously stated in the introduction to this chapter, estimating a portfolio of comparison for the optimal portfolio derived later in this thesis is necessary and an important factor to carry out the analysis. From the portfolio allocation data of the individual pension funds, a weighted average has been calculated to obtain the most accurate view of the sector’s actual portfolio\(^{28}\). For calculations and estimation of the actual portfolio, see Appendix D - The Actual Portfolio.

The return data is based on yearly return before tax on pension returns on policyholders’ savings (N1\(^F\)) on portfolio level (Announcement Regarding Financial Reporting for Insurance and Pension Funds, 2011), referring to section 1.5.3 Overall Conditions of the Thesis. This data covers the same five year period from 2006 to 2010.

The actual portfolio of the Danish pension fund sector including the expected return, the standard deviation and Sharpe ratio looks as follows in table 2.3.3.1.

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\(^{28}\) Data on the market shares is based on numbers from 2009 (The Danish Insurance Association, 2011). It is acknowledged that these can be different from the ones in the remaining years from which the asset allocation data is based upon.
The sector has, on average, allocated 17,94% to equities and 76,84% to bonds. The share in equities has, on average, been divided with one fourth in Danish and three fourth in foreign equity, approximately. The table also shows that the average annual return on the pension sector’s portfolio is 3,75%. The standard deviation on the actual return is 5,25%, revealing the volatility of returns on the funds’ investments. The Sharpe ratio measuring the performance of the portfolio is 0,11. This measure will be described in section 3.4 Performance Measures. It will be elaborated further upon in the comparison between this actual portfolio and the theoretical optimised portfolios.

<table>
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<th>Asset Class</th>
<th>4,38%</th>
<th>13,56%</th>
<th>18,15%</th>
<th>39,67%</th>
<th>7,69%</th>
<th>11,33%</th>
<th>5,22%</th>
<th>100,00%</th>
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<td>Danish Equity</td>
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<td>Government Bonds</td>
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<td>Inflation-Linked Bonds</td>
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<td>Corporate Bonds</td>
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<td>Others</td>
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<td></td>
</tr>
</tbody>
</table>

Table 2.3.3.1 The Actual Portfolio

Source: Own contribution, based on data from annual reports of the eight pension funds, 2006-2010.

The Danish pension fund sector’s portfolio has a Sharpe ratio of 0,11. The sector allocates 17,94% in equities and 76,84% in bonds.

In this section, the outset for the further analysis of the investments of the Danish pension fund sector has been presented. How the sectors’ actual portfolio is comprised and how it performs has been presented. This actual portfolio will serve as the point of comparison to the optimal portfolio which will be modelled by use of the theoretical model developed by Fischer Black and Robert Litterman (1992).
2.4 Conclusion on Chapter 2

Chapter 2 - *The Danish Pension Fund Sector* has provided the reader with a description of the Danish pension fund sector. It is a capital intensive sector being important because of its size relative to the Danish economy. The commercial part of the overall pension fund sector has been defined and chosen as the relevant subsector. Further, the defined benefit products have been assessed as the primary interest with respect to the aim of this thesis. The legal environment in which the pension funds operate sets certain boundaries for the investments carried out by the sector. The Danish pension sector is highly regulated by national legislation and the coming European Solvency II directive further increases focus on protecting the pension contributors. The legislation limits the pension funds’ liberty of action in executing their investment strategies.

To demonstrate how the Danish pension fund sector allocates its investments and how it performs, an estimation of the actual portfolio picturing the sector’s investment strategy is put forward. Approximately 18% of the investments are allocated in risky assets and more than 76% are allocated in bonds. The portfolio of the sector has an expected return of 3,75% and an associated standard deviation of 5,25%. Measuring the performance, the portfolio has a Sharpe ratio of 0,11. This portfolio serves as point of comparison to the theoretical optimal portfolios estimated in chapter 4 - *The Theoretical Optimised Portfolio* in investigating whether the Danish pension fund sector optimise its portfolio to the highest degree seen from the pension contributors’ point of view.

With the foundation provided in chapter 2 - *The Danish Pension Fund Sector*, the focus in the thesis now turns to provide a theoretical review which serves as a framework for further analysis. This includes a background for both modelling the optimal portfolio and analyse the performance of the Danish pension fund sector. The chapter will commence by defining the pension funds as long term investors and introduce the time aspects which are found highly relevant to include in the analysis. This will be based on both theory and practise however it still chosen to be included in the theoretical review since its primary purpose is to serve as a framework for the further analysis.
Chapter 3 - Theoretical Review

This chapter will serve as the theoretical foundation and frame of reference to the in-depth analysis in the following chapters (chapter 4 and chapter 5), including the theoretical portfolio optimisation and the incorporation of the time perspective in the investigation of the pension funds’ investment strategy. Below, the content in this chapter is illustrated for the purpose of depicting each of the individual sections’ relation to the in-depth analysis, referring to figure 1.5.6.1 Structure of Analysis in section 1.5.6 and to each other within this chapter. The reasoning behind the below illustration is that the theoretical review consists of independent theoretical sections which not necessarily are related to each other even though they follow each other in the chapter.

Figure 3.1 Structure of Theoretical Review

As illustrated in figure 3.1, some of the sections, individually, are relevant theoretical aspects to include in the in-depth analysis. Other sections, describing the standard mean-variance and CAPM, serve as input providers to the section covering the selected theoretical optimisation.

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29 The dark grey blocks indicate sections contained in this chapter. The light grey block illustrates the following chapters. The dotted lines represent the connection between theoretical aspects addressed in this chapter and the in-depth analysis. The full-lines indicate a connection between specific blocks within this chapter.
model derived by Black and Litterman (1992). This section provides input to the following in-depth analysis.

Firstly, in section 3.1 an introduction of the pension funds sector as long term investors is conducted. In the section focus is on the time perspective from a theoretical point of view, including the equity premium and time diversification. The challenge the pension fund sector face with respect to time and a theoretical framework for long term investments for risk investors follow.

Secondly, a presentation of modern portfolio theory is conducted in section 3.2. In this section the standard mean-variance model is addressed and a description of the CAPM follows subsequently. Thirdly, the Black-Litterman model, which is the selected model for solving the optimisation problem, and the theory behind the model is described in section 3.3. Fourthly, the theoretical foundation of measuring the performance of investments is introduced in section 3.4. Finally, theory on investing internationally including a brief note to ICAPM and a theoretical review of hedging is touched upon in section 3.5.

### 3.1 Pension Funds as Long Term Investors

Pension funds are defined as long term investors facing an infinite investment horizon. The primary function is to earn adequate returns on their investments to meet the obligations towards their customers, the pension contributors. Pension funds have a dual aim when setting their investment strategy; Apart from seeking to optimise returns, this strategy should also be designed to secure all future pension liabilities. Therefore, different risks related to time are an integrated part of the pension fund’s strategy. Both the level of risk in the investments themselves and the risk of a shortfall are important.

Pension funds are long term investors facing an infinite investment horizon.

When investigating investments, it is appropriate to define two terms which are related to the time aspect; (1) the investment horizon and (2) the planning horizon. Firstly, the investment
horizon defines the period of time of an investment. An investment can be short term, long term or even infinite depending on the investor. With respect to pension funds, as previously stated, they are assessed facing an infinite investment horizon. Secondly, the planning horizon refers to how often an investment is evaluated. Even though the investment horizon is long or even infinite, the planning horizon of the same investments is most likely shorter. From explorative interviews it is known that the portfolio management in pension funds is executed on a continuous basis and the investment strategy is revised on yearly basis as a minimum. This refers to the planning horizon of the Danish pension fund sector, which is assessed short term.

In this section, issues related to the long term investment strategy of the Danish pension funds are touched upon. Firstly, the focus is drawn to the equity premium. Secondly, the appliance of time diversification in pension fund investment strategy with respect to investments in risky assets is touched upon. Both of these issues are theoretically founded. With the equity premium and time diversification aspects taken into consideration, at last the challenges of the pension fund sector are outlined.

3.1.1 The Equity Premium

In the article The Equity Premium, A Puzzle, Mehra and Prescott (1985) investigated the long run equity premium. This is defined as the return earned by a risky asset in excess of that earned from investing in a risk-free asset. They found that what standard neoclassic economic models rationalise as a premium for bearing risk does not fully explain the actual excess annual returns on equity. However, using actual U.S. data from 1889-1978, over 20 years, the empirical excess annual return is considerably greater than the risk premium (Mehra & Prescott, 1985, p. 146). This pattern of excess returns is not unique to the U.S. - it is observed when significant capital markets exist (Mehra & Prescott, 2003 p. 7). For the purpose of this thesis, it is assumed that Mehra and Prescott’s findings are valid for the markets in which the Danish pension fund sector invests.

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30 This differs with respect to the length of the period over which the investment horizon is measured (Mehra & Prescott, 1985).
The observation of excess returns on equity has come to be known as the *equity premium puzzle*. What puzzles researchers is why investors put capital into risk-free assets when the return is higher on equity on a long term basis. Since the original article that was published in 1985, many have attempted to solve the equity premium puzzle and the puzzle has become a major research topic within economics and finance. This paper will not go any further into details on the different findings.

*On a long term basis, equity outperforms risk-free investments more than what the risk premium can explain. This is referred to as the equity premium.*

The research question under investigation, questions the portfolio management of the Danish pension funds, including the time perspective in the analysis. The long term perspective underpins the wealth building potential of the equity premium (Mehra & Prescott, 2003)\(^\text{31}\). From this, it is clear that the equity premium should be of central importance in for example portfolio allocation decisions.

For the purpose of this thesis, it is found relevant to include the long term opportunities. Previously it is stated that through legislation, the Danish pension fund sector is restricted to act with a short term perspective in its investments. At the same time it is defined as a long term investor with an infinite horizon.

Historically, there has been a substantial equity premium, but from an investment perspective the future is interesting, why looking at the equity premium that is *expected* to prevail in the future is relevant. Mehra and Prescott (2003 p. 57) state that over a long run horizon, the equity premium is likely to be similar to the one experienced in the past. Thus, the returns on investments in equity are expected to continue dominating the return on the risk free asset for investors with a long investment horizon.

In summary, the excess returns on equity are too large to be consistent with economic theory and reasonable levels of risk aversion. This calls for attention in relation to portfolio allocation

\(^{31}\) Under the following assumptions: all payments to the underlying assets, dividend payments to stocks and interests payments to bonds, are assumed reinvested and there is no taxes are paid (Mehra & Prescott, 2003).
decisions. It is expected that returns on investments in equity will continue to be higher than the returns in risk-free assets, why this perspective is interesting and relevant with respect to investigating the portfolio performance of Danish pension fund sector.

It is known from investigating the investments in the pension funds in section 2.3 *The Actual Portfolio of the Danish Pension Fund Sector* that 17.94% of the overall investments in their existing portfolio are allocated in risky assets, including domestic and foreign equity. Prior to modelling the optimal asset allocation in the sector’s portfolio, 17.94% does not seem to be a large proportion, the equity premium and the long investment horizon taken into consideration.

In the next section, the purpose is to continue the underlining of the importance of the time perspective from a theoretical point of view. The section describes the effect of time with respect to diversifying the investment portfolio - how the length of investment horizon affects the allocation of the investments in risky assets.

### 3.1.2 Time Diversification

As stated in the above section, it is a widespread argument that the equity premium is substantial over time. This speaks in favour of that an investor should hold a higher concentration of equity, if the investment horizon is far-sighted.

Analyses show that over a long horizon, above-average returns will cancel out below-average returns. The volatility in annually returns will decrease over time, meaning that the annually returns are mean-reverting (Kritzman, 1994). Longer investment horizons with lower expected standard deviation of annually returns should therefore, ceteris paribus, allow for more risk taking. These two arguments mean that the probability of a shortfall decreases with time. A shortfall is defined as the probability $p$ of an indexed stock portfolio providing a return lower than a specific target rate, e.g. the return on a risk-free investment maturing at the horizon date (Bodie, Z., 1995, p. 18). This vindicates the classic argument that investing in the equity market is less risky in a long run perspective and speaks in favour of increasing the equity-bond ratio in portfolio decisions for the Danish pension fund sector since it is stated, in section 1.5.3 *Overall Conditions of the Thesis*, that the sector has an infinite investment horizon.
Chapter 3 - Theoretical Review

Volatility in returns on equity decreases with time. This speaks in favour of increasing the equity-bond ratio in portfolio decisions facing an infinite investment horizon.

The above mentioned probability of equity holdings outperforming the safe investment, does however not say anything about the size of the loss an investor will occur in case of improbability $1-p$. In improbability, the return of a risky investment will be lower than the risk-free return. Mathematically, it is proven that the costs, if a loss occurs, increase with the investment horizon (Kritzman, 1994). Therefore, perhaps a more reliable measure of the long term risk is the cost of insuring against a shortfall. The price of such option increases with the length of investment horizon (Samuelson, 1971). Even though the probability of incurring these costs is small in a 20 years perspective, the size of loss should not be neglected since this loss may outweigh the improbability of realising it. This has led to criticism of the time diversification theory, as the costs connected to obtaining the lower volatility may cancel out the benefit.

In spite of the critiques of the time diversification argument, Bodie (1995, p. 2) illustrates that the probability of not incurring loss over a period of 20 years is 96%. This is assessed sufficiently high to accept the arguments of time diversification. This combined with the fact that the equity premium is shown substantial in a long run perspective leads to one obvious question with respect to the research question brought forward in this thesis - Is it possible for the Danish pension fund sector to enhance its portfolio management and benefit pension contributors to a higher extent than it does?

From deepening into the equity premium and time diversification, it is concluded, that from a theoretical point of view, it is possible for the Danish pension fund sector firstly to gain higher returns by investing more risky, and secondly obtain a lower risk, if investing long term. In the next section the challenges of the Danish pension fund sector, with respect to the possibilities in incorporating the time perspective in its investments strategy, is addressed.
3.1.3 The Challenge of the Danish Pension Fund Sector

As illustrated above, the pension fund sector is assessed to have an infinite investment horizon which rationales the pension funds to invest in more risky assets to obtain higher returns. This is supported by the fact that, historically assets have shown to pay higher returns than what even risk premiums can explain combined with the fact that time diversification decreases volatility in annual returns over time.

Investigating the pension fund sector reveals, that the pension funds actually act much more short termed than theory suggests. One of the explanations is that the pension funds are restricted by legislation and have large liabilities towards the pension contributors, why exposures to losses due to changes in the markets, must be secured against. The pension funds are responsible for the pension contributors’ future wealth and they must act according to this responsibility. The pension funds are obliged to pay their liabilities to the pension contributors in retirement. This does not constrain the possibility of investing on a long term horizon. The solvency requirement enforced by Danish law (article § 159(1) in the Financial Business Act) however limits this, since it restricts the pension funds to be solvent for all future liabilities at all points in time, referring to section 2.2 Regulations within the Danish Pension Fund Sector. Also the Solvency II directive enforces a short term risk focus. This will also affect how the pension fund sector allocates its investments. To secure against losses (meaning the pension funds are not able to satisfy the day-to-day solvency requirement) and to obey the short term risk requirement, the pension funds must actively manage their portfolios to match the current economic environment. This explains a short term approach to investments.

In spite facing an infinite investment horizon, the Danish pension fund sector acts very short termed in setting its investment strategy.

Whether the pension fund sector’s short term investment approach benefits the pension contributors to the highest extent possible is found interesting to investigate.

The pension funds hold a dispersed group of customers. As stated earlier, for some pension contributors, time to retirement is short, and for others there is a long time to go. Pension
contributors with many years to retirement might prefer pension funds to apply an investment strategy based on a longer term since this opens up for possible higher returns whereas contributors close to retirement are only interested in the short term perspective. This, even though all pension contributors hold the defined benefit product and hence are assessed equally risk averse. This will be elaborated upon in section 5.1.1 Managing Long Term and Short Term Interests.

3.1.4 The Theoretical Foundation of Long Term Investments

The challenges regarding investments on a long term basis has received quite some attention from financial theorists. They attempt to develop models incorporating the long term perspective opposite the static mean-variance optimisation models. In these theories it is recognised that the expected return and risk will change over time and thereby enable the investor to act on these changes. The models capture this aspect. In the theory; The Strategic Asset Allocation, it is argued that long term investors should take both short term and long term risk into account when selecting portfolios, due to the changes in expected return and risk over time (Campbell & Viceira, 2006).

Long term investors should take into account both the short term and long term risk when carrying out investments.

Theory argues how investors should invest strategically. For example, Campbel and Viceira (2006) suggest that investors who are risk averse and have a long term investment horizon, such as the Danish pension fund sector, should increase investments in mean-reverting risky assets since such assets will hedge the variations on its own expected return. Research concludes mean-reversion in risky assets over time (Kritzman, 1994), (Fama & French, 1993)\textsuperscript{32}. This suggests that a long term approach to risk will allow for a long term investment strategy.

\textsuperscript{32} It is acknowledged that some academic work supports the mean-reverting assessment.
Practical tools for implementing this multiple period strategic asset allocation into optimisation models raise complexity extensively, why this is delimitated from in this thesis.

This section has focused on the time perspective, by including theory, supported by empirics, related to investment opportunities in the long run, and how these theories speak in favour of investing in risky assets. Further the challenges that the Danish pension fund sector is subject to with respect to incorporating the time perspective in carrying out investments have been outlined and lastly, the strategic asset allocation of risk averse, long term investors is brought forward, supporting investments in risky, mean-reverting assets. Having addressed the time perspective to support the in-depth analysis carried out in chapter 4 and 5, in the next section the foundation for this analysis will be further strengthen by investigating modern portfolio theory, focusing on the standard mean-variance model and CAPM.

3.2 Modern Portfolio Theory

As a background for the analyses of the actual and optimal portfolio allocation and evaluation of the performance of the Danish pension fund sector, a review of fundamental theory on portfolio allocation will be presented. This background will contain a brief description of financial theory on asset allocation in decisions under uncertainty and risk. Furthermore, the review contains an examination of the effects on portfolio risk following diversification. The modern portfolio theory states the art of the standard mean-variance model.

3.2.1 The Standard Mean-Variance Model

The all-important aim in portfolio decisions is the maximisation of wealth accumulation of a given portfolio. Markowitz (1952) found that under the assumption of normal and random
distribution of expected returns, among others, portfolio optimisation is not only a question of
the expected returns - the risk related to this maximisation problem of the investor is equally
important. These are the necessary factors to consider in portfolio decisions. In general, an
investor will always prefer more return to less and less risk to more.

In portfolio decisions, the expected return and the volatility in the returns is equally
important.

Expected Return
As a part of the portfolio theory, the concept of expected return is one of the two core aspects in
the standard mean-variance model derived by Markowitz (Markowitz, 1952). The expected return
of a portfolio containing a number of assets equals
\[ E(r_p) = E\left(\sum_{i=1}^{N} X_i r_i\right) \]
where \( X_i \) is the fraction of the investor’s capital invested in the \( i \)th asset, \( r_i \) is the expected return on asset \( i \) and \( N \) refers to
the number of assets.

Variance in the Portfolio - Systematic and Unsystematic Risk
As Markowitz deduced in 1952, besides focusing on the expected return, it is important how
much the return of the investment differ from the average return - the standard deviation (\( \sigma \)). The
volatility in the returns can also be referred to in terms of the variance (\( \sigma^2 \)). One thing is the risk
of a single asset, another is the risk of a combination of assets – these two are very different. The
variance of the combination of a number of assets may be less than the variance of the individual
assets that comprise the portfolio. This does not imply that an undiversified portfolio will never
be superior to the alternative. It might be the case that a security will have higher return
combined with lower variance speaking in favour of a less diversified portfolio.

Holding a portfolio, it is widely known that the investment is subject to two specific types of risk.
(1) The first type of risk refers to the systematic risk. This is the risk associated with the market
returns in general. To some degree, the values of the assets in the same market, e.g. the equity

33This model builds on several assumptions (Markowitz, 1952). The normality and randomness will be tested in data
applied to the model used in this thesis. The remaining will not be elaborated further upon, however, it is
acknowledged that these might not be realistic when applying the model to the real world.
market, move in same direction either decreasing or increasing\textsuperscript{34}. This means that normally the correlation coefficient will lie between 0 and +1. The correlation affects the covariance, which measures how the returns on assets in a given portfolio, vary together (Elton et al., 2007, p. 54). If the returns on two assets deviate from the mean; one positively or one negatively, the covariance between the two is negative.

In every market there is always some correlation and thus covariance between the assets. Hence, every portfolio is exposed to some degree of risk that cannot be eliminated by diversification. The (2) unsystematic risk in a portfolio is, though, possible to eliminate. This is the risk inherent in the different individual assets that are held in the given portfolio. Holding a portfolio with e.g. two assets which have their high and low returns in rather opposite times, it is possible to hold some combination of these two which yields an almost constant return. Hence, when return patterns of the two assets are almost independent, so that the correlation coefficient and covariance approach zero, the variance and thus risk of the portfolio is less than the one of the assets individually.

Thus, it is possible to diversify some of the economic risk away, however, only to a minimum level of variance in the returns. This minimum level of variance refers to the systematic risk, described above, and the investor cannot eliminate this in conducting a specific investment strategy. This is illustrated below in figure 3.2.1.1.

\textsuperscript{34} Events such as Black Monday in 1987 or macroeconomic events might cause such outcome to a higher or lower degree.
The portfolio variance of $N$ assets looks as follows:

$$\sigma_p^2 = \sum_{j=1}^{N} (X_j^2 \sigma_j^2) + \sum_{j=1}^{N} \sum_{k=1 \neq j}^{N} (X_j X_k \sigma_{jk})$$

where $X_j$ is the proportion of the portfolio invested in the $j^{th}$ asset, $\sigma_j^2$ is the variance of asset $j$ and $\sigma_{jk}$ is the covariance between asset $j$ and asset $k$.\(^{35}\)

The contribution of the individual asset’s variance to the portfolio variance approach zero as $N$ – the number of assets in the portfolio – gets large. By contrast, the contribution of the covariance in determining the variance of the portfolio approaches the average covariance (Elton et al., 2007, pp. 58-59).\(^{36}\)

As stated above assets move together to some degree in a market and are thus not entirely independent of each other. This condition sets a limit to the benefits from diversification. As long as the assets are less than perfectly correlated, i.e. the coefficient is less than one, diversification

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35 The covariance is given by the following equation: $\sigma_{jk} = \rho_{jk} \sigma_j \sigma_k$, where $\rho_{jk}$ is the correlation coefficient between asset $j$ and $k$ and $\sigma_j$ and $\sigma_k$ each defines the risk contained in the individual assets.

36 Following equation clarifies the mentioned effect of diversification on portfolio risk: $\sigma_p^2 = \frac{1}{N} \left( \sigma_j^2 - \sigma_{kj} \right) + \overline{\sigma_{kj}}$. As assets are added to the portfolio, the effect of the difference between the average risk on the individual asset and the average covariance is reduced ($\left( \overline{\sigma_j^2} - \overline{\sigma_{kj}} \right)$ approaches zero).
helps reducing portfolio risk without lowering expected returns. The most diversified portfolio one is able to hold is the *market index portfolio*. This will be elaborated further upon in the next section 3.2.1.1 *The Efficient Frontier*.

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**Diversification enables lower volatility in a portfolio compared to a non diversified portfolio.**

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In portfolio allocation decisions it is rarely a question of diversifying one’s portfolio within a single market, e.g. in domestic stocks. One of the decisions an investor faces, is the allocation of capital in different markets, e.g. between equity and bonds. By placing some of the invested capital in each market it is possible to reduce the risk exposure. Another allocation decision is between domestic and foreign equity. Correlations are lower between equities across different national markets, meaning foreign investments improve the portfolio performance. These are just some of the alternatives available in selecting a well diversified portfolio – other possible assets could be property, different types of bonds, renewable energy, currencies etc.

### 3.2.1.1 The Efficient Frontier

From the standard mean-variance model originated by Markowitz, it is possible to plot portfolio possibilities in a risk-return space. See figure 3.2.1.1.1. Portfolios on the efficient frontier have maximum return at a given level of risk or, alternatively, minimum risk at a given level of return. From the assumptions of the portfolio theory, a rational investor will select a portfolio on the efficient frontier rather than any of the portfolios that lie outside this frontier. The efficient frontier contains all portfolios that lie between the minimum variance portfolio $M$ and the maximum return portfolio (Merton, 1972)$^{37}$.

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$^{37}$There is only a maximum return portfolio if short sales are not allowed; otherwise there are unlimited return possibilities.
Introducing a riskless component, more possible opportunities of investing one’s capital occur. The tangent portfolio is illustrated by the $T$ in the risk-return space in figure 3.2.1.1.1 and is graphically the tangency point between the efficient frontier of risky portfolio combinations and the line passing through the risk-free rate on the vertical axis. From figure 3.2.1.1.1, the straight line contains all possible combinations of the risk-free asset in terms of either lending (to the left of the tangent portfolio) or borrowing (to the right) and a risky portfolio (Markowitz, 1952)\(^{38}\).

In equilibrium, where all investors are rational, hold same beliefs and capital asset prices have adjusted among other assumptions, the tangent portfolio is named the *market index portfolio*. All investors hold this portfolio combined with the option to lend or borrow at the risk-free rate. These portfolios are located on the *capital market line (CML)* which is the linear line going from the intercept, $r_f$ and is tangent to the efficient frontier. Not one single portfolio is concluded to be the efficient one – many alternative combinations of the *market index portfolio* and the risk-free asset are efficient (Sharpe, W. F., 1963). This thesis delimitates from including the domestic money market (referring to section 1.5.5 *Delimitations*), why investments in the risk-free asset market and the risk-aversion related to this will not be elaborated further upon.

\(^{38}\) Assuming the same risk-free rate in lending and borrowing is not considered to be realistic, since in the real world, the borrowing rate will naturally be higher than the lending rate. Hence, the cost of borrowing money exceeds the payoff from lending money.
Chapter 3 - Theoretical Review

Markowitz’s standard mean-variance model is widely applied in portfolio selection problems. Alternative models to this one are, among others, the Arbitrage Pricing Model based on the law of one price derived by Ross in 1976-1977 (Elton et al., 2007) and the classic expected utility theory, only taking the return and hence not volatility in returns into account. A third alternative is the Black-Litterman optimisation and portfolio selection model. This is elaborated upon section 3.3 The Black-Litterman Optimisation Model. The critique of the standard mean-variance model is included in that section.

3.2.2 CAPM

Under some specified assumptions, Sharpe (1964) set up a model for pricing assets in market equilibrium – this is known as the CAPM. From the previous subsection, it is clear that there is a linear relationship between return and risk for efficient combinations of risky assets. This part of the portfolio theory deals with the relationship between expected return and risk on the individual assets. In this case there is a consistent relationship between their expected returns and the systematic risk (Sharpe, W. F., 1964, p. 436). As mentioned in subsection 3.2.1 The Standard Mean-Variance Model, the unsystematic risk in a well-diversified portfolio will approach zero, why focus is on the systematic risk. The systematic risk is defined as beta. Assets which are more responsive to changes in $E(r_m)$, the return on the market portfolio, will naturally have higher expected returns than those which are less responsive. Investors get rewarded for bearing systematic risk – the risk that the investor cannot diversify away, referring to section 3.2.1 The Standard Mean-Variance Model. In equilibrium, prices on capital assets will adjust until there is a linear relationship between this degree of responsiveness and expected return (Sharpe, W. F., 1964) – this relationship, named the security market line (SML) is visualised in figure 3.2.2.1.

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39 This model builds on several assumptions (Elton et al., 2007, pp. 284-285). These will not be elaborated further upon however it is acknowledged that these might not be realistic when applying the model to the real world.

40 Lintner and Mossin are also widely cited for their independent works on this model in 1965 and 1966 respectively (Elton et al., 2007).

41 $\beta = \frac{\sigma_{im}}{\sigma_{m}^2}$, where $\sigma_{im}$ is the covariance between the market and the individual asset and $\sigma_{m}^2$ variance in the market. Beta equals one in the market portfolio.

42 Hence a beta value higher than one. A beta value less than one indicate a lower risk and hence a lower expected return.
The following CAPM equation describes the expected return for all assets as well as portfolios in the economy:

\[
E(r_i) = r_f + \beta_i(E(r_m) - r_f)
\]

The expected return on an asset or a portfolio equals the risk-free return that investor is sure to acquire, plus the beta times the expected excess return from holding the market portfolio.

**In CAPM, investors only get rewarded for bearing systematic risk.**

The CAPM theory has been subject to critique over time. One point of criticism is that the CAPM model is derived on the basis of strong assumptions. These have made the model difficult to test empirically and during such testing, the observed returns are not consistent with the standard CAPM. Different modified forms of the equilibrium relationship have been developed with less restrictive assumptions, e.g. allowing for short sales, difference in lending and borrowing rates, introduction of taxes etc. (Elton et al., 2007).
Summing up on the modern portfolio theory, Markowitz (1952) was the first to suggest that an investor should select portfolios based on two factors – the expected return and the risk both related to the individual assets and to the portfolio. This is known as the standard mean-variance model. In a risk-return space, all feasible and efficient portfolios give rise to the efficient frontier. When markets are in equilibrium, CML illustrates all efficient combinations of holding the risky market portfolio and the risk free rate. Rational investors will always hold an equilibrium portfolio. Sharpe (1964) took his outset in the single asset riskiness in CAPM. Since the unsystematic risk can be diversified away, the systematic risk is the risk measure of interest. In equilibrium, the linear relationship between the systematic risk and the expected return on a single asset make up the SML.

Investors, like the Danish pension funds sector, diversify its portfolio internationally. Solnik (1974) developed an international version of Shapes’ CAPM theory – the ICAPM. This theory will be touched upon when the theoretical review outline the international outset from an investor’s point of view, after the Black-Litterman optimisation model is addressed in the next section.

3.3 The Black-Litterman Optimisation Model

The Black-Litterman optimisation model was introduced in 1992 by Fischer Black and Robert Litterman. They recognise that the popular standard mean-variance model of Markowitz (see section 3.2.1 The Standard Mean-Variance Model) has some shortcomings when applying it in practice (Black & Litterman, 1992). Firstly, the standard mean-variance model requires expected returns on all assets in the portfolio. Investors typically only have knowledge regarding a smaller segment of assets and therefore cannot set expected returns for all assets43. Secondly, the standard mean-variance model is very sensitive to even small changes in the expected returns. Such changes result in portfolios with large long and short positions, which is not very intuitive (Black & Litterman, 1992). These facts combined make it reasonable to question the appropriateness of the standard mean-variance model in practice and this is the motivation behind the work of an alternative approach to optimisation.

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43 In practice, this is often done by averaging the historical returns.
3.3.1 The Intuition of the Black-Litterman Optimisation Model

The Black-Litterman model is a framework based on (1) Markowitz’s mean-variance model (1952) and (2) the CAPM by Sharpe (1964), see sections 3.2.1 *The Standard Mean-Variance Model* and 3.2.2 *CAPM*. The model setup allows investors to include their opinions on how the markets will develop. Further, the model incorporates the investor’s level of confidence in his opinions. The Black-Litterman model is the selected approach to estimate the theoretical optimal portfolio in this thesis.

The first thing to do in the modelling is to set a benchmark portfolio. The CAPM equilibrium portfolio is outlined to be a neutral reference. Alternative benchmark portfolios are possible when an explicit CAPM benchmark does not exist (Black & Litterman, 1992, p. 39).

The expected returns on the benchmark assets are partly based on the anticipated total benchmark portfolio return. The individual expected returns on the benchmark assets are referred to as the implied equilibrium returns. The implied equilibrium returns serve as the outset for the investor to implement his opinion on developments in the different markets. This is partly what differentiates this model from the standard mean-variance model.

In the standard mean-variance model an expected return for each asset should be decided upon from the start of the modelling, whereas in the Black-Litterman model the outset is given by the CAPM equilibrium portfolio and returns on this. If the investor does not have an opinion on a specific asset, the expected return of this asset from the CAPM equilibrium benchmark constitutes the expectation (Black & Litterman, 1992).

The Black-Litterman model is the selected approach to solve the theoretical optimisation problem in the analysis of the Danish pension fund sector.

No restrictions, on how the investor’s views should be expressed, are imposed. Both absolute and relative views can be implemented in the model. The absolute views are expressed as specific
returns of an asset whereas the relative views express e.g. that one asset is expected to outperform another (Litterman & He, 1999, p. 4).

As mentioned above, it is possible to implement the confidence level of the investor’s opinions. If the investor is not 100% certain whether the opinion will be realised, this confidence level is implemented to adjust the opinion-adjusted expected returns. This will affect the weights allocated to the different asset opportunities in the optimal portfolio.

When the opinions of the investors are implemented, the new opinion-adjusted expected returns emerge. This return vector is used in the portfolio optimisation. From this point in the modelling, the optimisation process corresponds to the standard mean-variance model.

3.3.2 The Black-Litterman Model

The Black-Litterman opinion-adjusted returns include both the implied equilibrium returns and the opinions of the individual investor. The applying this approach the implied equilibrium returns are identified by use of the benchmark portfolio and the covariance between the assets. The formula is as follows (Litterman & He, 1999, p. 17):

\[ \Pi = \lambda \Sigma w_{eq} \]  

where

- \( \Pi \) is the implied excess equilibrium return vector. This is the equilibrium created by the anticipated total benchmark portfolio return expectation;
- \( \lambda \) is the risk aversion coefficient;
- \( \Sigma \) is the covariance matrix of excess returns;
- \( w_{eq} \) is the market capitalisation weight of the assets, hence the benchmark portfolio proportions.
As mentioned above, having identified the implied equilibrium returns, next step is to include the opinions of the individual investor to obtain the new opinion-adjusted expected returns needed in the optimisation process. This step is expressed in the following formula (Litterman & He, 1999, p 17):

\[
E(r) = [(\tau \Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau \Sigma)^{-1}\Pi + P'\Omega^{-1}Q]
\]  

(2)

where

\(E(r)\) is the new opinion-adjusted expected return vector;
\(\tau\) is a scalar, which with the \(\Omega\) decides how much weight should be put on the market returns and the investor’s opinions;
\(\Sigma\) is the covariance matrix of excess returns;
\(P\) identifies the assets involved in the opinions;
\(\Omega\) is the diagonal covariance matrix of error terms from the expressed opinions representing the uncertainty in each opinion;
\(\Pi\) is the implied excess equilibrium return vector. This is the equilibrium created by the anticipated total benchmark portfolio return expectation;
\(Q\) is the opinion vector.

Now the needed return data is estimated in terms of the opinion-adjusted expected returns derived in (2)\(^{44}\), and the optimisation process can be commenced. The formula is as follows (Litterman & He, 1999, p. 18):

\[
w_{opt} = (\lambda \Sigma)^{-1}E(r)
\]  

(3)

where

\(w_{opt}\) are the weights of the individual asset in the optimisation;
\(\lambda\) is the risk aversion coefficient;
\(\Sigma\) is the covariance matrix of excess returns;

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\(^{44}\) Since this thesis delimits from including the confidence in the opinions when modelling the theoretical optimal portfolio of the Danish pension sector, the \(\tau\) and \(\Omega\) are excluded in the modelling.
\( E(r) \) are the new opinion-adjusted expected returns.

By the above formulas and the approach brought forward, the Black-Litterman model seeks to solve the optimisation problem of an investor in an alternative way compared to the standard mean-variance optimisation model.

Like the theory which the Black-Litterman model takes its outset in, the model is a static, one-period solution to the portfolio optimisation problem, referring to section 1.5.2 Data Review. It is not possible to conclude anything about future portfolio holdings beyond one period. The incorporation of an investor’s opinions and his confidence in these, bring forward the advantages of this model. Arguably, it is questioned how precise the estimates are. Firstly, how the opinions and additional input parameters are set and secondly how an investor is to estimate his own confidence in his opinions. Thus, the advantages of implementing opinions and confidence levels in the theoretical setup might limit the appropriate use of the model to solve practical investment management problems.

The description of the selected theoretical model for use in the quantitative part of the analysis serves as the background for the practical modelling executed in Excel, see appendix C - The Theoretical Optimised Portfolios. The intuition behind the practical use of the theory is brought forward in chapter 4 - The Theoretical Optimised Portfolio. In the next section performance measurement is presented. This is important in order to analyse and conclude on the performance of the portfolio held by the Danish pension fund sector.

### 3.4 Performance Measures

When optimising portfolios, comparing the performance of the different portfolios identified is essential. This enables the investors to select the most beneficial portfolio. Expected returns are not an appropriate measure since different expected returns are associated with different levels of risk. Therefore, it is necessary to use performance measurements that take both the expected return and risk into account.
3.4.1 Sharpe Ratio

For the purpose of this thesis, the risk-adjusted performance measure, which is based on mean-variance criterions developed by William Sharpe (1966), is selected. This performance measure enables the investor to rank portfolios and thereby identifying the portfolio with the highest performance. Mathematically, the Sharpe ratio divides the excess return ($\overline{r}_p - \overline{r}_f$) by the standard deviation of the portfolio ($\sigma_p$) (W. F. Sharpe, 1966, p. 130).

$$\text{Sharpe Ratio} = \frac{\overline{r}_p - \overline{r}_f}{\sigma_p}$$

The ratio measures the reward-to-total volatility-trade-off. Therefore, the goal for the investor is to maximise expected return per total risk unit - the portfolio with the highest Sharpe ratio is the best choice.

Interpreting how much better one portfolio is performing to another based on the difference in the Sharpe ratios is not possible, why other measures such as the $M^2$ should be applied for this purpose. The risk adjusted performance measure $M^2$ interprets how the portfolio performs relative to the benchmark index portfolio. This enables both a ranking but also a possibility to say how much better one portfolio is performing relative to another. When optimising in section 4.5 Theoretical Portfolio Optimisation & Comparison to the Actual Portfolio, more and more restrictions are added to approach to actual conditions the Danish pension fund sectors is affected by. Therefore, it is not relevant to use $M^2$.

The relevant measure for the purpose of this thesis is identified to be Sharpe ratio, why other measurements are not described into detail. Though, a few, well-known performance measurements are briefly addressed. Treynor’s measure uses the systematic risk instead of the total risk applied in the Sharpe ratio, which seems appealing from an intuitive point of view. The measure is used to evaluate the performance of a given portfolio when adding specific assets/ sub portfolios to the overall investment portfolio (Bodie, Kane, & Marcus, 2008). The purpose of his thesis is to identify one general portfolio for the Danish pension fund sector, why taking stands
on specific assets/sub portfolios are not relevant. Therefore, Sharpe ratio is identified as the relevant performance measure.

The measure of Jensen’s alpha shows how a given portfolio performs compared to a CAPM portfolio (Bodie et al., 2008). Since the Black-Litterman model assumes a CAPM equilibrium benchmark the Jensen’s alpha performance measure could have been used.

**Sharpe ratio is the chosen measurement to evaluate the performance of the portfolios.**

From focusing on how portfolios are measured in terms of performance, the theoretical review turns to another aspect important for the analysis. The Danish pension funds diversify their portfolios internationally. Therefore, it is found relevant to - firstly provide a short elaboration on hedging and secondly how this aspect is dealt with in the data set extracted for use in the theoretical optimisation model.

### 3.5 Investing Internationally

The Danish pension fund sector diversifies its portfolio internationally. In this section, the international CAPM is presented including the connected exchange rate risk and the financial derivative instruments that are applicable to minimize this risk. Further, it will be stated how the implications of investing internationally is dealt with in this thesis.

#### 3.5.1 ICAPM

The traditional CAPM theory, which the Black-Litterman partly is based on, is an equilibrium model on investments in national markets. However, the globalisation of capital markets has increased and investors cross borders to attain international diversification. Introducing foreign assets into the portfolio, (Solnik, 1974) developed a model of the international capital market in the framework of CAPM, called ICAPM. A fundamental dimension of this international market is the existence of exchange rate risk and mechanisms providing protection to investors unwilling to carry that kind of risk.
Chapter 3 - Theoretical Review

An international market implies heterogeneous viewpoints of investors due to different nationalities. This means that each individual investor’s portfolio selection depends on how much each asset in the portfolio contributes to the expected excess return and variance measured in the domestic currency. Solnik’s main hypothesis is that investors are indifferent between investing in domestic assets or in foreign, since the prices are equal when the exchange rate risk is hedged against (Solnik, 1974, p. 500)\(^\text{45}\).

The Danish pension fund sector invests abroad, why the performance of the currencies must be taken into account in the asset allocation decision. The risk of changes in the exchange rates that the sector primarily hedge against are related to the USD, GBP and JPY. This is known from explorative interviews. These currencies held up against the DKK hence the exchange rates have changed over the period of interest, 2001-2010, see Appendix C – The Theoretical Optimised Portfolios.

In this thesis, it is assumed that the Danish pension fund sector does not speculate in exchange rate changes and the development in international currencies – this is confirmed by investigating the pension funds’ portfolios in the annual reports. The sector’s sole interest is to cover the risk involved in investing in foreign assets nominated in other currency than DKK (Annual reports, 2010). This is done partly to minimise the risk of losing returns and partly in order to oblige to article § 165, which states how much of the total investments should be held in congruent currencies, referring to section 2.2 Regulations within the Danish Pension Fund Sector.

The exchange rate risk, which the pension funds are exposed to when investing internationally, is included in the optimal portfolio covered in chapter 4 - The Theoretical Optimised Portfolio. Much of the foreign assets in the sector’s portfolio are nominated in EUR. However, following the fact that Denmark is a member of ERMII\(^\text{46}\) and DKK is pegged to EUR with a maximum span of +/- 2.25%, it is assessed that, if any, only a minor part of the sector hedge against this. Therefore, this is not included in this thesis.

\(^{45}\) He shows that the risk premium on assets, over the national risk free rate, is proportional to their international systematic risk as modelled in the following equation for the one-country.

\(^{46}\) The European Exchange Rate Mechanism.
3.5.2 Exchange Rate Hedging

When investing abroad pension funds must consider the development in exchange rates as described in the previous section 3.5.1 ICAPM. A way of doing this is by hedging the economic exposures that are related to this. The modelled portfolios in this thesis contain international investment opportunities which cause risks both in the underlying assets and in the exchange rates. In this section hedging is introduced, different hedging strategies are briefly touched upon and hedging instruments, available in the market, are outlined. Subsequently it is explained how exchange rate risks are managed in the data set used in the modelling of the theoretical optimal portfolio in this thesis.

3.5.2.1 What is Exchange Rate Hedging?
Hedging is the use of financial derivative instruments to reduce e.g. exchange rate risk. Hedging is an important part of risk management, and pension funds use hedging to secure against the risk of changes in cash flows in foreign markets as a result of changes in the exchange rate. If these risks are not addressed, uncertainty will increase in the investments. The financial instruments within the hedging framework can either fully or partially neutralise the risk created by these uncontrollable changes. In practice, hedging is done by taking offsetting positions against the different currencies (Cusatis & Thomas, 2005).

3.5.2.2 Hedging Strategies and Derivative Instruments
Basically investors must assess whether the expected currency market developments will benefit or hurt the investor’s foreign positions.

Four derivatives are widely used when hedging against exchange rate risk. *Currency forwards* are over-the-counter products where the holder is obliged to buy/sell a specified amount of currency at a specified date at a specified price. The pension funds are thereby sure not to lose money on the possible exchange rate changes. *Currency futures* are very similar to currency forwards. The main difference is that the futures contracts are standardised products traded on exchanges whereas terms of forwards are negotiated for each individual contract. Furthermore, the obligation of futures is managed by a clearing house why counterparty risk is eliminated (Cusatis
Chapter 3 - Theoretical Review

& Thomas, 2005) Currency swaps are used when recurring cash-flows are wished to be hedged. Two parties agree to exchange one currency for another at a specified swap rate on one or several future dates. Finally, currency options provide the pension funds’ possibility to limit downside risk but still retain the possibility to gain (Cusatis & Thomas, 2005, pp. 244-246).

Investigating the annual reports of pension funds of interest for this thesis, only few specify which types of derivatives they apply. Among the few pension funds providing this data, forward contracts and swaps are the ones most applied in securing against exchange rate risk (Danica Pension, Alm. Brand Liv & Pension A/S, SEB Pensionsforsikring A/S, Annual reports, 2010).

3.5.2.3 The Chosen Hedging Approach in the Data Set

It is chosen to adjust for exchange rate changes directly in the data set used for modelling the theoretical optimal portfolios and thus not include hedging instruments. In the chosen approach the returns generated from exchange rate changes in the foreign investments are subtracted. The corrections of exchange rates are made on the three most represented currencies in the foreign indices: USD, JPY and GBP⁴⁷,⁴⁸. These three currencies are “fully hedged” in the data set since any gains or losses, as a result of changes in the exchange rates, are subtracted from the returns on the underlying assets. The reasoning behind choosing this approach is that subtracting returns caused by exchange rate changes, will approximately have the same effect as including the hedging instruments. It is acknowledged that the potential upside from hedging is not included in this approach. Hedging is not part of the main focus of this thesis, which justifies the chosen approach. To see how exchange rates changes are controlled for in the foreign investments in the data set for the modelling of the optimal portfolios, see Appendix L.

In this thesis the exchange rate changes are fully hedged. Returns from exchange rate changes are subtracted from returns on the underlying assets.

⁴⁷ Specific calculations of the currency corrections are in Appendix C - The Theoretical Optimised Portfolios.
⁴⁸ The proportions of USD, JPY and GBP corrected for, is decided upon through investigation of similar indices. The foreign equity index includes 48% equity nominated in USD, 10% in JPY and 10% in GBP. The inflation-linked bond index includes 38% bonds nominated in USD, 5% in JPY and 23% in GBP, see appendix C - The Theoretical Optimised Portfolios.
3.6 Conclusion on Chapter 3

The theoretical review is now presented. This serves as the theoretical frame of reference in the in-depth analysis following in chapter 4 and 5.

Equity is shown to outperform risk-free assets over time. Time diversification implies that above-average returns outset below-average return with time. These things combined with the fact that annual returns on assets are mean-reverting over time, the theory on strategic asset allocation suggests that risk averse, long term investors should allocate more of its investment in risky, mean-reverting assets. The Danish pension funds are defined as long term investors facing an infinite investment horizon, however due to e.g. legislation enforced the investments are carried out rather short term based. It is concluded that the time perspective is interesting in the investigation of whether pension funds possibly can benefit the pension contributors to a higher extent than it does.

The Black-Litterman optimisation model is selected for solving the Danish pension fund’s optimisation problem in practice. The Black-Litterman optimisation model builds partly on the standard mean-variance model and partly on CAPM. It is found an advantageous optimisation model since it incorporates an investor’s opinions on future market developments. Having said this, some drawbacks are acknowledged. From focusing on the theoretical optimisation, interpreting the outcomes of such is important for the purpose of the analysis of the performance of the Danish pension fund sector - Sharpe ratio is selected in order to measure performance on investments, hence portfolios.

The Danish pension fund sector diversifies its portfolio by investing across countries. This implies a need for focus on exchange rates. The exchange rate risk and how this affect the cash-flow from a foreign investment can be hedged against by use of different hedging instruments. In this thesis hedging is included in the theoretical modelling by subtracting the returns on the changing exchange rates from the returns on the underlying assets.

The Foundation for Continuing the Analysis

The purpose of this section is sum up on the previous chapters in order provide an overview of where in the analysis the reader is by now.
Referring to figure 1.5.6.1 *Structure of Analysis* in section 1.5.6 the first five blocks are addressed. Chapter 2 and lately chapter 3, constituting the first five blocks, serve as the foundation of the in-depth analysis carried out in chapter 4 and chapter 5 which make up block six.

The foundation contains a variety of aspects; (1) different descriptive sections of the Danish pension fund sector in order to state the context in which the analysis is carried out, (2) the legal environment, which is a highly relevant aspect in relation to the investment strategy of the sector. Further (3), an estimation of the actual portfolio which is important to the analysis. (4) The theoretical review presented in this chapter, constituting the theoretical frame of reference, contains different and individual theoretical aspects. Some of them are applied to the case of the Danish pension fund sector directly in the sections to make it contextual. Others partly serve as stating the foundation of classic modern portfolio theory and partly as input providers to the selected optimisation model applied in the analysis.

Next, the in-depth analysis follows. Chapter 4 firstly provides the quantitative part of the in-depth analysis. Here theoretical optimised portfolios are estimated and compared to the actual portfolio of the Danish pension fund sector.
4 Chapter 4 - The Theoretical Optimised Portfolio

In this chapter focus is on the optimal portfolio. First, the data used to estimate the optimal portfolio allocation will be presented in section 4.1 including relevant information in relation to the overall data set. Secondly, the statistical inference on the data will be presented in section 4.2. Thirdly, in section 4.3 the individual asset markets available for investments will be presented. Fourthly, a critique of the use of historic data as a predictor of the future is conducted and a change in data is performed in section 4.4. Fifth, different optimal asset allocation portfolios for the Danish pension sector are presented in section 4.5.

The optimal portfolios are based on the optimisation model developed by Fischer Black and Robert Litterman in 1992 presented in chapter 3. In section 4.5, besides the theoretical optimised portfolios are presented, it contains the comparison of the outcomes of these and the actual portfolio, including analysis on the performance. Finally, in section 4.6 there will be shed light on some factors that are found important to note with respect to the diverging performance and the basis for comparison.

4.1 Data for the Theoretical Portfolio Optimisations

Optimal portfolios of the Danish pension fund sector are estimated in section 4.5. Before turning to this, it requires a presentation of the data on the different classes of assets that the portfolio comprises and providing the reader with information on how the overall data set is handled for the purpose of this thesis. Firstly, the asset classes available to the optimisation of the theoretical optimal portfolio are presented, shortly. A thorough description of the individual data sets is provided in appendix E - Description of Data for the Theoretical Optimisation Modelling. Secondly, general information on the overall data set and how this data set is dealt with is provided.

4.1.1 Presentation of Data for the Theoretical Portfolio Optimisations

The asset classes relevant for the purpose of this thesis are selected in order to hold a portfolio opportunity set that is comparable to the actual portfolio to the largest extend possible. As stated
in 2.3 *The Actual Portfolio of the Danish Pension Fund Sector*, the actual portfolio serves as the point of comparison when estimating the optimal portfolio.

The asset classes included are Danish and foreign equity and different types of bonds; Danish government bonds, Danish mortgage bonds, international inflation linked – and European corporate bonds. These are approximately the classes included in the actual portfolio why these are found appropriate to include as investment opportunities in the optimal portfolio.

It is acknowledged that the actual portfolio of the pension fund sector to a higher degree comprises of foreign investments than what is depicted from the data for use in the optimisations. This is e.g. in terms of investments in foreign government bonds. According to the Danish FSA, the Danish pension fund sector placed more than 70% of bond investments in Danish bonds in 2010, why international government and mortgage bonds are assessed reasonable not to include (The Danish Financial Services Authority (FSA), 2011a).

Below the asset classes, included in the opportunity set, are listed. The listing contains the Bloomberg ticker codes to illustrate the specific data indices extracted.

- **Danish Equity** (KAXGI)
- **Foreign Equity** (NDUEACWF)
- **Government Bonds** (NDEAGVT)
- **Mortgage Bonds** (NDEAMO)
- **Inflation-Linked Bonds** (BCIW1G)
- **Corporate Bonds** (MSBIURTR)
- **Risk-free Asset** (CIBO03M)
- **Exchange Rates** (other source)

As stated above, a thorough description of the individual data sets is provided in appendix E - *Description of Data for the Theoretical Optimisation Modelling*.

In the next section, information and other related comments are provided in relation to the overall data set.
4.1.2 Information on Data for the Theoretical Optimisations

This section provides firstly, information on the overall data set.

The extracted data series on the asset classes cover the period January 2001 to December 2010\(^{49}\). In general, data extracted from a specific period is sensitive to the developments in the different markets in this period. This also affects the estimates calculated from the data. The period of interest is affected by different developments and states in the global economy and hence capital markets, starting with the aftermath of the IT bubble, which busted in 2000-2001. The 9-11 attack on the United States highly affected the markets in 2001-2003. From that, markets were characterised by optimism and an upward trend followed for some years. In late 2007-2008 the global economy experienced a massive down turn. Thus, the 10 years that are used for the analysis contain both up- and downturns in the market. It is expected that the latter is more pronounced, why return data is dominated by low values. Caused by the turmoil in markets over the last 10 years, most likely, the volatility is affected negatively. Further, the exchange rate changes are affected by the global economic turmoil.

For the purpose of this thesis, data on monthly basis has been applied. Of course daily observations would increase the robustness of the results with regards to statistical tests on the data and for estimation purpose, however daily data has not been available for all asset classes, why monthly data is used. The data sets count to 119 observations. For all data apply, that the last price value\(^{50}\), index number or return are extracted, either on monthly basis directly or as the value on the first day of each month. From this, inter-month returns have been calculated. The conversions of data are provided in appendix C - The Theoretical Optimised Portfolios. All outcomes will be stated in yearly numbers as stated in section 1.5.3 Overall Conditions of the Thesis.

The data sets, nominated either in last price values or indexed, are converted to returns by use of the formula for discrete compounding rate of return: \( r_t = \left( \frac{P_t - P_{t-1}}{P_{t-1}} \right) \), where \( r_t \) is the rate of return.

\(^{49}\) Frequency distributions of returns for all asset classes are shown in appendix F – Frequency Distribution of Returns.

\(^{50}\) Last price is defined as the most recently traded price. This is the closing price if this is the last price traded at the given trading day (Bloomberg, 2011).
at time $t$, $P_t$ is the price at time $t$ and $P_{t-1}$ is the price at time $t-1$. A conversion to continuous compounding rate of returns is also a possibility. The difference between the values of the two is usually small (Benninga, 2008, p. 258). The first method is chosen since the latter has one serious drawback; the return on a portfolio comprising of at least two assets does not equal the weights of the individual assets times the return on the individual assets (Brealey et al., 2008). Since the theoretical optimisation model dictates this, it is argued that the discrete rate is more suitable for this analysis.

Three of the selected indices are nominated in foreign currency, why the returns are converted to DKK to make the comparable to the Danish asset markets in the opportunity set, see appendix L – Returns in DKK.

This section presented information on the overall data. In the coming section, the data set is investigated statistically.

4.2 Statistical Analysis of Data for the Theoretical Optimisations

Having described the data in the above section and before applying it to the theoretical portfolio optimisation, this section provides a numerical data analysis, in terms statistical interpretation of the data used in the following modelling. The modern portfolio theory, overall, prescribes some assumptions; see section 3.2 Modern Portfolio Theory. Among others these refer to normality and randomness in the expected returns. This assumption is tested in this section.

All test results are based on a significance level of 5% unless else is stated. SAS Enterprise guide 4.2 software package is used to run the statistical analyses. The results build on test statistics and probability values provided by SAS and look-up in the specific test statistic tables. See appendix G – SAS Output to see the SAS output.

---

51 $r_t = \ln \left( \frac{P_t}{P_{t-1}} \right)$ $P_t$ is the price at time $t$ and $P_{t-1}$ is the price at time $t-1$.

52 $p_t = p^a_t + p^b_t, r = \ln \left( \frac{p_t}{p_{t-1}} \right) = w^a r^a + w^b r^b$, where $p$ is the price of the asset at a given point in time, $r$ is the return at a given point in time and $w$ is the weight of the given asset $a$ or $b$ in the portfolio of two assets (Brealey et al., 2008).

53 From this point referred to as SAS.
4.2.1 Normality Test

As stated, one of the assumptions underlying the portfolio theory, which is applied in this thesis, is that the returns on the different assets, comprising the given portfolio, must be random and normally distributed. If this assumption is violated, interpretation and inference may not be valid. Non-normality is a common problem in empirical data. In spite of that data is often used in analyses. For the purpose of this thesis, it is found relevant to test for normality in the different return time series. The outcome and how this is dealt with moving forward in the analysis will follow in section 4.6 Addressing the Actual vs. the Theoretical Optimised Portfolio after the results of the tests are presented.

In order to investigate whether the time series of return data fulfil the normality-part of the assumption, the numerical Anderson-Darling (A-D) test for distribution adequacy on normality is used\(^{54}\). Other numerical tests, such as the Shapiro-Wilk (S-W) W test or the Jarque-Bera, are applicable (Gujarati & Porter, 2009, pp. 131-132)\(^{55}\). The latter is probably the most well-known, however since it is a large sample test, it is assessed that the alternatives are more applicable in this case where the individual samples count to 119. For the purpose of this thesis, the first mentioned A-D test is used and the S-W W test is further applied to confirm the robustness of the results\(^{56}\). There are different advantages and drawbacks on the different tests. These will only be mentioned according to their relevance. Both tests of normality set up a null-hypothesis of normality.

The A-D test statistic looks as follows:

\[
A-D = -n - S \\
S = \sum_{i=1}^{n} \frac{(2i - 1)}{n} [\ln F(Y_i) + \ln (1 - F(Y_{n+1-i}))]
\]

\(^{54}\) This is sometimes referred to A\(^2\) test (Gujarati & Porter, 2009). In this thesis it is called the A-D normality test.

\(^{55}\) Also more graphic and hence intuitive and descriptive tests are applicable. These are not included in this analysis.

\(^{56}\) The results from the latter test; the S-W W test are shown in appendix H - Sharpio-Wilk W test & Skewness and Kurtosis. This test requires a sample size less than 2000, why it has been found applicable to use for testing the robustness of the A-D test of normality.
Where \( n \) is the sample size, \( F \) is the cumulative distribution function and \( Y_i \) is the individual data point.

The results from testing normality in the data set by use of the A-D test is shown in the below table 4.2.1.1.

Table 4.2.1.1 Statistical Test of Normality in the six Asset Classes, 2001-2010

<table>
<thead>
<tr>
<th></th>
<th>Danish Equity</th>
<th>Foreign Equity</th>
<th>Government Bonds</th>
<th>Mortgage Bonds</th>
<th>Inflation-Linked Bonds</th>
<th>Corporate Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anderson-Darling test</td>
<td>0.215</td>
<td>0.325</td>
<td>0.635</td>
<td>0.102</td>
<td>0.589</td>
<td>0.502</td>
</tr>
<tr>
<td>A-D Probability (p-value)</td>
<td>&lt;0.0050</td>
<td>&gt;0.2500</td>
<td>0.097</td>
<td>0.011</td>
<td>0.127</td>
<td>0.212</td>
</tr>
</tbody>
</table>

Source: Output from SAS Enterprise Guide 4.2
Note: The returns statistically tested are nominated in DKK

As shown in table 4.2.1.1 the computed A-D test statistic for foreign equities is 0.325, with a p-value over 0.25, which is well above the 5% significance level chosen to be the required to assume normality of foreign equity returns. The same is true for corporate bonds. The S-W W test confirms normality in these. Both the Government and inflation-linked bond returns show normality when testing the returns with the A-D test, however the S-W W test rejects the hypothesis of normality in these. Both tests reject normality in returns on Danish equity and mortgage bonds. Both show p-values below the 5% significance level.

From investigating normality by looking at the skewness and kurtosis, normality in all data series is rejected. See appendix H - Sharpio-Wilk W test & Skewness and Kurtosis for results and further elaboration.

All in all, the tests on the assumption, demanding normality in returns, show mixed results.

Returns tend to be overestimated compared to the standard deviation in presence of less or not normally distributed returns. Thus, there is a risk of over-allocation to Danish equity and mortgage bonds by use of this data in the selected optimisation model.

Even though evidence of normality is weak or non-existing for the returns in several markets, the analysis will proceed with the acknowledgement that this may have implications for the validity of the results stemming from the analysis. A discussion on use of historical data in general, with
respect to the optimisation problem of interest for this thesis will follow in section 4.4 
*Challenging Historical Data*.

### 4.2.2 Test for Autocorrelation

When dealing with time series data, the successive error terms are possibly correlated with each 
other, meaning that they are not independently distributed over time. By use of the ARIMA 
methodology enables analysis of the properties of the individual time series. The Ljung-Box test 
is chosen, since it is regarded as a powerful statistical test with more small-sample properties than 
competing tests (Gujarati & Porter, 2009, p. 754). This test has drawbacks, however it is assessed 
proper to use in this case. The Ljung-Box follows the chi-squared distribution with \( m \) degrees of freedom.

\[
L-B = n(n+2) \sum_{k=1}^{m} \left( \frac{\hat{\rho}_k^2}{n-k} \right) \sim \chi^2 m
\]

\( \hat{\rho}_k \) is the sample autocorrelation coefficient. If the time series is purely random as assumed, it 
exhibits white noise and the autocorrelation coefficient approximately follows a normal 
distribution with zero mean and a variance equal to one over the sample size \( n \). This coefficient 
measures how much dependence there is between two neighbouring returns in the time series, in 
this context from one month to another. The Ljung-Box test allows for testing the joint 
hypothesis that all \( \hat{\rho}_k \) to a certain lag \( m \) are zero simultaneous in time meaning that the error 
terms are uncorrelated (Gujarati & Porter, 2009, p. 753).

If the computed \( L-B \) test statistic exceeds the critical value from the chi-square distribution, \( \hat{\rho}_k \) 
equal to zero is rejected. For this purpose \( m \) is chosen to be 6 corresponding to the numbers of 
degrees of freedom in the test. Test statistics higher than the critical value of 12.592 lead to a 
rejection of the hypothesis that no serial correlation exists. This means that at least one serial 
correlation coefficient up to the sixth lag is significantly greater than zero. The results from 
testing for autocorrelation are shown in table 4.2.2.1.

---

57 It is assessed that e.g. there are some conflicting elements in the robustness of the test. As \( n \) increases, the 
distribution approximately is chi-squared. On the other hand, the power of the test decreases with \( n \).
58 The critical value is looked up in an *Upper Percentage Points of the Chi-Squared Distribution*. 

71
From Table 4.2.2.1 it is clear that all time series of returns in the six different asset markets show strong evidence of autocorrelation. This means that positive returns will very often be followed by positive returns in the subsequent period. Of course, the same holds the opposite way around with negative returns.

Like in the case of non-normality, the result of autocorrelation test leads to biased and thus not valid expected returns. It is a common problem in empirical data analysis. Continuing the analysis with the data, the violation of randomness in returns is acknowledged when the outcomes of the analysis are interpreted.

<table>
<thead>
<tr>
<th></th>
<th>Danish Equity</th>
<th>Foreign Equity</th>
<th>Government Bonds</th>
<th>Mortgage Bonds</th>
<th>Inflation-Linked Bonds</th>
<th>Corporate Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ljung-Box Q test</td>
<td>26,330</td>
<td>19,580</td>
<td>36,570</td>
<td>45,830</td>
<td>37,020</td>
<td>54,570</td>
</tr>
<tr>
<td>L-B Q Probability (p-value)</td>
<td>0,0002</td>
<td>0,003</td>
<td>&lt;0,0001</td>
<td>&lt;0,0001</td>
<td>&lt;0,0001</td>
<td>&lt;0,0001</td>
</tr>
</tbody>
</table>

Source: Output from SAS Enterprise Guide 4.2

Note: The returns statistically tested are nominated in DKK

From Table 4.2.2.1 it is clear that all time series of returns in the six different asset markets show strong evidence of autocorrelation. This means that positive returns will very often be followed by positive returns in the subsequent period. Of course, the same holds the opposite way around with negative returns.

Like in the case of non-normality, the result of autocorrelation test leads to biased and thus not valid expected returns. It is a common problem in empirical data analysis. Continuing the analysis with the data, the violation of randomness in returns is acknowledged when the outcomes of the analysis are interpreted.

The assumption of random and normally distributed returns for use in the theoretical optimisation model is not fulfilled. Necessarily, this will affect the outcomes.

Having interpreted the data for use in the modelling of the theoretical optimal portfolio, the statistical tests show that the returns in the individual asset markets are not normally distributed neither random. In the next section, a presentation of the individual asset markets available in the portfolio decision is conducted, including a look at their individual risk and return and the performance of investments in the individual markets.

4.3 The Individual Asset Markets

This section presents the individual investment opportunities - graphically how each of them is located in a risk-return space and comments on the numbers as well as their performance as individual non diversified portfolios.
From section 3.2.1 *The Standard Mean-Variance Model*, it is known that a portfolio allocating different proportions in the asset opportunities, hence diversifying across different markets, often makes it possible to attain lower risk at a given level of expected return. In figure 4.3.1 the location of the different investment opportunities, hence markets available for the pension funds in the research setup is illustrated.

Figure 4.3.1 *Risk-Return Space*

As previously stated, the individual asset portfolios are based on ten years data making the returns and volatility very sensitive to this rather narrow time span. Starting at the two extremes, it is expected that bonds have the lowest standard deviation. The market for corporate bonds has the lowest standard deviation of 3,16%. At the other extreme, Danish equity has the largest volatility in returns corresponding to 19,44%. When investigating the returns, the dispersion among these is notable. Investing in Danish equity provides a yearly return of 8,98% on average, whereas investing in the foreign equity market on average, based on ten years data, gives a yearly return of -2,53%.
When comparing to theoretical predictions, it is peculiar how the markets are located in the risk-return space. It might not be that surprising that three of the bond markets are located as low risk investments. Without going into details, it is found peculiar that corporate bonds exhibit lower risk than government bonds. This is assessed from an intuitive point of view. It was expected that government bonds would be the least risky investment since it is assessed that the default risk of corporations is higher than it is for economies themselves.

Table 4.3.1 *Expected Return, Standard Deviation & Sharpe Ratio for the Six Asset Classes*

<table>
<thead>
<tr>
<th></th>
<th>Danish Equity</th>
<th>Foreign Equity</th>
<th>Government Bonds</th>
<th>Mortgage Bonds</th>
<th>Inflation-Linked Bonds</th>
<th>Corporate Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expected Return</strong></td>
<td>8,98%</td>
<td>-2,53%</td>
<td>5,62%</td>
<td>6,05%</td>
<td>3,58%</td>
<td>5,44%</td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
<td>19,44%</td>
<td>5,18%</td>
<td>4,01%</td>
<td>3,66%</td>
<td>7,65%</td>
<td>3,16%</td>
</tr>
<tr>
<td><strong>Sharpe ratio</strong></td>
<td>0,30</td>
<td>-1,11</td>
<td>0,60</td>
<td>0,78</td>
<td>0,05</td>
<td>0,71</td>
</tr>
</tbody>
</table>

Source: Own contribution
Note: The returns are nominated in DKK

That foreign equity exhibits negative return combined with a low volatility, to such degree, is found peculiar. It is found reasonable that this market show lower volatility than the Danish equity market since it is a more diversified investment being a portfolio of different international equity markets. However, the difference in volatility of the two markets is pronounced.

It might also be true that this investment itself has a lower return, since globally the equity markets have endured hard times. The Danish markets in general are also affected by the downturns. The foreign equity index comprises approximately 50% investments (see appendix C - *The Theoretical Optimised Portfolios*) in the United States. As the US economy has endured a number of negative incidents described in section 4.1.2 *Information on Data for the Theoretical Optimisations*, close to half of the foreign equity index is highly affected by this fact. It is assessed that this explains a part of the negative return.
The individual asset markets are affected by the development in the global economy from 2001 to 2010. The foreign equity market shows most unexpectedly. This market shows low volatility combined with a negative expected return in an unexpected degree.

Summing up, the foreign equity index imposes the biggest surprise with low volatility and negative average return. Government-, corporate- and mortgage bonds are not precisely located in risk-return space in accordance to what was expected.

To briefly comment on the performance of the different markets, it is seen in table 4.3.1 that the bond markets, except the inflation-linked bonds, exhibit the highest performance. These markets have the highest return per unit of overall risk. The Danish equity market is very volatile, why this market does not outperform the bond markets in spite of the fact that it exhibit highest return. It has a Sharpe ratio of 0,30. From what has emerged with respect to the foreign equity market, it is not surprising that this exhibits a negative Sharpe ratio, -1,1, why this portfolio itself is not efficient, since it is located below the minimum variance portfolio, referring to section 3.2.1.1 The Efficient Frontier.

Having presented the individual investment opportunities in the portfolio each constituting individual non diversified portfolios, the analysis investigating the diversification possibilities now proceeds. Before turning to this matter, it is found necessary to comment on the challenges in using historical data in the theoretical portfolio optimisation problems.

4.4 Challenging Historical Data

Investigating the data, has shed light on the problems of using historical mean returns as proxies for future expected returns. It is widely known that history cannot predict the future and one must be critical to conclusions reached when applying such data. However, it is the an approximation and if the data set does not turns out extreme, from what was expected, historical data is often used with the improprieness in mind.
When applying historical data, the range of period is important. The wider time range, the more cycles in economies the data contain. Some argue that this provides a more accurate picture of how economic factors come into existence. On the other hand, it is questioned if 30 year data says much about the current development. Today, the development in economies happens so fast that the historic cycles may not be that saying with respect to the cycles of newer dates. It is assessed that more often than prior, events or other things causing shock in economies happens. Referring to section 4.1.2 Information on Data for the Theoretical Optimisations, it is seen that just over the last 10 years (2001-2010) several factors have implied different shock in the economies.

From the 10 year data in this research setup, investing in foreign equity on average has paid out a negative return. This makes the standard mean-variance optimisation model useless due to mathematical circumstances. When constructing the minimum variance portfolio, the model setup will place a large proportion of the investment in foreign equity. Such an investment, clearly, does not make sense at all.

It is important to state, that the historical data outcomes of the other assets are just as poor and misleading as the foreign equity index. The standard mean-variance model does not capture this, since none of the other assets show negative expected returns as foreign equity does59. This, among other factors, indicates that the standard mean-variance model may not be applicable in solving optimisation problems. The Black-Litterman model is used in this thesis, referring to section 3.3 The Black-Litterman Optimisation Model. This theoretical model is selected to reach as well-authenticated outcomes as possible in the research setup. Among other factors, this namely allows for incorporating opinions to the expected returns, hence these are not based on the historical data.

The data set on foreign equity is changed and it is decided the only data input changed. It is acknowledged that this is not optimal, but a reasonable action due to the theoretical optimisation

59 The standard mean-variance optimisation model is not used in the analysis in this thesis, however see appendix I – Standard Mean-Variance Model to see how the negative returns affect the standard model.
model selected in this analysis. An argumentation will follow; For the purpose of the application of the Black-Litterman model, the implied equilibrium returns are the model’s outset. After applying the investor’s opinions of the development in the markets, the opinion-adjusted expected returns will serve as the return input for the continuation of the optimisation process. The change of foreign equity is made in order to obtain an expected standard deviation, rather than the one the historical data defines. Obviously, changing the standard deviation also implies a change in the expected return. However, the return is adjusted for opinions in the optimisation process anyway, why the change has no effect on the outcome. The change in standard deviation will affect the analysis since the portfolio weights, the returns on the portfolio etc. are based on the covariance matrix.

Practically, an expected/mean return and a standard deviation are estimated from the rationale described below (a monthly mean return of 0.40% and a standard deviation of 4.45%). Then 119 random numbers have been drawn by use of the Excel add-in Data Analysis. These random numbers represents the returns on the foreign equity market from this point on in the analysis.

The choice of the specific values is addressed both from a theoretical and intuitive approach. Theoretically, it is assessed that equities have higher risk than bonds. Furthermore, international diversification reduces the volatility in the portfolio hence it is reasonable, that, in this context, the foreign equity index is located more to the left in the risk-return space compared to the domestic, Danish equity index. This corresponds to what is seen in figure 4.3.1 Risk-Return Space in section 4.3 The Individual Asset Markets. Theory also assumes that equity pays a higher return than bonds, as a consequence of the higher risk implied. This speaks in favour of locating foreign equity with a higher return than bonds.

Seen from a more intuitive point of view it is found reasonable that Danish equity has outperformed the foreign index with regards to expected returns. In the 10 year period of interest (2001-2010) the global economy has been highly affected by the turmoil in the financial markets, among other factors. This has also affected the Danish economy hence the Danish equity markets, but not to the same extent. Denmark had a more stable and positive economic outlook.
prior to the crisis; high employment, positive public finances etc, rather low debt to foreign countries etc. than many other countries in the world (Central Bank of Denmark, 2011b). Several aspects should be mentioned with respect to the lower return on the foreign equity index. As stated earlier, approximately 50% of the investments in the selected foreign equity index are placed in the USA. The US economy has been in recession in more periods over the last ten years, the country still experience high unemployment and the economic downturn has not turned around yet (Bloomberg, 2010). Approximately 10% of investments in the index have been allocated in Japan and United Kingdom respectively (see appendix C - The Theoretical Optimised Portfolios). It is known that especially United Kingdom has also been remarkably affected by the financial crisis.

**Historical data is a poor predictor of the future. This affects the outcome of the theoretical optimised portfolio.**

Having addressed the impropriety of using historical data as predictor of the future as well as conducted and justified a change in the data set, the next section contains the more in-depth quantitative analysis. Theoretical optimal portfolios are derived and they are compared to the actual portfolio held by the Danish pension fund sector in parallel. This enables an assessment of whether it is possible for the Danish pension fund sector to optimise its portfolio, to benefit pension contributors to a higher extent than it does.

### 4.5 Theoretical Portfolio Optimisation & Comparison to the Actual Portfolio

In this section more in-depth analytical unravelling will be made and it will be investigated, whether the Danish pension fund sector from a theoretical point of view might be able to optimise its portfolio to a higher extent than it does. The aim of the optimisations is to illustrate different investment environments by implementing different restrictions and see how these affect the outcomes of the optimisations. In the two first optimisations, the variance in the expected returns of the portfolios is minimised. The following optimisations will be subject to maximisation of the return on the portfolios. For both approaches
the aim is to see what effect implementing legislation will have on the portfolio allocation and performance of the portfolios. In the last optimisation, it is found relevant to implement the risk level of the sector’s actual portfolio in the investigation of whether it is possible to hold a higher performing portfolio than the sector does.

The aim of the optimisations is to illustrate different investment environments by implementing various restrictions and see how these affect the outcomes of the optimisations.

Throughout the optimisations, comparison to the sector’s actual portfolio is carried out.

From section 3.3 The Black-Litterman Optimisation Model, it is stated that the Black-Litterman optimisation model is the one applied in this thesis. The setup of Simon Benninga (2008) is applied to implement the Black-Litterman optimisation model. Benninga’s setup splits the optimisation process of estimating the optimal weights in a slightly different way compared to the formulas stated in the original article of Black and Litterman from 1992. Therefore, the formula of the optimal weights included in this section differs from the formula provided in section 3.3 The Black-Litterman Optimisation Model. The outcome of the modelling is the exact same as one reached by using the original formulas.

The first part in the Black-Litterman optimisation process reaching the estimated opinion-adjusted expected returns is the same for all the different optimisations is firstly explained. In the following sections, the different optimisations are provided. The first optimisation and how it is estimated will be explained rather in-depth to illustrate how the model is applied in practice by use of matrices\(^60\). Having presented the first optimised portfolio, hereafter only the outcomes of the remaining optimisations are presented. The different optimisation processes are shown in appendix C - The Theoretical Optimised Portfolios.

\(^{60}\) It will provide the reader with an understanding of the model which is presented mathematically in section 3.3 The Black-Litterman Optimisation Model.
The first part of the Black-Litterman optimisation process, estimating the opinion adjusted expected returns, is the same for all the different optimisations conducted.

From having introduced the application of the Black-Litterman optimisation model, the next section will deep into the optimisation process, more specifically to the part of the process which is the same for all optimisations conducted in this analysis.

4.5.1 First Part of the Optimisation Processes

When applying the Black-Litterman model, the investor must initially decide upon which benchmark portfolio, the optimised portfolios should take their outset in. The theory states that in need of a CAPM benchmark, an anticipated neutral portfolio is also applicable (see section 3.3 The Black-Litterman Optimisation Model). In the present context, the actual portfolio of the Danish pension fund sector has been chosen as the benchmark portfolio in the optimisations. This is the portfolio composition presented in section 2.3 The Actual Portfolio of the Danish Pension Fund Sector. This portfolio, ceteris paribus, must represent the most appropriate investments in the current markets however it is acknowledged it is not a neutral outset.

As seen in section 2.3 The Actual Portfolio of the Danish Pension Fund Sector, an allocation of 5.22% goes into the asset class “others” in the actual portfolio. These include as described, investments in hedging among others. For modelling purpose, it is found reasonable to split this amount equally across the asset classes that are included.

Further the anticipated benchmark portfolio return must be estimated. The actual portfolio of the sector shows an annual return of 3.75%. Acting as the investor, this thesis assumes a return corresponding to this though with an expectation of an increase. The increase is based on the fact that the actual portfolio’s return is based on five year data including 2007 and 2008 where investment returns were largely affected by the financial crisis. 2009 and 2010 showed remarkably higher returns. However, it is assessed that the downturns in the previous years influence the average return more than the latter years’ higher returns (see appendix D- The Actual Portfolio to see the returns achieved over the period of interest). It is therefore expected
that it is possible to achieve higher return than the actual portfolio’s average return indicates. Hence, the anticipated benchmark portfolio is set to a yearly return of 5.00%.

Having identified the benchmark found relevant for the present portfolio optimisation problem, the implied expected returns are calculated.

Identifying the individual opinions of investors’ is often done by leaning on analyses and forecasts on future expected returns in the individual markets (for example, GMO provides a seven years asset class return forecast\(^{61}\)). For this purpose, a relative approach, referring to section 3.3 *The Black-Litterman Optimisation Model*, is selected.

Having estimated the opinion-adjusted expected returns, the optimisations of portfolios with different restrictions follow. These restrictions are the ones found relevant to include in order to investigate the research objective.

In the following sections, the optimal and actual portfolios are presented and compared assuming perfect basis for comparison. In section 4.6 *Addressing the Actual vs. the Theoretical Optimised Portfolio*, a critique of such comparison will follow.

**4.5.2 Portfolio One – The Minimum Variance Portfolio\(^ {62}\)**

The theoretical optimisations conducted in this section are based on minimising the variance in the optimised portfolios. The reasoning behind this is to focus on the risk contained in the portfolios since it is recognised that the Danish pension fund sector highly focuses on risk in its investment strategy.

Firstly, it is found interesting to investigate, how much return the sector is possible to achieve if it wants to minimise risk as much as possible and if no legal restrictions are enforced. The

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\(^{61}\) At GMO's homepage it is stated that these forward-looking statements are based upon reasonable beliefs of GMO and are no guarantee for future performance (GMO, 2011).

\(^{62}\) In practical terms the variance is minimised in the modelling. This could equally well have been the standard deviation. The outcome of all models included in this thesis is disclosed in terms of the standard deviation. This is done to make it consistent with the modern portfolio theory which primary takes its outset in the standard deviation as the measure of volatility.
calculations are based on matrix algebra following the Black-Litterman theory (see appendix C - *The Theoretical Optimised Portfolios* for the detailed modelling).

Calculating the covariance matrix of returns is a part of the optimisation process applying the Black-Litterman model. It is found relevant to make a short note on this. With the adjusted foreign equity market, the covariance between the asset classes follows in table 4.5.2.1.

**Table 4.5.2.1 Covariance Matrix**

<table>
<thead>
<tr>
<th>Covariance Matrix Yearly</th>
<th>Danish Equity</th>
<th>Foreign Equity</th>
<th>Government Bonds</th>
<th>Mortgage Bonds</th>
<th>Inflation-Linked Bonds</th>
<th>Corporate Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Danish Equity</td>
<td>0,03779</td>
<td>-0,00103</td>
<td>-0,00199</td>
<td>-0,00091</td>
<td>0,00065</td>
<td>0,00011</td>
</tr>
<tr>
<td>Foreign Equity</td>
<td>-0,00103</td>
<td>0,02378</td>
<td>0,00070</td>
<td>0,00069</td>
<td>0,00097</td>
<td>0,00048</td>
</tr>
<tr>
<td>Government Bonds</td>
<td>-0,00199</td>
<td>0,00070</td>
<td>0,00161</td>
<td>0,00119</td>
<td>0,00124</td>
<td>0,00091</td>
</tr>
<tr>
<td>Mortgage Bonds</td>
<td>-0,00091</td>
<td>0,00069</td>
<td>0,00119</td>
<td>0,00134</td>
<td>0,00114</td>
<td>0,00090</td>
</tr>
<tr>
<td>Inflation-Linked Bonds</td>
<td>0,00065</td>
<td>0,00097</td>
<td>0,00124</td>
<td>0,00114</td>
<td>0,00585</td>
<td>0,00098</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>0,00011</td>
<td>0,00048</td>
<td>0,00091</td>
<td>0,00090</td>
<td>0,00098</td>
<td>0,00100</td>
</tr>
</tbody>
</table>

Source: Own contribution

The covariance matrix shows the covariance between the different asset markets. The diagonal represents the variances of the individual markets. The covariance between Danish equity and both foreign equity, government bonds and mortgage bonds is negative. This means that while returns on Danish equities are above their expected value, the other asset classes’ returns tend to be below their expected returns. Negative covariance has positive effect on diversification opportunities. All other markets vary positively with each other. In the optimisation process, the covariance matrix, based on historical data, stays unchanged regardless of the opinions implemented on the future in the different market returns. It is assessed that this affects the outcomes of the theoretical optimal portfolios. This will be elaborated upon in section 4.6 *Addressing the Actual vs. the Theoretical Optimised Portfolio*.

The next step in the optimisation process is to calculate the inverse covariance matrix and identify the opinion-adjusted expected excess returns on the different asset markets\(^{63}\). These are

---

\(^{63}\) The opinion-adjusted expected return vector is depicted in appendix C - *The Theoretical Optimised Portfolios*. Also the excess returns give an idea of the level of the opinion adjusted expected returns.
used in calculating the optimal weights. Given a risk-aversion level of one, the minimum variance portfolio is given by:

\[
\sum^{-1} = \begin{bmatrix}
378.83 & 7.83 & 1045.18 & 50.25 & -118.47 & -929.67 \\
7.83 & 513.28 & -47.71 & -179.81 & -40.38 & 0.65 \\
1045.18 & -48.71 & 26262.22 & -15808.00 & -1123.85 & -8694.30 \\
50.25 & -179.81 & -15808.00 & 34356.06 & -6623.2 & -15896.09 \\
-118.47 & -40.38 & -1123.85 & -6623.2 & 25909.5 & -900.41 \\
-929.67 & 0.65 & -8694.30 & -15896.09 & 34356.06 & 35380.52 \\
\end{bmatrix}
\]

This matrix shows the order of the assets in the following matrices:

\[
\sigma^{-1} \cdot (E(r) - \tau) = \begin{bmatrix}
0.38 \\
0.34 \\
0.01 \\
0.09 \\
0.09 \\
0.14 \\
\end{bmatrix}
\]

\[
w_{opt} = (\sum)^{-1} \cdot (E(r) - \tau) = \begin{bmatrix}
3.28 \\
1.94 \\
11.84 \\
13.96 \\
0 \\
68.99 \\
\end{bmatrix}
\]

\(\sum^{-1}\) is the inverse covariance matrix and \(E(r) - \tau\) is the opinion-adjusted excess returns\(^{64}\). \(w_{opt}\) is the optimal portfolio weight vector representing the optimal allocations in the different asset classes in the minimum variance portfolio. Before reaching the optimal weights, different restrictions are imposed in the optimisation.

In the Excel setup (see appendix C - The Theoretical Optimised Portfolios), these restrictions are implemented by use of the Solver add-in to reach the optimal portfolio weights. In this example minimising the portfolio variance with a constraint on the individual weights to be at least zero and restricting the weights to sum to one in total (since short sales are not allowed, see section 1.5.4 Specific Comments to the Theoretical Optimisation Model) are imposed. These two restrictions are imposed in all optimisation and will not be mentioned when the other restrictions, imposed to the different optimisations, are outlined. In the following optimisations, the Solver is equivalently applied to estimate the optimal portfolios.
In table 4.5.2.2, the asset allocation constituting the above optimal portfolio, the expected return on the portfolio, the standard deviation and the performance measurement in terms of the Sharpe ratio of the minimum variance optimised portfolio are illustrated. Also the actual portfolio held by the Danish pension fund sector is shown for comparison (see section 2.3 The Actual Portfolio of the Danish Pension Fund Sector for estimation of the actual portfolio\(^{65}\)).

Table 4.5.2.2 Portfolio 1a

<table>
<thead>
<tr>
<th>Portfolio 1a: Minimum variance</th>
<th>Asset Class</th>
<th>Optimal Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Danish Equity</td>
<td>3,28%</td>
<td></td>
</tr>
<tr>
<td>Foreign Equity</td>
<td>1,94%</td>
<td></td>
</tr>
<tr>
<td>Government Bonds</td>
<td>11,84%</td>
<td></td>
</tr>
<tr>
<td>Mortgage Bonds</td>
<td>13,96%</td>
<td></td>
</tr>
<tr>
<td>Inflation-Linked Bonds</td>
<td>0,00%</td>
<td></td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>68,99%</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100,00%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Expected return</th>
<th>Standard deviation</th>
<th>Sharpe ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio 1a: Minimum variance</td>
<td>4,78%</td>
<td>3,05%</td>
<td>0,52</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Actural Portfolio</th>
<th>Asset Class</th>
<th>Expected Return</th>
<th>Standard deviation</th>
<th>Sharpe ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Danish Equity</td>
<td>5,25%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Foreign Equity</td>
<td>14,43%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government Bonds</td>
<td>19,02%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mortgage Bonds</td>
<td>40,54%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation-Linked Bonds</td>
<td>8,56%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>12,20%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100,00%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Own contribution

From 4.5.2.2 it is seen, that the minimum variance portfolio achieves a higher return of 4,78% at a lower volatility, compared to the actual portfolio of the Danish pension fund sector achieving a return of 3,75%. The minimum variance portfolio has a Sharpe ratio of 0,52, whereas the actual portfolio has a Sharpe ratio of 0,11. In line with the properties of the Sharpe ratio, it is concluded that the performance of the minimum variance portfolio, without any legal restrictions imposed, is higher than the actual portfolio (for further information see section 3.4 Performance Measures).

Investigating the equity ratio, the optimised portfolio allocates 5,22% in equity. This is much lower than the 19,68% in the actual portfolio. The high allocation of investments in mortgage bonds is surprising intuitively. Since the aim of the optimisation is to obtain an optimal portfolio with the lowest possible variance, it is clear that much is allocated in bonds. Because of a high

\(^{65}\) The asset class "others" which appears in the actual portfolio in section 2.3.3 is split across the remaining asset classes to enable a comparison, see section 4.5.1 First Part of the Optimisation Processes.
correlation between the different bond markets, much of the investments in bonds are placed in mortgage bonds, which show the highest expected returns. This implies a less diversified portfolio. The correlation matrix is provided in appendix J – *Correlation Matrix*.

The legal restrictions, which limit the investment possibilities of the Danish pension fund sector, are of course incorporated in the actual portfolio. The actual portfolio depicts the real investments carried out by the sector and through that the conditions under which these are undertaken. The optimised portfolio, 1a, does not include these restrictions. It is found relevant to include legal restrictions in the theoretical optimisation model to obtain an as true and fair picture of the investment environment as possible.

The next step in the analysis is to find the optimised portfolio, implementing two allocation restrictions, regulated by Danish legislation. As stated in section 1.5.4 *Specific Comments to the Theoretical Optimisation Model*, the legislation possible to implement in the theoretical optimisations refers to the two allocation restrictions. These dictate maximum 70% of the total investments allocated in equity and maximum 40% in mortgage bonds66. Table 4.5.2.3 illustrates the optimised minimum variance portfolio subjected to the two allocation restrictions.

<table>
<thead>
<tr>
<th>Portfolio 1b: Minimum variance with legal restric.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Asset Class</strong></td>
<td><strong>Optimal Portfolio</strong></td>
</tr>
<tr>
<td>Danish Equity</td>
<td>3.28%</td>
</tr>
<tr>
<td>Foreign Equity</td>
<td>1.94%</td>
</tr>
<tr>
<td>Government Bonds</td>
<td>11.84%</td>
</tr>
<tr>
<td>Mortgage Bonds</td>
<td>13.96%</td>
</tr>
<tr>
<td>Inflation-Linked Bonds</td>
<td>0.00%</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>68.99%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>100.00%</td>
</tr>
<tr>
<td><strong>Expected return</strong></td>
<td>4.78%</td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
<td>3.05%</td>
</tr>
<tr>
<td><strong>Sharpe ratio</strong></td>
<td>0.52</td>
</tr>
</tbody>
</table>

Source: Own contribution

---

66 The actual portfolio allocation in mortgage bonds actually exceed the 40% restricted by law. This is reasoned by the fact that the 5.20% invested in “others” are split across the remaining asset classes, referring to footnote 67. When investigating the actual portfolio in section 2.3, the 5.20% invested in “others” is separated in reality.
Including the two allocation restrictions does not affect the outcome of the optimisation. Table 4.5.2.3 shows the exact same outcome as the prior optimised portfolio. Looking at table 4.5.2.2 (portfolio 1a) it is evident that the two allocation restrictions are already obeyed before implementing them into the theoretical optimisation. Since this is the only feature included in this portfolio, obviously the same result will emerge.

The two allocation restrictions are not the explanatory factors of, why the Danish pension fund sector does not achieve higher performance in its portfolio, when focus is to minimise the variance in the expected return of the optimal portfolio.

From an intuitive point of view and not with the data input taken into account, the restrictions would imply more investments in less risky assets following the 70% limit in equity if this restriction was not obeyed before imposing it. According to modern portfolio theory, higher diversification in the portfolio, following an allocation change in the total investment, would lead to lower volatility at the same level of expected return. On the other hand, if investments are re-allocated to low risk assets, only as a result of the restrictions imposed, the expected return will probably decline together with the volatility since assets with low volatility has a lower return from a theoretical point of view. The theoretical gain from diversification in such case will be reduced caused by the restrictions imposed. Therefore, both lower volatility and return is expected from such scenario.

By minimising the variance in the expected return of the theoretical optimised portfolio and obeying the two allocation restrictions, it is possible to achieve higher return and lower volatility implying a higher performing portfolio than the actual portfolio.

4.5.3 Portfolio two - Maximise Return

The following optimisations focus on maximising return, whereas the above portfolios minimised the volatility. In investigating whether the Danish pension fund sector benefit the pension
contributors to highest possible extent, it is found relevant to focus on the other relevant aspect in portfolio composition; the expected return.

If the aim only is to maximise return without any concerns about the volatility, the entire investment will logically be allocated in Danish equity since this asset has the highest expected return, as seen in section 4.5.2 Portfolio One – The Minimum Variance Portfolio. This 100% allocation in Danish equity constitutes portfolio 2a, illustrated in table 4.5.3.1. The non diversified portfolio shows a high return of 8,00% and a large standard deviation of 19,44%. The Sharpe ratio is lower than the Sharpe ratios of portfolio 1a and 1b, why the performances of these portfolios are better. In portfolio 2a the rise in expected return is proportionally lower than the rise in the standard deviation. The expected return per unit of total risk is thus higher in portfolio 1a and 1b.

Table 4.5.3.1 Portfolio 2a

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Optimal Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Danish Equity</td>
<td>100,00%</td>
</tr>
<tr>
<td>Foreign Equity</td>
<td>0,00%</td>
</tr>
<tr>
<td>Government Bonds</td>
<td>0,00%</td>
</tr>
<tr>
<td>Mortgage Bonds</td>
<td>0,00%</td>
</tr>
<tr>
<td>Inflation-Linked Bonds</td>
<td>0,00%</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>0,00%</td>
</tr>
<tr>
<td>Total</td>
<td>100,00%</td>
</tr>
</tbody>
</table>

Expected return 8,00%
Standard deviation 19,44%
Sharpe ratio 0,25

Source: Own contribution

Such an investment is unrealistic since it is assessed that the Danish pension fund sector would act rather conservatively in their investment strategy, even if the allocation restrictions were not enforced by law. This conservative approach to investments is caused by the fact that they are responsible for managing pension contributors’ retirement savings.

This optimised portfolio is included to strengthen the theoretical argument of diversifying portfolios and thereby improving the performance. This is seen from the low Sharpe ratio of portfolio 2a compared to portfolio 1a and 1b.
The outcome of optimising a maximised return portfolio, where the allocation restrictions are imposed, is illustrated in table 4.5.3.2.

Table 4.5.3.2 Portfolio 2b

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Optimal Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Danish Equity</td>
<td>70,00%</td>
</tr>
<tr>
<td>Foreign Equity</td>
<td>0,00%</td>
</tr>
<tr>
<td>Government Bonds</td>
<td>0,00%</td>
</tr>
<tr>
<td>Mortgage Bonds</td>
<td>0,00%</td>
</tr>
<tr>
<td>Inflation-Linked Bonds</td>
<td>0,00%</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>30,00%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100,00%</strong></td>
</tr>
<tr>
<td><strong>Expected return</strong></td>
<td><strong>7,06%</strong></td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
<td><strong>13,66%</strong></td>
</tr>
<tr>
<td><strong>Sharpe ratio</strong></td>
<td><strong>0,28</strong></td>
</tr>
</tbody>
</table>

Source: Own contribution

As earlier stated, one of the allocation restrictions dictates a maximum allocation in equity of 70%. In portfolio 2b, all 70% is allocated into Danish equity due to its high expected return. The remaining 30% must be invested in bonds. This amount will be allocated to the bond investment opportunity with the highest expected return. It is investments in corporate bonds referring to section 4.5.2 Portfolio One – The Minimum Variance Portfolio.

Compared to portfolio 2a, both the expected return and the volatility are lower. Investigating the Sharpe ratio it is concluded that portfolio 2b performs better than portfolio 2a with a ratio of 0,28 compared to 0,25. Comparing this portfolio to the actual portfolio instead, the optimised performance is higher than how the sector’s portfolio performs. All legal regulations are not included in the theoretical optimisation model. This will be elaborated upon later.
When maximising the expected returns and imposing the two allocation restrictions, it is proven possible to hold a theoretical optimal portfolio performing better than both the optimal portfolio, not subject to forced diversification, and the actual portfolio held by the Danish pension fund sector.

The final optimisation reflects the conditions, under which the pension fund sector operates, to the highest possible extent in the research setup. The aim of the optimisation is to maximise return subject to the two legal allocation restrictions like it was done in modelling portfolio 2b. Furthermore, the volatility of the portfolio is restricted to reflect the corresponding volatility of the actual portfolio\(^\text{67}\). The reason for equalling the volatility levels is based on the assessment that the pension fund sector carries out investments very conservatively. This means that the level of risk incurred in its portfolio somehow will reflect the low volatility under which they invest. Indirectly, this indicates the sector’s risk aversion. Practically, when equalling the volatility level in the two portfolios compared, focus will solely be on the return opportunities and through that the performance of the two portfolios.

Table 4.5.3.3 *Portfolio 2c*

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Optimal Portfolio</th>
<th>Actual Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Danish Equity</td>
<td>17,33%</td>
<td>Danish Equity</td>
</tr>
<tr>
<td>Foreign Equity</td>
<td>22,53%</td>
<td>Foreign Equity</td>
</tr>
<tr>
<td>Government Bonds</td>
<td>0,00%</td>
<td>Government Bonds</td>
</tr>
<tr>
<td>Mortgage Bonds</td>
<td>0,00%</td>
<td>Mortgage Bonds</td>
</tr>
<tr>
<td>Inflation-Linked Bonds</td>
<td>0,00%</td>
<td>Inflation-Linked Bonds</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>60,14%</td>
<td>Corporate Bonds</td>
</tr>
<tr>
<td>Total</td>
<td>100,00%</td>
<td>Total</td>
</tr>
<tr>
<td>Expected return</td>
<td>6,02%</td>
<td>Expected Return</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>5,27%</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0,54</td>
<td>Sharpe ratio</td>
</tr>
</tbody>
</table>

Source: Own contribution

\(^{67}\) A small difference in the standard deviation occurs due to the fact that the *Solver* add-in in Excel could not find an exact solution to this problem, why a converged solution is provided.
From table 4.5.3.3, it is evident that the theoretical optimised portfolio achieves a higher expected return and thereby a higher Sharpe ratio corresponding to 0,54. The actual portfolio exhibits a ratio of 0,11. This shows that from optimising the theoretical portfolio, it is possible to achieve higher return, hence, a better performing portfolio in the sector than what it does.

The optimised portfolio, portfolio 2c, is not very diversified across the different asset classes, referring to table 4.5.2.6. Approximately 60% of the total investment is allocated to mortgage bonds and the remaining is invested in equity. Intuitively, without taking the data input into account, it is expected that the Danish pension fund sector at least will diversify its investments in bonds across the opportunities in this subgroup of assets. It was expected that the pension fund sector should invest in government bonds and mortgage bonds to larger extent than what portfolio 2c suggests.

It is evident from the actual portfolio that a relatively large amount of the sector’s actual investments are allocated to government bonds and in particular mortgage bonds. Government bonds are assessed to be the safest investment as the default risk on such investments is little. It would imply that the given economy, in which investments are made, goes bankrupt.

As mentioned in section 2.2 Regulations within the Danish Pension Fund Sector, the mortgage bond market is very important to the Danish economy and pension funds hold large positions in mortgage bonds. Mortgage bonds are considered safe investments and the 40% limit on investments in mortgage bonds is almost fully utilized in the actual portfolio (excl. the surplus of “others”). Therefore, a high allocation in mortgage bonds was expected in the optimal portfolio. The investments in mortgage bonds will possibly be limited further due to the coming solvency II directive requiring a maximum of 15% originating from one issuer. If this rule is enforced, it will

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68 At least this was the assessment before the Greek, Irish, Portuguese and not to forget the US cases.
affect the investments strategy of the Danish pension fund sector because of the limited number of issuer of mortgage bonds. This might affect the sector’s ability in utilizing the 40% limit.

All in all, it was expected that more investments should be allocated into mortgage and government bonds in the optimised portfolio, 2c. The explanation of why the theoretical optimal portfolio does not diversify across the different bonds is reasoned by the data set. This will be addressed in section 4.6 Addressing the Actual vs. the Theoretical Optimised Portfolio.

A comparison of the asset allocation of the two portfolios shows, that a higher proportion of equity is held in the optimal portfolio, 39.86% whereas the actual portfolio only allocates approximately 20% of investments to equity. From this it is concluded, that the pension sector should allocate more to risky assets since it is assessed that such investments contribute to the higher return.

When depicting the investment environment to highest possible extent and maximising expected return, the theoretical optimal portfolio allocates more investments to equity, than is done in the investment strategy of the Danish pension fund sector. The pension funds should allocate more investments to risky assets.

Even though the proportion of equity is higher in the optimal portfolio, it is seen that the limits of how much the investor maximum must invest in equity and mortgage bonds are not close to be reached in the optimal portfolio.

When depicting the investment environment to highest possible degree and maximising expected return, the two allocation restrictions, imposed by law, are not close to be reached.

Holding portfolio 2c would imply a higher expected return with the same volatility compared to the actual portfolio. This is assessed partly caused by the fact that the solvency requirements (article § 159(1) in the Financial Business Act described in section 2.2.1 The Danish Legislation
– The Financial Business Act) is not incorporated in modelling the optimal portfolio. Therefore, it is found reasonable to conclude that the solvency requirements limit the possibility to achieve higher returns.

### The solvency requirements imposed to the investment strategy of the Danish pension fund sector, limits the possibility to achieve higher returns.

It is assessed that the rather tight risk management in terms of the traffic-light system and the current draft of the solvency II directive has an effect on the investment strategy of the Danish pension fund sector. At least a part of the excess return and better performance obtained in the theoretical optimal portfolio stems from a different and less restricted investment strategy with respect to risk management.

### The high focus on risk management in investment strategy, as a result of the traffic-light system and coming Solvency II directive, hamper the performance of the Danish pension fund sector.

Having conducted a number of theoretical optimisations and compared the outcomes to the actual portfolio, different interesting conclusions have emerged. The different optimisations were conducted in order to investigate how different investment environments possibly affect the outcomes of the theoretical optimal portfolios. A sum up all outcomes is presented in table 4.5.3.4.

<table>
<thead>
<tr>
<th>Portfolio #</th>
<th>Actual</th>
<th>Minimum Variance</th>
<th>Maximum Return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>Portfolio 1a</td>
<td>Portfolio 1b</td>
</tr>
<tr>
<td>Expected Return</td>
<td>3,75%</td>
<td>4,78%</td>
<td>4,78%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>5,25%</td>
<td>3,05%</td>
<td>3,05%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0,11</td>
<td>0,52</td>
<td>0,52</td>
</tr>
<tr>
<td>Equity Ratio</td>
<td>19,68%</td>
<td>5,22%</td>
<td>5,22%</td>
</tr>
</tbody>
</table>

Source: Own contribution
Chapter 4 – The Theoretical Optimised Portfolio

The table clarifies and gathers the outputs from the optimisations conducted in this section. The foundations of the optimisations; minimising variance or maximising return, affect the outcomes. Different restrictions imposed (the two allocation restrictions and - in portfolio 2c - the equality in variance) imply different outcomes both in relation to the performances of the portfolios and the proportion of allocation to equity. The portfolio showing the lowest performance is actually the actual portfolio with a Sharpe ratio of 0,11. As concluded in this chapter, this means that it is possible to hold better performing portfolios than the sector does. The highest performing portfolio is portfolio 2c with a Sharpe ratio of 0,54. Compared to the remaining optimised portfolios, this shows higher allocations in equity than portfolio 1a and 1b and lower equity ratio than portfolio 2a and 2b with its level of 39,86%. This comparison between the different theoretical optimised portfolios should not be taken literally, since different restrictions are imposed to them individually, making them incomparable.

The next section addresses different factors relevant to comment on in relation to the analysis conducted so far.

4.6 Addressing the Actual vs. the Theoretical Optimised Portfolio

In the above section it was concluded that by applying the theoretical Black-Litterman model to the optimisation problem, a higher return and performance (because of an equal volatility) can be achieved compared to the actual portfolio held by the Danish pension fund sector. Further, it was concluded that some of this excess performance is ascribable to the solvency requirement and the risk-based focus in investments. This is not incorporated in the optimisation model, but it naturally affects the actual portfolio.

The aim of this section is twofold. Firstly, it aims to address some of the other aspects that naturally affect the diverging performances of the two portfolios that are compared. By this, it is believed that the diverging performances of the two portfolios stem from other factors besides what has been extracted from the above comparison. Secondly, it is important to elaborate on the challenges with regards to the appropriateness of comparing the theoretical portfolio with the actual one, the latter depicting reality. Theoretical models are applied in practise, when portfolios
are selected. However, this does not necessarily mean that portfolios estimated from theoretical modelling are comparable to actual portfolio compositions and performances. The two points of interest will be elaborated upon in parallel. It is acknowledged that many factors have an impact on the question of basis for comparison and the diverging performances. Only some of these are included.

(1) At first, it is found important to appoint the inference of the statistical tests on the data obtained for use in the optimisation process. See section 4.2 Statistical Analysis of Data for the Theoretical Optimisations. It was concluded that the data exhibits weak or non-normality and is not randomly distributed. These facts affect the outcomes of the optimised portfolios. The Black-Litterman model assumes random and normally distributed returns. The consequences of the fact that this assumption is not fulfilled imply that results are biased and hence not valid, referring to section 4.2 Statistical Analysis of Data for the Theoretical Optimisations. This affects the basis of comparison of the optimised and actual portfolio. If the estimates extracted from the theoretical optimised portfolio are not true a comparison to real thus valid values is not appropriate. Moreover, as a consequence of the less accurate estimates in the optimised portfolio, this affects the outcomes, including the performance. Therefore, a part of the divergence between the performances of the two portfolios is also explained by this fact.

(2) Another condition important to address, is how the covariance is applied in the Black-Litterman model and how that affects the outcome of the modelling. Speaking in practical model terms, when implementing the investor’s opinions into the model, only the expected returns are affected, while the volatility remains the same without any “opinion adjustment”. As the covariance matrix is included in the calculation of optimised weights, the unchanged covariance matrix put together with the opinion-adjusted expected returns possibly result in misleading optimised portfolios. From portfolio 2c it is evident that, on portfolio level, diversification is low and further investments in bonds are not diversified across this subgroup of assets. This is assessed partly due to changes in expected return and maintained levels of volatility. As an example, corporate bonds have a relatively high opinion-adjusted expected return at the retained low volatility. This combined with the fact that the bond markets mutually are highly correlated imply that all investments are made in this group of bonds.
Alternatively, a covariance matrix could be estimated according to the investor’s opinions, however, it demands advanced knowledge within the capital markets to estimate these in a proper way. Further, the theoretical model setup (Benninga, 2008) ascribes the use of historical data with regards to the covariance, why this is done in this analysis with the awareness of the drawbacks in relation to the factor appointed here and the use of historical data to predict the future. It is assessed that a part of the divergence between the outcomes of the actual and theoretical optimised portfolio stems from the absent “opinion adjustment”.

(3) The comparability of the theoretical and actual portfolios is relevant to question with respect to the time span over which data is obtained. The actual portfolio is conducted by use of five year data; 2006-2010. The reason for this is addressed in section 2.3 The Actual Portfolio of the Danish Pension Fund Sector. The optimal portfolio is conducted on data covering 10 years; 2001-2010. This is rationalised by a wish to achieve highest possible robustness in data, see section 4.1.2 Information on Data for the Theoretical Optimisations. This difference in number of years is assessed not necessarily a reason why the returns diverge from each other. Yet, the difference in number of years is criticised with respect to the basis of comparing the portfolios.

(4) Another critique of the appropriateness of comparing a theoretical model with a real world portfolio is the question of the validity in the theoretical model applied to conclude on the performance of the pension fund sector (see section 1.5.2 Data Review). The appropriateness of comparing “two worlds” is found reasonable to question. Nonetheless, since there is no other alternative than applying theory to the real world, it is done anyway with the acknowledgement of the problems attached to this.

(5) The theoretical model uses indices as asset opportunities in the portfolio. It is argued that both the divergence in the returns on the two portfolios as well as the comparability between them is affected by this. First of all, it is only possible to depict the real portfolio of the pension fund sector to some extent. The use of indices provides a rather simplified picture. In reality the portfolios are managed actively. For example, investments in foreign equity are assessed not carried out in a global index only, but rather, at least to some degree, divided into the different markets that are assessed beneficial to invest in from the pension fund’s point of view. For this
reason it is the performance of the individual investments, rather than the index’s performance, that subscribes the returns achieved on the actual investments carried out by the sector.

It should be noted that, some types of investments carried out by the pension fund sector can be approximated to the performances of indices since many investors benchmark this type of investments up against indices. Secondly, as addressed earlier not all real investment opportunities have been possible to include in the model - for example investments in property. This has been taken into account in estimating the actual portfolio \( D - \text{The Actual Portfolio}. \) Yet, it is not possible to extract all “noise” why the two portfolios, ceteris paribus, are less comparable with respect to this factor.

All in all, having analysed the theoretical opportunities in heightening the performance of the Danish pension fund sector, it is clear that some factors, beside the solvency requirement and the high risk-management with respect to investments, explain the divergence between the theoretical optimal portfolio and the actual portfolio held by the sector. Further, there are circumstances, which make the comparison between the theoretical portfolio and the actual one inappropriate. These refer partly to the model applied for the purpose and partly to other practicalities, which constraint the comparison potential.

- Comparing theoretical optimised portfolios to actual portfolios is questionable.
- The violation of the statistical assumption affects the basis of comparison of the two portfolios negatively and it explains a part of the divergent performances.
- The absent “opinion adjustment” of the volatility in returns affect the divergence of the outcomes of the two portfolios.
- The difference in the number of years which the two portfolios are estimated from makes the comparability of them misleading.
- The use of indices in estimating the theoretical based portfolio can both explain divergence between the two portfolios’ outcomes and affect the comparison.
4.7 Conclusion on Chapter 4

From the above analysis it is concluded that it is possible to achieve higher returns and hence performance than what the Danish pension fund sector does – the sector could change its portfolio management and through that benefit the pension contributors to a higher extent. This conclusion is based on the comparison of the actual portfolio of the sector and the theoretical optimised portfolios derive by applying the Black-Litterman model.

The data applied in modelling the theoretical optimised portfolios comprises of six investment opportunities selected to resemble the actual portfolio’s asset classes to the highest possible extent. The theoretical model assumes independent and normally distributed returns in the individual assets to estimate as true estimates as possible. This assumption is not fulfilled, yet with this acknowledgement and condition taken into account, the analysis proceeds.

Individually, the six asset classes are non diversified portfolios. Naturally, the time series, representing the markets, are affected by the economic development over the 10 years of interest (2001-2010). Using historical data to predict the future (partly done in the theoretical optimisation model) is subject to much criticism. Yet, in lack of better approximations of the future, historical data is often applied. The selected theoretical optimisation model opens up for incorporating opinions to the development in the individual markets. Therefore, changing one of the assets, foreign equity, to resemble the expectations to the future volatility in this market, is found reasonable and also necessary to proceed with the analysis.

The theoretical optimisations include different restrictions to exemplify different investment environments and how these affect the portfolio composition and performance. (1) When minimising the variance, the two allocation restrictions do not affect the investment strategy and it is concluded that theoretically it is possible to achieve higher return and a lower variance than the pension fund sector does. (2) Maximising the expected return in the optimisation increase focus on expected return in contrary to reality, where the risk is assessed the primary factor in setting the investment strategy. Imposing the two allocation restrictions to the optimisation problem, it is found that diversifying the theoretical portfolio by investing less in equities, increases the performance. This optimised portfolio also shows higher performance than the actual portfolio.
To resemble the real investment environment to the highest possible extent in the theoretical optimisation, the legislation and a standard deviation equal to the level present in the actual portfolio are incorporated. The optimised model suggests an allocation of approximately 40% in equities, whereas the pension fund sector allocates about 20% in these. The two allocation restrictions, enforced by law, do not restrict the sector in heightening the performance since the allocations in equity and mortgage bonds are not close to go beyond the limits. The pension sector should allocate more into risky assets since it is assessed that such investments contributes to the higher return. The theoretical optimised portfolio shows higher return and performance, why it is concluded that it is possible for the pension fund sector to benefit the pension contributors to a higher extent than it does through its investment strategy.

Not all legislation is possible to implement in the theoretical model, including the solvency requirement, which is imposed to the pension fund sector’s actual investment strategy by Danish legislation and the risk focus in the portfolio management e.g. enforced by the coming European Solvency II directive. Some of the performance opportunities, which are concluded not exploited in the pension fund sector, are argued to be partly a result of the both the solvency requirement and the enforced focus on risks in investments themselves.

Besides the conclusions, possible to extract from the quantitative comparison of the two portfolios, it is identified that several factors affect, firstly the divergence in the performances of the two portfolios and/or secondly the basis for comparison.

(1) The fact that the assumption of the theoretical model implying randomness and normality in returns is not fulfilled implies that outcomes of the theoretical optimisations are not based on true estimates. This affects the divergence in the performances of the two portfolios. Further, this makes the appropriateness of comparison to the actual portfolio questionable.

(2) The Black-Litterman model includes opinion-adjusted expected returns. No changes are implemented in the volatility of the individual asset class and no change in the covariance between the asset markets in the portfolio. This means that a higher or lower opinion-adjusted expected return will be associated with the same volatility and the same covariance, which is not what theory predicts. This possibly explains a part of the divergence between the outcomes of the theoretical optimisations and the actual portfolio.
(3) The actual portfolio stems from five year data, whereas the theoretical optimisation model is based on 10 year data. This disparity in basis for extracting data affects the basis for comparison.

(4) It is acknowledged that theoretical models are useful in practical portfolio optimisation problems to some extent. However, the fact that outcomes of a theoretical model in this context are compared to a real life portfolio is questionable with respect to the appropriateness in doing so.

(5) The use of indices cannot fully resemble the world in which the pension funds carry out their investments. Pension funds conduct portfolio management in a much more active way than the simplified depiction, use of indices, can possible express. Lack of availability of every investment market, in which the pension fund sector invests, is partly taken into account in estimating the actual portfolio. However, noise will still appear with respect to this. Both the divergence and basis for comparison are relevant to address with respect to this.

The above stated factors contribute to the divergence in the performances of the two portfolios and contribute to question the basis of comparison. Some of the factors affect both of these two aspects, while others affect only one of them.

Having concluded on this chapter, the analysis will turn to a new aspect highly relevant to include in the analysis of the Danish pension fund sector and its investment strategy.

The theoretical optimal portfolios are static solutions to the optimisation problem, meaning that this part of the analysis depicts the portfolio management of the pension fund sector, with respect to the time perspective, only on a short basis. However, pension funds are assumed long term investors facing an infinite investment horizon, why it is found relevant to address the investment strategy of the pension funds, the time perspective taken into consideration. Therefore, in the next chapter, the analysis continues, providing the more conceptual part of the analysis, by addressing the investment strategy of the Danish pension fund sector, with the time perspective as the pivot.
5 Chapter 5 – The Time Perspective in Portfolio Management

Having conducted the more quantitative part of the in-depth analysis, this chapter turns the in-depth analysis in a more conceptual direction. The primary purpose of this chapter is to investigate the importance of the time perspective in the pension fund’s investment strategies with the aim of benefitting the pension contributors to a higher extent.

It has been concluded that legislation is an important factor in investigating how the sector’s portfolio management is executed. It restricts the pension fund’s liberty of action and it further constrains them in achieving higher performance in their portfolios. The pension funds are long term investors facing an infinite investment horizon. This underpins the potential of taking the time perspective in the investment strategies into account.

In figure 5.1, the structure of chapter 5 is illustrated to give the reader an overview of the chapter.

Figure 5.1 Structure of Chapter 5

Source: Own contribution
Firstly, in section 5.1, two central aspects regarding the time perspective are addressed; (1) Section 5.1.1 *Managing Long Term and Short Term Interests* consists of two subsections, including how the pension contributors have divergent interests in the time aspect of their retirement savings and whether an entirely short termed investment strategy hence is the most beneficial to the dispersed group of pension contributors. (2) In section 5.1.2 *Managing Long Term Investments on a Short Term Basis* focus is brought to the execution of the investment strategy and how the time perspective comes into relevance with respect to this. Secondly, in section 5.2, suggestions, with respect to the two central aspects, are brought forward. In (Ad. 1) a model incorporating the time perspective is set up, enabling the pension funds to hold both a short term and a long term focus on risk and investments in selecting its investment strategy. Lastly in (Ad. 2) it is suggested that the long term perspective, as the revised investment strategy incorporated, should also apply to the incentive based remuneration packages.

5.1 Time Perspective – Central Aspects

In an analysis of the Danish pension fund sector, the time perspective is a very important factor. The pension funds are defined as long term investors since they are responsible to invest pension contributions on a long term basis, achieving at least the guaranteed return corresponding to the defined benefit. Further, they face an infinite investment horizon based on the accession of pension contributors happens on a continuous basis. At the same time people enter retirement. It is assumed that this cycle continues in infinite time, referring to section 1.5.3 *Overall Conditions of the Thesis*. This section seeks to address and discuss two central aspects that are found relevant for the Danish pension fund sector to include in its investment strategy with respect to the time perspective, see figure 5.1. The investigation of the two aspects is conducted since the overall aim of this thesis is to explore, if the pension funds benefit the pension contributor to highest extent with focus on optimisation of their portfolios.
5.1.1 Managing Long Term and Short Term Interests

As illustrated in figure 5.1 in this section, two subsections are included. These two, combined, explain the challenges the Danish pension fund sector faces in benefitting all pension contributors to the highest possible extent.

5.1.1.1 Long versus Short Term Horizon

The purpose of this section is to illustrate the disparity with respect to the time perspective seen from both the Danish pension fund sector’s point of view and among the pension contributors as a group.

One thing is the investment horizon another thing is the planning horizon. This distinction is very pronounced in the investment perspective of pension funds. On one hand they are assessed to have an infinite investment horizon, however the planning horizon is very distinct from this infinity (see section 3.1 Pension Funds as Long Term Investors).

The Danish pension fund sector’s infinite investment horizon is very distinct from the short term planning horizon it is subject to.

As stated before, it is known from explorative interviews that the portfolio management in pension funds is executed on a continuous basis and the investment strategy is revised on yearly basis - as a minimum. Furthermore, the interviews confirm that the reason for holding such a narrow planning horizon is mainly due to legislative restrictions forcing the funds to manage the pension portfolios on a very short term basis. This is seen in relation to the solvency requirement imposed by Danish legislation, as well as the risk focus, including the coming European Solvency II directive. The sector carries the entire risk, as the pension contributors have chosen the defined benefit product to avoid risk.

It is assessed that what is of interest to pension contributors is how much wealth they will have when they reach retirement age. Pension contributors, who chose the defined benefit product, are all risk averse and thus very conservative in their attitude to investments of their retirement
None of them are willing to accept any risk with regards to their retirement savings. This holds for all customers in this group no matter how many years to retirement. They will rather be sure of a certain, but possibly lower, return from the guaranteed return plus the extra possible gain from the bonus potential, than accept risk and hereby being able to achieve higher returns.

Even though an equal level of risk aversion is held across the entire group of pension contributors, they are still assessed to have different views on how important their retirement savings are to them at specific points in time. A man who has five years to retirement is assessed to care much about the near future development of his retirement savings, whereas a 27-year old girl, who just entered the labour market is more interested in her retirement savings on a long term basis meaning maybe 40 years into the future.

The pension contributors have the same level of risk aversion. Nevertheless, they have diverging views on the importance of their retirement saving since this view is defined by the number of years to retirement.

In this section, it is concluded that the number of years to retirement influences how pension contributors view the importance of their retirement savings. This suggests that the present short term investment strategy conducted in the pension fund sector is not ideal to benefit the pension contributors to the highest extent. In the next section, this is investigated further and it is highlighted why the Danish pension fund sector is precluded from acting more long term.

5.1.1.2 Is a Short Term Investment Strategy the Most Beneficial to Pension Contributors?

The previous section brought forward that the pension funds focus on complying with the guaranteed return and the rate of interest (also referring to section 2.1.2.1 The Defined Benefit Products). Obviously, the portfolio will be rebalanced in accordance with this focus. From the assessment of how the time horizon is valued from the different pension contributors’ point of

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69 Alternatively, the individual pension contributor has either no interest or knowledge to choose the most appropriate product matching his profile.
view, it is worth questioning, whether this yearly focus, in terms of the rate of interest, is equally important to all contributors.

Obviously the group of clients, including the man mentioned just above, close to retirement, is interested in a short term focus, whereas, the group including the 27-year old girl, where retirement lays 40 years away, might not have an equal interest in the short term performance. Arguably this group is more focused on the performance on a long term basis. Furthermore, the 27-year old girl’s retirement savings only amounts to a small portion of capital at this point in time, why the return in money-terms is little. She is more interested in the returns achieved in the future when her savings accumulates. By then the returns in money-terms are higher. This underpins the difference in interests with respect to time.

Present legislation precludes satisfying these distinct interests. The solvency requirement stated in article § 159(1) in the Financial Business Act (see section 2.2.1 The Danish Legislation – The Financial Business Act) is enforced on a day-to-day basis. From the quantitative analysis conducted in chapter 4 - The Theoretical Optimised Portfolio, this solvency requirement limits the possibilities of obtaining portfolios with higher performance. The day-to-day solvency requirement limits the possibilities of investing more risky from a theoretical point of view, referring to section 3.1 Pension Funds as Long Term Investors. Further, the coming risk-based Solvency II directive indirectly limits risky investments due to the strict risk management and demand on solvency with respect to these investments.

The above exemplifies the short term risk horizon, both in terms of the risks related to the incapability of being solvent at all points in time and risks related to investments themselves. To a high degree, this is assessed as limiting the possibilities of exploiting the high return opportunities on a long term basis. From the outcomes of portfolio 2c in section 4.5.3 Portfolio two - Maximise Return, it is concluded that even on a short term basis (since the theoretical optimisation model is derived on a one-period basis), it is not possible to obtain a portfolio performing as high as the theoretical optimised. Thereby, the solvency requirement actually limits the investment strategy at all points in time.
It is questioned whether the present narrow portfolio planning horizon necessarily is the best solution in managing even risk averse customers’ pension contributions. The short termed focus suppresses the opportunities of taking an investment perspective on longer horizons since the legislation in terms of the solvency requirement and the future Solvency II directive precludes this.

A suggestion in terms of a different view on the time perspective and hence an alternative approach to the investment strategy in the Danish pension fund sector is presented below in section 5.2 Bringing Suggestions Forward Addressing the Time Perspective. In section 5.2, only national legislation hence the solvency requirement is considered. The Solvency II directive will not be further elaborated upon or included in the suggestion, however it is acknowledged, that this part of the legislation is viewed as a limitation from the sector’s point of view with respect to its investment strategy aiming at benefitting the pension contributors to highest possible extent.

This and the previous section have addressed a central aspect in the analysis of the Danish pension fund sector. Defined by the number of years to retirement, pension fund contributors have diverging interests with respect to the importance of their retirement savings. The younger part of the group is more interested in the returns able to achieve in the long run. The Danish pension fund sector hampered to incorporate a long term approach to its investment strategy by the day-to-day solvency requirement, as well as the short term risk focus on investments themselves enforced by the coming Solvency II directive. Referring to figure 5.1, the second aspect of the time perspective is dealt with in the next section.
5.1.2 Managing Long Term Investments on a Short Term Basis

The second aspect found relevant, when it comes to the time perspective and the argued short term focus in the management of pension contributions, is the principal-agent relationship in the pension funds. Different from the labour market pension funds, the commercial pension funds are not owned by its members\(^{70}\). This means that there are diverging interests, between the owners and the pension contributors. A pension fund is a company like any other, operating a business. The prime function of a pension fund is to obtain as high a return as possible on the pension contributions, obviously with an overall aim of achieving satisfying financial results.

It is found interesting to question, how the incentive remunerations programs are defined from a time perspective point of view. In general, the principal-agent theory addresses the relationship between the principal, e.g. the board or other party representing the owners and the agent hence the top management – the CEO or other responsible party for operating the daily business (Bebchuk & Fried, 2003). This relationship can also be applied at lower stages in the company hierarchy e.g. between the top management (principal) and the responsible of the execution of the company’s investment strategy, hence the investments (agent).

From the principal’s point of view there is a great interest in making sure that the interests of the agent are aligned with the ones of the principal. In order to be sure of this alignment, incentive remunerations programs are widely used. The aim of these is obviously to obtain a joint perception of the direction of the company. This is often done through performance targets aligning the two parties towards the same goals.

The incentive based remunerations often take form of economic bonus agreements as a part of a remuneration package\(^{71}\). However, there are found two reasons to question these in this context. (1) One thing is the fact that pension funds are assessed long term investors facing an infinite horizon. The agent’s employment in the company is very short compared to this infinity, implying a natural short term focus from the employee’s point of view. It is believed that people

\(^{70}\) Of course if it is a public limited company hence the clients have the opportunity to hold stocks.

\(^{71}\) A presentation of what different incentives one can apply and a discussion of whether these economic incentives are the most appropriate to align interests will not be discussed in this thesis.
act in the interest of the given company they work for while employed. From the pension contributors’ point of view this natural conflict of interests with regards to the time perspective may constrain the contributors’ wealth accumulation opportunities. This conflict however is not possible to do anything about.

(2) The construction of the incentive based remuneration is the other problem, which rightly can be questioned. From explorative interviews it is known that such incentive based remunerations are based on yearly results, measured by some performance targets that are aimed to reach in accordance with what is demanded from the principal.

It seems inappropriate that pension fund investments, which are assessed as a long term investments from the pension contributor’s point of view, are managed by the agents who seek to reach short term incentive goals. The agent is encouraged to achieve short term goals and is rewarded economically by doing so. A typical target of a bonus agreement could, for example, be the financial results of the investment department the current year. Such a target is not encouraging the agent to consider long term investments as a goal in itself, as it might contradict the short term incentive included in the bonus agreement and thereby lowering the achieved bonus payoff. These diverging interests between the agent and the principal on one side and the pension contributors on the other side, possibly harming them as the possible gains from investing long term will not be achieved.

The Danish pension fund sector’s possibility to invest according to its infinite investment horizon, is affected by the short term focus of both the principals, setting, and agents, executing, the investment strategies.

The two aspects focused on with respect to the time perspective have now been presented. Presently the Danish pension fund sector is not able to incorporate the long term perspective into its investment strategy due to the present legislation, even though it is argued that it would benefit the pension contributors to a higher extent. The time perspective in the investment strategy is highly important both in relation to selecting the strategy hence deciding on the business model and in relation to the execution of the investment strategy. One thing is the fact that employment is very short termed compared to the fact that pension funds are long term
investors. Another thing is that the incentive based remuneration packages are short term focused. The next section will bring forward to suggestions to these two aspects with respect to benefit the pension contributors to the highest extent possible.

5.2 Bringing Suggestions Forward Addressing the Time Perspective

From having addressed the central aspects regarding the time perspective, suggestions are brought forward in this section. Firstly, in section 5.2.1 Incorporation of the Long Term Perspective it is addressed how the pension fund sector is enabled to invest on a longer term basis than it is the practice today. This is done by suggesting a more flexible solvency requirement. Secondly, the second central element with respect to the time perspective is addressed. Initiatives which could be implemented to secure that management and employees work towards the same goals determined for the pension funds are suggested in section 5.2.2 Alignment of Strategy and Execution. The focus in relation to this is how the pension funds’ investment strategy can benefit the pension contributors.

5.2.1 Incorporation of the Long Term Perspective

In this section, a suggestion of how the Danish pension fund sector can incorporate the long term perspective into its investment strategy is presented. The suggestion takes its outset in a relaxation of the present day-to-day solvency requirements.

5.2.1.1 Risk in a Long Term Perspective

As the outcomes from section 4.5 Theoretical Portfolio Optimisation & Comparison to the Actual Portfolio show, it is possible, theoretically, to hold more beneficial portfolios than what the pension fund sector has achieved. As stated in the same section, a part of the divergence in outcomes is due to the fact, that the legal restriction, dictating that pension funds must be solvent on a day-to-day basis, was not included in the Black-Litterman optimisation model. This indicates that relaxing the solvency requirement will make it possible for the Danish pension fund sector to obtain higher performance in its portfolio. Exactly this solvency requirement is also argued to be one of the main reasons why the pension fund sector approaches the investment
strategy with such a short planning horizon. The need for a short planning horizon stems from the fact that portfolios are assessed to be evaluated and possibly often rebalanced to comply with the solvency requirement.

The need for a short planning horizon stems from the fact that portfolios are evaluated often and possibly rebalanced to comply with the day-to-day solvency requirement.

The proportion of investments allocated to Danish and foreign equity is proven to be lower in the actual portfolio than in the optimised portfolio (portfolio 2c) in section 4.5.3 Portfolio two - Maximise Return. The difference can be explained by the fact that pension funds cannot hold large amounts of equity due to the related volatility and thereby risk of a potential loss. It is known that the volatility in risky asset returns is higher on a short term basis. Relaxing the solvency requirement will enable the pension funds to hold more risk, so that they can enlarge size of investments in Danish and foreign equity and thereby achieving a higher return over time. Moreover, the relaxation, enabling the sector to invest more risky, consequently will open up for obtaining higher returns on a long term basis and through that achieving possible higher benefits of a long term view on investments.

Relaxing the day-to-day solvency requirement enables the Danish pension fund sector to hold more risk, enlarge size of investments in equity and thereby achieve higher return over time.

A higher return would benefit all pension contributors, possibly in the form of higher yearly rate of interest and bonus potential bonuses. From theory it might even enable the pension funds to raise the guaranteed rates of return on future products, since theory states that higher returns are reachable in the long run, when investing more in risky assets (referring to section 3.1 Pension Funds as Long Term Investors). It is acknowledged that there is divergence between theory and reality. An assessment of the future of these products will be presented in chapter 7 - Bringing the Analysis into Perspective.
It is highly recognised that a form of solvency requirement is necessary in the Danish pension fund sector to secure an appropriate management of the pension contributions. The legislation is first and foremost a statutory instrument enforced to protect the pension contributors and protecting the consumers is accepted as very important. Further, the financial crisis has shown that such restrictions are necessary in the financial markets, when people’s capital is managed by third party.

The question found relevant to ask, is not whether a solvency requirement is necessary but rather a question of, whether it is appropriate to require solvency for all liabilities at all points in time. An alternative approach to the solvency requirement, such as relaxing these, might benefit the pension contributors to a higher degree than it is the case today. The time to retirement hence time to payout of pension contributions is found relevant to include in the suggestion. The suggested model should be accepted as an outset to view this problem in an alternative way, rather than it should be adapted as a definitive solution to the problem.

5.2.1.2 The Suggested Model

In this section, the suggested model is presented. The model is based on the idea, that relaxing the solvency requirement enables the Danish pension fund sector to benefit the pension contributors to a higher extent.

Dividing the customer group into subgroups, depending on time to retirement, is the outset. The simplistic division into two subgroups is chosen, since the following suggestion is seen as a qualitative and conceptual presentation of how the pension funds’ investments could be approached differently than today. The first subgroup (subgroup A) contains the pension contributors with 20 years or less to retirement. The second subgroup (subgroup B) includes all contributors that have more than 20 years to retirement. The choice of 20 years as the split between the two groups is based on following reasons; The equity premium is rationalised on a 20 year horizon. Moreover, it is assessed reasonable to assume that 20 years cover a business cycle from a macroeconomic point of view.

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72 It is acknowledged that if more quantitative analyses were applied to the problem, a more fragmented setup might be more appropriate. However, for illustrative purposes, this division is chosen.
Chapter 5 - The Time Perspective in Portfolio Management

The focus is drawn to the risk taking from the pension funds’ point of view. Whether people, with 20 years to retirement, care a lot about their future retirement is not found important with respect to how this split is made – the focus of interest in this thesis are the pension funds’ total investments and not the individual pension contributors’. Hence, in setting the 20 years split, the only things that matter are the factors affiliated with the pension funds.

With regards to subgroup A, the present solvency requirement should apply. This means that the amount of capital required to cover these liabilities should be handled as the pension fund sector does it today, being very conservative with almost no risk taking in order to fulfil the current day-to-day solvency requirement. With regards to subgroup B, it is suggested that a solvency requirement of 80% of the liabilities in this subgroup is introduced\textsuperscript{73}. A lower solvency requirement could be suggested, however, the aim of the suggested change in the solvency requirement in this thesis is to produce a rather realistic setup, why the requirement is set conservatively. The 80% is chosen for the purpose of this illustration\textsuperscript{74}.

The relaxation of the solvency requirement opens up for the possibility to exploit some of the opportunities that follow from a less restricted investment strategy, seen from a solvency and time perspective. Hence, it opens up for taking a long term approach to risk in combination with the existing short term risk handling.

It is assessed that a possible loss from rebalancing the portfolio on a short term basis, when capitalising some of the risky investments that may show unprofitable, will be outweighed by corresponding gains in other periods on a long term basis, referring to the fact that equity is assessed mean-reverting (see section 3.1.2 Time Diversification). Moreover, holding risky investments over a long period of time will, most likely, earn higher returns referring to previous stated (in section 3.1.1 The Equity Premium). Because of the infinite horizon, risky investments showing negative returns can be kept in the portfolio until the investment earns a positive return.

\textsuperscript{73} It is recognised that this problem can be modelled mathematically. However, due to high complexity this is delimited from in this thesis.

\textsuperscript{74} If a quantitative model was applied such would be able to estimate the appropriate level. That might differ from the chosen level.
It is important to stress that the above suggestion will not cause any transfer of risk to the pension contributor, as they have purchased a risk-free pension product. With the suggestion brought forward, the pension funds still hold the entire risk. From a business point of view, no company is willing to take any excessive risk for their consumers for free. This makes the investments, that they are responsible for, very conservative by nature\textsuperscript{75}.

In the model, it is suggested that the enhanced risk, which the pension funds take on, is attached to the liabilities connected to subgroup B - the pension contributors with more than 20 years to retirement. Each year the pension funds evaluate and possibly rebalance their investment strategy to include pension contributors with exactly 20 years to retirement in the group of liabilities, to which the pension fund must be solvent at all times – subgroup A. People entering the labour market, constituting new pension contributions, are placed in the group of liabilities that the solvency requirement does not fully restrict. By that, the pension fund sector pushes the enhanced risk forward at all points in time. The infinite investment horizon enables this.

The age composition naturally affects the size of this risk. This composition will be elaborated upon shortly. Even though the possibility of enhanced risk taking stems from the group of pension contributors with more than 20 years to retirement, the higher returns will benefit the pension contributor group as a whole. As expressed above, the pension contributors still hold a risk free product, why everyone should also have an equal share of the return\textsuperscript{76}.

The amount available to invest on a long term basis depends on the age composition, since this defines how many pension contributors are placed in the two subgroups. To give an example, the

\textsuperscript{75} Alternatively, the costs of this product would be higher. However, this is delimitated from in this thesis, referring to section 1.5.5 \textit{Delimitations}, why this will not be elaborated further upon.

\textsuperscript{76} This is recognized that the contribution principle affects this.
pension contributors could be split with 50% in each of the two subgroups. However, naturally this does not mean that the capital available for investments, hence the liabilities have the same split. The example only serves an illustrative purpose, why the specific numbers are not important. The age and capital composition is shown in figure 5.2.1.2.1.

As illustrated in figure 5.2.1.2.1, the liabilities for the two groups differ, whereas the number of pension contributors in each subgroup is the same. Subgroup A has been saving up for more years than subgroup B, resulting in a larger sum of liabilities for subgroup A.

The suggested solvency requirement scheme can be justified as only partly flexible since the sizes of the age groups define the amount of capital available to invest more freely. For example, when the number of people in subgroup B is large, a bigger amount is released for more risky hence more long term based investments due to the lower solvency requirement on this subgroup.

This amount will be the foundation for the possible increase in the total benefit of all pension contributors.

The sizes of subgroups in age define the amount of capital available to invest freely.
Figure 5.2.1.2.2 illustrates the amount of capital paid in by the contributors with more than 20 years to retirement (subgroup B). This constitutes the light grey area in the capital column in figure 5.2.1.2.1.

![Figure 5.2.1.2.2 Capital of Subgroup B](image)

Source: Own contribution.

The extra amount of capital possible to invest freely, corresponding to the 20% of the liabilities of subgroup B, is illustrated in figure 5.2.1.2.1. It is acknowledged that this amount is limited, taking the total liabilities of all pension contributors into account. It is acknowledged that the economic effect on investments may not be large in real money terms, but it will enable an alternative investment strategy, making it possible to earn higher returns.

Overall, the suggested revision of the solvency requirement enables the Danish pension fund sector to combine the present short term perspective on risk and hence investments with a more long term perspective. The short term perspective is mainly to ensure fulfilment of the liabilities attached to subgroup A - pension contributors with 20 years or less to retirement. The inclusion of more relaxed solvency requirement for subgroup B, the Danish pension fund sector is able to take a long term perspective on risk. This opens up for allocating more investments in equity.

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It is acknowledged that the pension funds also invest part of their equity (in Danish: egenkapital). This equity is free to be invested in risky assets. The suggestion of revised solvency requirement hence makes it possible to lose more capital before the solvency limit is reached.
From the above suggestion, it is assessed that no matter how the investment- and planning horizon are defined - incorporating a long term perspective with respect to a part of the investments - the pension funds will still have a conservative approach to the investment strategy after all. This means that the long term investments should be allocated in risky assets. This follows from what was rationalised in section 3.1.4 *The Theoretical Foundation of Long Term Investments*.

**Even though the legislation is relaxed, a conservative approach to investing pension contributors’ retirement savings is still important.**

Looking forward, the consequence of a relaxation of the solvency requirements makes it possible to incorporate risk and hence investment strategies on a long term basis combined with the short term approach that is applied presently. This possibly opens up for achieving higher returns on investments in the Danish pension fund sector. This will, ceteris paribus, benefit the pension contributors to a higher extent as the return obtained from the portfolios will be higher due to more investments in assets such as equity. Whether this increased benefit will result in higher guaranteed returns on future products, higher yearly rates of interest or higher bonuses is difficult to say.

Having suggested a model that enables The Danish pension fund sector to incorporate the long term perspective in its investments, focus is turned to how the implementation of the long term perspective also should be incorporated in the alignment of setting the investment strategy and the execution of it (see figure 5.1 *Structure of Chapter 5*).
5.2.2 Alignment of Strategy and Execution

As concluded above, extending the investment strategy with a long term approach on top of the short term focus is assessed to benefit the pension contributors. This means that it is very important that the management, in the pension funds, incorporates this long term view into the business model, and that the responsible parties, for the investments, implement it into the portfolio asset allocation process.

If the pension funds as companies, overall representing the principals, approach investments including a long term focus, it is important to ensure that the agents act in alignment with this. Therefore, it is suggested that the incentive based remuneration packages fit this short- and long term founded investment strategy. Instead of primarily focusing on short term performance, it is suggested that these remunerations should incorporate both short term performance targets as well as long term targets as a foundation of the bonuses. This ensures alignment, not only between the principal and agent, but also third party, aligning what is found most beneficial to the pension contributors.

If the short term focus, with respect to this aspect, is kept unchanged, it is recognised that the alternative strategy will not change anything with respect to the portfolio management executed. Agents will, to higher degree, align own goals with the principal’s goal if economic incentives are applied.

It is important to incorporate the long term view on investments into the business model of the pension funds. Further, the responsible parties for the investments must implement this long term view into the portfolio asset allocation process too.

The short and long term focus in the investment strategy should be incorporated in the incentive remuneration packages.

Summing up on this section, it is suggested, that the Danish pension fund sector implements the incorporated long term focus in investment strategies into the incentive based remuneration
packages to align the dual investment strategy, combining the short and long term focus, with the execution process.

Suggestions to the two aspects of relevance with respect to the time perspective have been presented in this part of the chapter. Firstly, a model including a relaxation of the day-to-day solvency requirement was suggested. This enables the Danish pension fund sector, as long term investors with an infinite investment horizon, to focus both on the short term risks and investments, however, also incorporate investments on a more long term basis. This allows for holding more risky investments and thereby, according to theory, obtaining higher returns. Secondly, this incorporation of long term focus in the sector’s investment strategies should also be incorporated into the execution process. Hence, with respect to the alignment of a combined short and long term strategy and execution, incentive based remuneration founded based on short and more long term goals should be incorporated into the incentive programs, which are set up to ensure that investments are executed responsibly in line with the strategies of the pension funds. This suggestion should, as everything else in this thesis, be seen from what can benefit the pension contributors to highest extent. It is acknowledged that the suggestion most likely conflict with what the shareholders of the pension funds find most important to them.

5.3 Conclusion on Chapter 5

In chapter 5 a number of central aspects have been addressed with regards to the time perspective, which is found of great importance with respect to the analysis of the Danish pension fund sector and how it performs in carrying out investments of Danish pension contributors’ retirement savings.

Even though the Danish pension fund sector is assessed to have an infinite investment horizon, it has been shown, that the sector is forced by regulations to act rather short term. Legislation is imposed to protect the pension contributors which are, by choosing this type of product, not willing to take on any risk. Conclusions from optimisation of the pension fund sector’s portfolio in chapter 4 - The Theoretical Optimised Portfolio show that it would be possible to gain a higher return at the same risk level by relaxing the solvency requirement.
Even though the entire group of pension contributors is risk averse, it is argued that it has divergent views on the importance of the retirement savings depending on how many years to retirement the individuals within the group have to retirement. It is argued that pension contributors, who have many years to retirement, are interested in pension funds to implement a long term investment strategy. Present legislation precludes the opportunity to have a combined short term and long term focus on investments. Danish and European legislation restrict the investment strategy directly and indirectly, among others, by setting a day-to-day solvency requirement and enforcing a short term risk approach to investments.

In this chapter an alternative model, where the solvency requirement is relaxed, is suggested. The group of pension contributors is divided into two subgroups, where the solvency requirement is relaxed on the liabilities of the group with more than 20 years to retirement. This enables the Danish pension fund sector to seek risk and investments in a longer term perspective and possibly achieve higher returns to the pension contributors. Opening up for more risky investments in a long term perspective does not mean that investments should be undertaken with no notice of risk. The suggested solution keeps the investments at a conservative level, however making it possible to combine the current short term and a long term perspective in the Danish pension fund’s investment strategy.

Another relevant aspect, when it comes to the time perspective and the argued short term focus in the management of pension contributions, is the principal-agent relationship in the pension funds. Two problems are found relevant to include with respect to this aspect; (1) Pension funds as investors face an infinite horizon, whereas the agents have very short employment compared to this. This problem is given and cannot be solved. (2) The construction of the incentive based remuneration packages to secure the alignment of the goals of the principal and agent are often based on targets set on yearly basis. If the Danish pension fund sector is allowed to adopt a long term perspective to risk and investments, in the form of relaxed solvency requirements, it is important to ensure that such incentive based remunerations reflect the new strategy.
Next chapter 6 provides an overall conclusion on the thesis and the analysis is brought into perspective in chapter 7.
Chapter 6 - Conclusion

The purpose of this thesis has been to investigate whether the Danish pension fund sector possibly can enhance the performance of its investments by changing its portfolio management, hence, increase the value creation for the pension contributors to higher extent.

By 2045, 50% of all pension payouts are estimated to come from private retirement savings. The Danish pension fund sector is capital intensive and the size of the sector in 2009 corresponded to 43.20% of the Danish GDP. These facts combined underline the relevance of investigating the investments strategies of the commercial pension funds with regards to the defined benefit product. By holding this product, the pension contributors place the responsibility of their future wealth in the hands of third party in order to sustain a certain standard of living during later stages of life. These customers are not willing to take on any risk and are, through the defined benefit product, offered a guaranteed return on their retirement savings. To protect the pension contributors, current and coming Danish and European legislation, restrict the investment strategy directly and indirectly - among others by setting a day-to-day solvency requirement and enforcing a short term risk approach to investments.

By averaging across the pension fund sector, defined by eight commercial pension funds, the actual portfolio of the Danish pension fund sector is estimated. Approximately 20% of the investments are allocated in equities, whereas the remaining is invested in bonds primarily. The actual portfolio has a Sharpe ratio of 0.11 measuring its performance. This serves as the basis of comparison with the theoretical optimised portfolio in investigating the performance of the sector.

From estimating the optimal portfolio by use of the theoretical Black-Litterman optimisation model, it is found that the Danish pension funds could benefit the pension contributors to a higher extent than it does - since the optimal portfolios derived show higher performances in terms of higher Sharpe ratios. The optimised portfolio, depicting the real investment environment to the highest extent possible, shows an allocation of approximately 40% in equities. This portfolio has a Sharpe ratio of 0.52. It is concluded that the two allocation restrictions do not
restrict the investment strategy. However, from optimising a portfolio containing the same level of volatility as the actual portfolio holds, some of the higher performance must stem from the solvency requirement as well as the increased focus on risk management, specific in relation to the current draft of the Solvency II directive. This underpins that legislation restricts the Danish pension fund sector in heightening the performance of its investments.

It is believed that the diverging performances of the two portfolios stem from other factors besides what has been extracted from the analysis. Further, there are challenges with regards to the appropriateness of comparing the theoretical optimised portfolio with the actual one depicting the investment strategy of the Danish pension fund sector. Firstly, it is assessed that theoretical models are useful in practical portfolio optimisation problems however the fact that theoretical models are compared with a real life portfolio, in this context, is questionable with regards to the appropriateness in doing so. Secondly, the actual portfolio and the theoretical optimisation portfolio are estimated based on different numbers of years. Both the appropriateness of comparing the two portfolios and the effect on the outcome of the optimised portfolio can be questioned.

Thirdly, features of the dataset are a part of the explanation for the divergence between the performances of the two portfolios. The data set used in the analysis is based on indices, which in itself makes a comparison with a much more actively managed portfolio questionable. Lack of availability of all investment opportunities in which the pension fund sector invest in is partly taken into account in estimating the actual portfolio. However, noise will still affect the outcome of the optimised portfolio. Fourthly, the theoretical model assumes randomness and normality in returns. This assumption is not fulfilled and possibly affects the outcome of the theoretical optimised portfolio.

Fifth, the Black-Litterman model includes opinion-adjusted expected returns but no changes are made in the expected volatility of the individual asset class or the covariance between the asset markets in the portfolio. This means that a changed opinion-adjusted expected return derived from the benchmark portfolio is associated with an unchanged level of volatility and covariance. This is assessed affecting the outcome of the theoretical optimised portfolio.
The time perspective in the investment strategy of the pension fund sector is found important to include in the analysis. Even though the entire group of pension contributors is risk averse, it is argued that they have divergent views on the importance of the retirement savings depending on number of years to retirement. This should be exploited and to benefit all pension contributors to highest extent, the Danish pension fund sector should focus on both short term and long term investments. The legislation in the form of the day-to-day solvency requirement and the short term risk management hamper the pension funds to act long term in their investments. This thesis suggests an alternative approach to the solvency requirement. The suggested model includes a relaxation of the solvency requirement for the group of pension contributors with more than 20 years to retirement and maintaining the current requirement for the group with less than 20 years. The suggestion proposes that only an 80% day-to-day solvency requirement should apply to the group with more than 20 years to retirement. By relaxing the solvency requirement, it enables the Danish pension fund sector to take on a more risky long term investment strategy, most likely resulting in higher returns hence benefitting the pension contributors to a higher extent. The proposed model setup entails that the investment strategy held by the Danish pension fund sector is still very conservative reflecting the great responsibility the sector have in investing pension contributors’ retirement savings.

The time perspective is equally important in setting the investment strategy and in executing it. Targets are set to align the interests of the principal and the agent. Pension funds are long term investors facing an infinite investment horizon, employment in a pension fund is very short term compared to this. This is a natural problem, which cannot be solved. However, it is known that incentive based remuneration packages are based on short term targets and this problem is manageable. Incorporating the time perspective, combining the short term targets with more long term targets in incentive based remuneration packages, enables the time integrated investment strategy to conclude in alignment between the principal selecting the investment strategy and the agent executing the investments.

The Danish pension fund sector is able to enhance the performance of its investments by improving its portfolio management and benefit the pension contributors to a higher extent than it does.
Chapter 7 - Bringing the Analysis into Perspective

The pension funds are challenged in obliging to the high guaranteed returns promised back in time, since the developments in the capital markets and the strict legislation on these products limits the opportunity in achieving high returns on investments.

From this, it is found relevant to reflect on the future of the defined benefit products. They still make up the largest share of the pension products sold in the market. The outlook of the defined benefit products will be elaborated upon referring to the overall pension fund sector.

It has been concluded that a long term approach to risk and thus investments will enable the pension fund sector to hold more beneficial portfolios. From this it has been concluded that in theory this might even enable the pension funds to raise the guaranteed rates of return on future products. This is reasoned by the fact that theory states that higher returns are more likely to be achieved in the long run when investing more capital in risky assets. As stated, one thing is theory another is reality.

It is assessed that the guaranteed returns are one of the parameters of competition in the commercial subsector. The scenario outlined in the analysis, suggesting a relaxed solvency requirement, combined with the competitive factor speaks in favour of increasing the guaranteed returns. However, it is believed that the pension funds would not set higher guaranteed returns in reality. This is reasoned by the fact that this would imply the pension funds themselves to impose a restriction on their investment strategy equal to what the pension funds struggle with today. The decreasing interest rates in the period over which these products have been offered has resulted in difficulties for the Danish pension funds to oblige to the guaranteed returns promised back in time.

The guarantees themselves, set by the pension funds, restrict the pension funds’ liberty of action as it requires high investments in low risk assets, which do not have such high returns as when the products were commenced. Further, large amounts of capital are tied up given the existing and coming legislation. Thus, the pension funds are both restricted by the guarantees themselves and by legislation. The challenges in obliging to the guarantees imply that the future of this product is assessed doubtful- arguably the products are outdated.
Presently, most pension funds have started to move pension contributors from the defined benefit product to the defined contribution products, which are based on market returns rather than guaranteed returns. Some pension funds have withdrawn the defined benefit product. An example is SAMPENSION who has cancelled all guarantees. The argument for doing so is that without these guarantees, the company does not fall under the strict legislation and can therefore invest more offensive, thereby increasing the ability to achieve higher returns in the long run.

Keeping the guarantees forces the company to invest in low risk assets resulting in such low returns that present guaranteed returns are not possible to oblige to (SAMPENSION, 2010). SAMPENSION forced the pension contributors into the new pension product agreement. SAMPENSION, as a labour market pension fund, is able to take this forcing action since pension contributors are bound to this specific pension fund through their membership in a certain labour union. Thus, such action will not imply loss of customers\textsuperscript{78}.

However, the commercial pension funds operate in a competitive environment hence it is not possible to force the customers in such manner. Most likely, the pension contributor would choose another pension fund to place his retirement savings if a cancellation of guarantees was enforced. Instead, the commercial pension funds move the existing pension contributors from the defined benefit products to the defined contribution products by promoting the latter products as more beneficial than the defined benefit products. However, this movement of customers demands acceptance from the pension contributors. It is assessed that the promotion of the defined contribution products towards new pension contributors is also pronounced because of the challenge the pension funds face with respect to their guarantees. If they move the customers to other products based on the market returns, this will of course entail that the pension contributors agree to take on risk to some degree - little or higher - depending on the choice of defined contribution product. However, it will also imply an increased liberty in carrying out investments. This increases the probability of achieving higher returns than what the limited investment opportunities of the defined benefit products entail. These limitations stem from the guarantees themselves and from legislation, which this analysis has shown imposing restrictions.

\textsuperscript{78} If a pension contributor resigns his membership in the labour union affiliated with the given pension fund, a loss of the pension contributor as a customer is possible.
of crucial importance with regards to the ability to achieve more beneficial returns to the pension contributors.

All in all, it is assessed that the defined benefit products are outdated. This is partly because of decreasing interest rates over the period of which these products have been offered and the fact that the guarantees themselves limit the liberty in investments of the pension funds, and partly because of the restrictions enforced by current and future legislation, which also limit a more offensive investments strategy. If, the defined benefit products should continue to exist, this would demand a less restrictive legislation, which would open up for increased long term risky investments and hence higher returns, so that the Danish pension fund sector is able to set reasonable level of guarantees and oblige to those already set.
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