Conservative Banks vs. Aggressive Venture Capitalists

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Executive Summary

In this paper I analyze two types of financial intermediaries that finance entrepreneurial firms: Banks and Venture Capital firms. The core of this work is a new mathematical model that explains why Venture Capital firms focus on financing high-risk firms with high growth potential in knowledge-intensive, high-tech industries. And why Banks, on the other hand, tend to be more conservative and focus exclusively on low-risk, stable, more mature firms.

In my model an endowed financial intermediary has a choice to structure either as a Bank or a Venture Capital firm. The advantage of structuring as a Bank is the higher monitoring effort the intermediary will exert when monitoring her investment portfolio. This benefit comes from the fact that Banks are financed by short-term demand deposits and are prone to bank runs if their claim holders observe a deteriorating performance. The advantage of structuring as a VC is the fact that this type of intermediary is not subject to bank runs because it is financed by long-term depositors. This long-term financial structure leads to lower monitoring effort on part of the Venture Capital firm, but the risk of inefficient liquidation is nonexistent. This trade-off drives the intermediary’s choice.

The results of my work state that intermediaries presented with low-risk investment projects will structure as Banks and intermediaries presented with high-risk ones will structure as Venture Capital firms. At high levels of risk the benefit of increased monitoring effort is simply overshadowed by the increased chance of inefficient liquidation.
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1 Introduction

Banks and Venture Capital firms (VCs) are very important sources of funding for startup ventures. Currently, bank financing is the dominant form of small business funding. Berger and Udell (1998) find that only about 2% of small firms in the United States are financed by VCs, as opposed to close to 19% that are financed by bank debt. However, this trend is rapidly reversing as VCs become an important source of funding to small businesses, especially in the United States and Canada. According to Ueda (2002), indicators of bank lending to small firms are constant or even fall after 1977, whereas venture capital investment is almost 100 times larger in 2001 than it was in 1977.

At first glance, Banks and VCs seem very similar. Both are financial intermediaries that perform essentially the same function: they take money from depositors with too much and lend them to businesses with a shortage of funds. However, if we scrutinize the two institutions further, we will find that there are two major differences between these two intermediaries. First, they differ in the types of firms they finance; and secondly, they have very different financial structures.

VCs tend to focus on financing high-risk firms with high growth potential in knowledge-intensive, high-tech industries. These firms are typically small and young, are plagued by high levels of uncertainty and usually have few tangible assets to pledge as collateral (Gompers and Lerner, 2001). The returns of firms with venture capital financing have a very risky and positively skewed distribution, with a high probability of weak or even negative returns and a very small probability of extremely high returns (Sahlman, 1990; Fenn, Liang, and Prowse, 1995). Gompers and Lerner (2001) find that a relatively modest fraction - historically, between 20 and 35 percent – of VC portfolio firms are successful, they account for the bulk of the investment returns of VCs. Banks, on the other hand, tend to be more conservative and focus exclusively on low-risk, stable, more mature firms. Bank financed firms usually have lower informational uncertainty, lower observable risk, smaller R&D expenditures and higher, tangible collateral than firms financed by VCs (Carey, Post and Sharpe, 1998; Carpenter and Petersen, 2002).
Another crucial difference between Banks and VCs are their financial structures. VCs are financed by long-term, illiquid claims from their depositors (called Limited Partners). Gompers and Lerner (2001) find that because VCs must make long-run illiquid investments in firms, they need to secure funds from their Limited Partners for periods of a decade or more. In addition to these liquidity restrictions, limited partners in U.S. private equity funds typically have very limited rights and incentives to influence or direct the funds’ activities. Even though, in rare cases, investors can dissolve the fund, this hardly ever occurs (Lerner and Schoar, 2004). Banks, on the other hand, are financed by short-term demand deposit claims. This means that the bank’s depositors can withdraw their funds at any time. This structure is subject to bank runs in which depositors simultaneously withdraw their holdings if they expect that the bank is on the verge of failing (Diamond and Dybvig, 1983).

It is worthwhile to also mention two more differences between these two financial intermediaries. First, they differ in the way they contract with the entrepreneurs of their portfolio firms. VCs usually demand an equity stake in the new firm in return for funding (Kaplan and Stromberg, 2001). Banks, on the other hand, exclusively use debt to finance their portfolio firms. The above reiterates that VCs tend to be more risk loving than banks as debt claims, on average, are safer than equity claims. Second, the two intermediaries also differ in the way they monitor their portfolio firms. Winton and Yerramilli (2007) point out that Banks mostly monitor for covenant violations, deteriorating performance, or worsening collateral quality that might jeopardize their loan. They exercise control by threatening to force default and possible liquidation. VCs monitor in a much different way; they usually hold seats on the boards of the borrowing companies\(^1\) and have strong voting rights\(^2\). These rights give VCs wide control over the firm’s activities such as hiring, evaluating, and firing top management and advising and ratifying general corporate strategies and decisions (Kaplan and Stromberg, 2003)

From here on I refer to an intermediary financed through short-term claims as a Bank and one financed through long-term, illiquid claims as a VC. This paper introduces a model that

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\(^1\) In Kaplan and Stromberg (2003) the authors find that VCs have the majority of the board seats in 25% of the cases, the founders in 14% of the cases, and neither in 61% of the cases. VC board control is less common for first VC rounds

\(^2\) In that same paper Kaplan and Stromberg (2003) find that that VCs have a voting majority in 53% of all financings in the minimum contingency case and in 41% of first VC rounds. In the maximum VC vote contingency cases, VCs control a voting majority in 69% of all financings and 61% of first VC rounds.
explains that an intermediary’s choice of financial structure is based solely on the degree of risk of the investment project to be financed. The model includes an investment project, an endowed intermediary and numerous depositors. The endowed intermediary already has a project on her balance sheet. This project is called the illiquid project. It matures at time 2 paying out a certain return. The project, however, can also be liquidated inefficiently at time 1 paying out less than at time 2. I also make an assumption that part of the payoff at time 2 is in the form of cash; meaning it cannot be verified by law enforcing institutions and can be appropriated by the intermediary. The endowed intermediary is presented with a new investment project and observes its degree of risk. If the intermediary agrees to undertake the new investment project she has to attract depositors since she is not big enough to finance the project herself. Here the intermediary has to make a choice: whether to offer short-term or long-term claims to her depositors (the liability holders). In other words, the intermediary has to choose to structure as a Bank or a VC. These two structures differ in case of failure of the investment project. Under short-term finance, after observing the signal of failure, the depositors run the intermediary forcing her to liquidate her illiquid project and give up its return at date 1 (since liquidating at time 2 will result in a payoff of zero for the depositors as per the assumption above). In the case of long-term finance a bank run is not a possibility since the depositors holding the VC’s long-term claims are not able to run the intermediary at time 1. This financial structure leaves the intermediary with a certain payoff from the illiquid project at time 2 regardless of the outcome of the new investment project. The main result of my research shows that intermediaries presented with low-risk investment projects structure as Banks. This structure choice puts them at risk of inefficient liquidation if the investment project fails, although it leads to high level of monitoring effort on the part of the intermediary (which leads to higher probability of success). Intermediaries presented with high-risk investment projects, on the other hand, structure as VCs. This leads to less monitoring effort on part of the VC, but the risk of inefficient liquidation is nonexistent.

The intuition for this result is the following: The disadvantage of Bank finance, as compared to VC finance, is the loss associated with inefficient liquidation by the depositors if they believe that the Bank's projects in its investment portfolio are going bad. On the other hand, there is also an advantage associated with this financial structure: The threat of inefficient liquidation causes the Bank to exert higher monitoring effort, which increases the probability of
a successful outcome of the investment project. The advantage of VC financing, as compared to Bank finance, is the fact that, due to their long-term financial structure, they are not subject to bank runs. On the other hand, this immunity to bank runs causes VCs to exert lower monitoring effort, which decreases the probability of a successful outcome of the investment project. The above trade-off results precisely because of the different financial structures that the two intermediaries possess. Risk of the potential investment projects plays a very important role here. At low risk levels, the advantage of Bank finance outweighs the disadvantage. This, however, is not the case at high levels of risk where the threat of inefficient liquidation becomes too great. My model predicts that high-risk projects will be financed by VCs; while safer, more conservative ones will be undertaken by Banks. The above results are consistent with the literature on the subject.

2 Literature Review

This paper is related to two branches of literature. First, there has been previous research on the topic of determinants of the trade-offs between VC and Bank financing. A very recent and important paper on this subject is Winton and Yerramilli (2007). The paper concentrates on the trade-off between VCs’ higher funding costs and their superior monitoring ability. The authors present a model in which an entrepreneurial firm can choose to seek financing either from a Bank or a VC. Banks can only monitor passively while VCs can monitor both passively and actively. Compared to Banks, VCs also demand a higher return from the firms they finance since they have a higher cost of capital. VCs have a higher cost of capital because they have to compensate their depositors for the illiquidity of their claims. The authors’ results state that entrepreneurs generally prefer Bank finance as it is always cheaper. Venture capital finance is only preferred when the average profitability of the firm’s continuation strategy is not too high, the incremental cost of active monitoring is not very high, and the firm’s strategic uncertainty is high. Winton and Yerramilli (2007) also mention random liquidity shocks that the Bank might experience from its depositors, but this leaves the analysis largely unchanged. These liquidity shocks do not raise the Bank’s funding costs above the VC and do not influence the Bank’s level of monitoring.
Another major paper in this field is Ueda (2002). Just like Winton and Yerramilli (2007) it concentrates on the trade-off between financing a new investment project through a Bank or a VC. The cost of financing from the Bank is asymmetric information and the cost of financing from a VC is the threat of expropriation. The author asserts that Banks are plagued by asymmetric information since they are less informed about the project. A VC specializing in the entrepreneur’s industry, on the other hand, becomes perfectly informed about the project and is able to fully capture the project’s rent. However, this almost perfect information in some cases enables the VC to continue the project without the entrepreneur. This threat of expropriation forces the entrepreneur to share some rents to the project with the VC. Ueda (2002), however, does not discuss monitoring effort of the two intermediaries, nor the differing degree of risk associated with different investment projects.

The second branch of the literature discusses the beneficial aspects of short-term financial structure of financial intermediaries. A major paper in this field is Diamond and Rajan (2001a, 2001b). The authors develop a model in which fragility of a bank’s financial structure is seen as desirable. This fragility is defined by a mismatch of liquid liabilities (demand deposits) and illiquid assets (loans and investments). The model consists of entrepreneurs who need to seek outside financing to undertake their projects. Once financed, the entrepreneur can threaten to leave the projects. In this case the project will be worth much less since as the entrepreneur has project specific expertise. This makes the project illiquid and presents a hold up problem for the Bank. To make matters worse, the Bank might also experience liquidity needs from its depositors and will be forced to sell the project to outsiders at a deep discount (because the outsiders are assumed not to have the same project specific know-how). Because of these complications, lending becomes expensive and the amount that the entrepreneur can borrow is limited. The authors argues that this problem is alleviated if the Bank borrows (issues demand deposits) against the cost of the entrepreneur’s project. These deposits are subject to bank runs; therefore, if the bank threatens to withdraw from the project, depositors will run it leaving it with no return.

Another important paper that discusses benefits of short-term financial structures of Banks is Calomiris and Kahn (1991). The authors claim that the short-term financial structure of Banks is crucial in solving the problem of diverging interests between a Bank’s depositors and its management. The authors develop a model where a banker has an investment opportunity, but
needs to attract outside deposits to be able to undertake the project. The project can either be a success and return a handsome payoff, or be a failure with a much smaller return. All agents come to find out the state of the project soon after the investment is made. In case of a bad state the banker has an incentive to appropriate the payoff instead of repaying it to the depositors. Thus, if the Bank is financed with demand deposits, it is run by its depositors if they observe that the bank’s investment project is failing. To curtail unnecessary bank runs, the Bank will set aside a reserve to service those depositors with urgent liquidity needs. However, their analysis does not take into account the now common system of deposit insurance which makes depositor monitoring less important.

Schure (2002) also discusses benefits of short-term debt that come in the form of higher monitoring effort on part of the financier. I incorporate the basic framework of Schure (2002) in my model; taking the assumption of an endowed intermediary that has to attract additional capital to finance a new investment project. My model comes to a similar result that short-term claims are beneficial in terms of higher monitoring effort from the financier. However, my model extends Schure (2002) by studying the role of risk of each investment project and how it affects an intermediary’s choice of financial structure. As it turns out, at high levels of risk the cost of inefficient liquidation associated with short-term finance exceeds the incentive benefit of short-term finance such that intermediaries financed with long-term debt become optimal at undertaking highly risky investment projects.

3 The Model

3.1 Overview and the Illiquid Project

This section describes the model in which an intermediary brings together deposits from agents in the economy and invests the accumulated cash into new start-up companies (investment projects) that differ in their level of risk (probability of successful outcome of the venture) and payoffs, with higher payoffs going to high-risk projects and vice versa. The depositors are assumed to be risk neutral and the expected rate of return is zero. The intermediary is already invested in a separate, illiquid project prior to investing into the investment project.
The illiquid project requires an investment of $I_0$ at time 0 and delivers $A_2 + C_2$ at maturity at time 2. $A_2$ is the asset component of the return, that is it can be measured and verified by courts or other law enforcing institutions. $C_2$, as in Myers and Rajan (1998), is the cash component that cannot be verified by courts and is therefore untraceable, making it possible for the intermediary to appropriate it. For example, the intermediary can invest the cash flows in some negative net present value project that is not modeled (Jensen, 1986). The illiquidity of the project comes from the fact that it can be liquidated prematurely at time 1 with a liquidation value of $A_1$. Since $A_1 < A_2 + C_2$, liquidation here is inefficient and undesirable. The illiquid project is illustrated in Figure 1 below:

![Figure 1: The Illiquid Project](image)

For the rest of the paper I assume that $A_1 = A_2$. As we will see shortly, this assumption simplifies the analysis by making the transfer payments back to the depositors equal regardless of the intermediary’s financial structure. It also does not compromise the “illiquidity” of the illiquid project since $A_1 < A_2 + C_2$, making the premature liquidation inefficient.

### 3.2 The Investment Project

The investment project requires an investment of $L_0$ at time zero. The intermediary is not big enough to finance any new investment project by herself. She has to attract a certain number of depositors to have enough funds. An investment in a new start-up company also requires monitoring effort on the part of the investors. The effort is only exerted by the main investor (the intermediary) since the effort of other agents fully duplicates the main investor’s effort (Diamond, 1984). Exerting effort also implies a disutility of $-e$. The investment project matures at time 2 and has a probability of success that depends on its risk class $r$ and the monitoring effort $e$ exerted by the main investor (the financial intermediary). The risk class parameter $r$ is positive and equal to or greater than one. This is an *ex-ante* risk measure of the different investment projects in the economy. The project returns $rR$ (an amount that can be verified by
courts) with probability $\frac{p(e)}{r}$, where $R$ is some constant, $p(e)$ is probability that depends on monitoring effort that increases at a decreasing rate and $r$ is the \textit{ex-ante} risk class parameter mentioned earlier. It is easy to see that the return increases (decreases) and the probability of success decreases (increases) as $r$ goes up (down). I assume that the expected profits of the investment project are positive and depositors find out the outcome of the project at time 1.

3.3 The Game

3.3.1 Description of the Game

In the first phase of the game the intermediary is presented with an investment project. She observes the project's \textit{ex-ante} risk level $r$ and chooses whether to invest in it or not. Since I have assumed that the expected profits of the investment project are always positive, the intermediary makes the investment. Next, the intermediary has to come up with $L_0$ by attracting enough depositors. She offers claims $k_i$ to the potential depositors. There are two versions of these claims; they can either be short-term demand deposit claims $k_{st} = \{L_0, D, 0\}$ or a long-term debt claims $k_{lt} = \{L_0, 0, D\}$ where $L_0$ is the amount needed for the investment and $D$ is the promised repayment back to the depositors at time 2. From now on I will refer to the former case as \textit{Bank finance} and the latter case as \textit{VC finance}.

Figures 2 and 3 (found in Section 10 at the back of this paper) show the financing game of Bank finance and VC finance respectively. The first stages of both games are the same: the intermediary offers claims $k_i$ to depositors who either accept or reject them. If the claims are rejected, the intermediary is left with the payoff from the illiquid project of $A_2 + C_2$ and the depositors are left with $L_0$. If the claims are accepted by the depositors, the game goes on and the investment project is financed. Also at this time the intermediary chooses her monitoring effort level $e$. Then, at time 1, Nature randomly chooses the outcome of the investment project at time 2 and depositors observe this signal. The next stage of the game differs between the two types of claims.

3.3.1 Bank Finance Game and Solution

In case of Bank finance, the successful outcome of the investment project yields a payoff of $rR + A_2 + C_2 - D - e$ to the intermediary and $D$ to the depositors. In case of failure of the
investment project, however, the depositors run the intermediary at time 1 (when the signal of failure is observed). The intermediary is forced to liquidate her illiquid project to pay back at least some amount to the depositors. The resulting payoffs are \(-e\) for the intermediary and \(A_1\) for the depositors. Here I make an assumption that \(A_1 \leq L_0\), that is the depositors do not recoup the total amount they have invested. The bank-financed intermediary’s problem is shown below:

\[
\max_{e,D} \left\{ \frac{p(e)}{r} \times (rR + A_1 + C_2 - D - e) + \left( 1 - \frac{p(e)}{r} \right) \times (-e) \right\} \quad \text{(Program BF)}
\]

Subject to:

\[
e \in \arg\max_{\tilde{e}} \left\{ \frac{p(\tilde{e})}{r} \times (rR + A_1 + C_2 - D - \tilde{e}) + \left( 1 - \frac{p(\tilde{e})}{r} \right) \times (-\tilde{e}) \right\} \quad \text{(IC)}
\]

\[
\frac{p(e)}{r} \times D + \left( 1 - \frac{p(e)}{r} \right) \times A_1 \geq L_0 \quad \text{(PC)}
\]

**Proposition 1:** Bank finance is viable (profitable) only if the above program has a solution. Under this financial structure the optimal monitoring effort level is \(e^*_s\). The proof is presented in the appendix.

The first constraint above is the Incentive Compatibility Constraint; it ensures that the intermediary will choose monitoring effort \(e\) so as to maximize her profits. The second constraint is the Participation Constraint, which if satisfied, ensures that the depositors agree to invest in the intermediary if, on average, they at least break even.

### 3.3.2 VC Finance Game and Solution

The case of VC finance looks more favourable to the intermediary. The outcome if the investment project is a success is the same as above; the intermediary receives \(rR + A_2 + C_2 - D - e\), while the depositors receive \(D\). In case of the investment project's failure, however, the depositors cannot run the intermediary at time 1 because they hold long-term claims. They run the intermediary when their claims become due (at time 2) forcing the intermediary to give up the verifiable part of the illiquid project's return, \(A_2\). While the intermediary receives \(C_2 - e\) because she is able to appropriate the unverifiable cash return of \(C_2\) from the illiquid project. This is the result of the long-term financial structure of the intermediary. The depositors cannot run the intermediary at time 1 when they observe the outcome of the investment project. Neither
can they get more than $A_2$ because the illiquid project returns a cash component that cannot be verified by courts. Thus the depositors are left with $A_2$ if the investment project turns out to be a failure. The VC-financed intermediary's problem is illustrated below:

$$\max_{e,D} \left\{ \frac{p(e)}{r} \ast (rR + A_2 + C_2 - D - e) + \left(1 - \frac{p(e)}{r}\right) \ast (C_2 - e) \right\} \quad \text{(Program VCF)}$$

Subject to:

$$e \in \arg\max_{\bar{e}} \left\{ \frac{p(\bar{e})}{r} \ast (rR + A_2 + C_2 - D - \bar{e}) + \left(1 - \frac{p(\bar{e})}{r}\right) \ast (C_2 - \bar{e}) \right\} \quad \text{(IC)}$$

$$\frac{p(e)}{r} \ast D + \left(1 - \frac{p(e)}{r}\right) \ast A_2 \geq L_0 \quad \text{(PC)}$$

**Proposition 2:** VC finance is viable (profitable) only if the above program has a solution. Under this financial structure the optimal monitoring effort level is $e^*_t$. The proof is presented in the appendix.

The constraints are same as in the case of Bank finance, except here $C_2$ is a given for the intermediary even in case of the investment project's failure.

## 4 Results

### 4.1 Optimal Effort Levels of the Two Intermediaries

Closer examination of the problems from Section 3 yields the optimal monitoring effort level $e^*$ of the two intermediaries and how they compare to each other.

**Proposition 3:** The short-term financed intermediary (Bank) will exert higher monitoring effort than long-term financed intermediary (VC). That is $e^*_{st} > e^*_{lt}$. The proof is presented in the appendix.

Figure 4 below gives a visual interpretation of the optimal monitoring level of the two intermediaries.
The intuition behind this result is the following: Banks have more at stake if their investment project fails, that is they completely lose their illiquid project when there is a bank run. This forces the Bank to exert higher monitoring effort. VCs, on the other hand, never lose the cash component \( C_2 \) of the illiquid project because of its non-verifiability. In case of failure of the investment project, a VC is run when the depositors’ claims become due, getting only the asset component of the return, \( A_2 \). Higher monitoring effort is a clear advantage of Bank finance. However, as mentioned earlier, this advantage becomes overshadowed by the increasing risk of inefficient liquidation as the degree of risk of the investment project raises.

### 4.2 First Best Effort Level

It is also worth noticing that the above optimal effort level choices are sub optimal. This sub optimality comes from the fact that both intermediaries’ incentives to exert effort are distorted. The distortions here are repayments back to the depositors. Since the intermediary is not able to appropriate the whole return of the investment project for herself, she will exert suboptimal monitoring effort level. To find the first best effort level, we have to forget about the illiquid project and focus solely on the investment project. We do this because liquidation of the
illiquid project is never socially optimal (see illiquid project description above). We have to choose \( e \) so as to maximize the expected value of the investment project.

\[
E(\text{return}) = \frac{p(e)}{r} * rR + \left(1 - \frac{p(e)}{r}\right) * 0 - e
\]

This simplifies to:

\[
\text{Max}_e P(e) * R - e
\]

**Proposition 4:** The first best effort level is \( e^{FB} \). It is greater than the optimal effort levels derived under Banks and VC finance. \( e^{FB} > e^e_{st} > e^e_{lt} \). This is the case unless \( C_2 \) is abnormally high.

### 4.3 Risk Parameter and Expected Profits of the Intermediaries

Taking into account each intermediary’s optimal monitoring effort level; let us look further into how the risk class \( r' \) of their investment projects affects their expected profits and their choice of financial structure. We can derive the expected profits function of the intermediary under both scenarios by looking back at the previous sections. Examining the intermediary’s problem under both financial structures yields the following expected profit functions for the Bank and the VC respectively:

\[
E(\pi_{st}) = p(e^*_st) * R + \frac{p(e)}{r} \{A_2 - D + C_2\} - e^*_st
\]

\[
E(\pi_{lt}) = p(e^*_lt) * R + \frac{p(e)}{r} \{A_2 - D\} + C_2 - e^*_lt
\]

Looking at the above profit function, we see the key piece of information is that the long-term financed intermediary gets the non-verifiable cash flow of \( C_2 \) regardless of the outcome of the investment project. To further understand how the *ex-ante* risk class affects the choice of financial structure, we need to derive the equilibrium transfer payment back to depositors and plug it into the above expected profit functions.

Let us go back to Figure 2; the only way a depositor will accept the intermediary's claim contract is if and only if the expected value if the depositor's payoff is equal to or greater than \( L_0 \). In equilibrium, under both Bank finance and VC finance, the transfer payment \( D \) is such that the
depositors just break even. Rearranging and setting the participation constraint (PC) equal to $L_0$ gives the equilibrium transfer payment $D^*$:

$$D^* = \frac{r}{p(e)} * L_0 - \frac{r}{p(e)} * A_2 + A_2$$

Substituting this into the expected profit function of the bank-financed intermediary will yield:

$$E(\pi_{st}) = p(e_{st}^*) \left( R + \frac{C_2}{r} \right) + A_2 - L_0 - e_{st}^*$$

Now let us refer back to Figure 3 for the scenario under VC finance. Here also the depositors will only accept the intermediary’s claim contract if and only if the expected transfer back to them will be equal to or greater than $L_0$. In equilibrium, under VC finance, the transfer back to the depositors will be same as under Bank finance. This is so because in the case of the investment’s project failure, the depositors will still run the intermediary, but only when their claims mature (at time 2) forcing the intermediary to give up $A_2$ from the return of the Illiquid Project. And as I have assumed earlier, $A_2 = A_1$. Substituting $D^*$ into the expected profit function of the VC financed intermediary will yield:

$$E(\pi_{lt}) = p(e_{lt}^*) * R + A_2 + C_2 - L_0 - e_{lt}^*$$

It is clearly seen that the VC finance expected profit function is immune to the risk class parameter $r$ as it is cancelled out when the substitution is made. The profit function of bank finance, however, is decreasing in $r$. This leads me to the following proposition:

**Proposition 5:**

- At low levels of $r$, the expected profits of the Bank-financed intermediary will be greater than or equal to the expected profits of the VC-financed intermediary: $E(\pi_{st}) \geq E(\pi_{lt})$
- At high levels of $r$, the expected profits of the Bank-financed intermediary will be less than or equal to the expected profits of the VC-financed intermediary: $E(\pi_{st}) \leq E(\pi_{lt})$
The original hypothesis that Banks finance low risk investment projects, while VCs finance high-risk ones is proven right if the above proposition holds. In the case of low-risk investment projects, the intermediary will structure as a Bank because her expected profits will be higher as opposed to structuring as a VC. This is so because at low levels of risk the threat of inefficient liquidation is small and the advantage of higher monitoring effort (higher probability of success of the investment project) outweighs the threat. However, with high-risk investment projects, the intermediary will structure as a VC because her expected profits will be higher as opposed to a Bank. This is so because at high levels of risk the threat of inefficient liquidation becomes too great and the higher monitoring effort is not enough to compensate. That is structuring as a VC becomes optimal since this structure is immune to bank runs. This result is illustrated in Figure 5 below.

![Figure 5: The division between the market for Bank Finance and VC Finance](image)

**4.4 The Threshold Level of the Risk Class \( r^* \) and Comparative Statics**

Given the above analysis we can further extend it by deriving a threshold level of the risk class \( r^* \). This is the level where Bank finance becomes less profitable than VC finance. The threshold \( r^* \) is found when we equate \( E(\pi_{st}) \) and \( E(\pi_{lt}) \). The result is:
This threshold risk class separates the market for Venture Capital financing and the Bank loan market. Projects with risk classes lower than $r^*$ will seek Bank financing, while projects with a higher risk class will exclusively seek VC finance. This market division is illustrated in Figure 5 above.

It is also of interest to see how this threshold level varies when we change other parameters of the model. These changes may have two effects on the threshold level of the risk class $r$. The first such effect is the direct effect. It is an effect that a change in a parameter has on the expected profits of the intermediary. The other is the indirect effect on the intermediaries’ choice of optimal effort level.

A parameter of great interest here is $C_2$, the unverifiable cash component of the return from the illiquid project. This parameter can be viewed as a measure of investor protection. When $C_2$ is large, it can be seen as an environment with weak investor protection. This is so because depositors in a VC fund will not be able to appropriate this component of VC’s assets if the investment project fails. Below we examine comparative statics of varying the degree of investor protection by studying both the direct and indirect effects.

- **The direct effect:** As $C_2$ increases, so does the expected profit function of the VC. As seen in Figure 6 below, the VC profit function shifts up, causing the threshold $r^*$ to decrease. This has the effect of expanding the market for VC finance and contracting the market for Bank finance.

- **The indirect effect:** However, as $C_2$ increases we also observe an increase in the Bank’s equilibrium monitoring effort $e^*_b$ ($e^*_l$ remains the same as the VC gets $C_2$ regardless of the outcome of the investment project). This increase has an effect of shifting the expected profit function of the Bank to the right, expanding the market for Bank finance.
The overall effect of increasing $C_2$ on the size of each market is uncertain. The magnitude of the first shift (VC profit function) is known, however; the magnitude of the second shift is ambiguous and depends on other parameters in the model and the proximity of $e_{st}^*$ to the first best effort level. Although following the arguments of La Porta, Lopez-de-Silanes and Shleifer (1999), weaker investor protection framework will lead to a less vibrant market for startup firms.

5 Discussions

5.1 Ex-ante Versus Ex-post Risk Parameter

In this section I would like to discuss and clarify two important issues in this paper. First, I will go into more detail about the way I measure risk in the model and clarify the difference between the *ex-ante* and *ex-post* riskiness of the investment project. Secondly, I will scrutinize evidence from other academics that conclude that VCs always monitor their investment portfolios more than Banks.

As we have seen above, each project has a different risk class $r$. This is an *ex-ante* risk measure of the different investment projects. In other words, the level of risk the financial
intermediary observes before undertaking the project. We can further derive the effect it has on
the probability of success of the investment projects by fixing the effort level across the two
financial intermediaries. Let’s assume that the fixed effort level for both intermediaries is $\bar{e}$. We
can see how the risk class influences the ex-ante riskiness of the investment project:

$$\frac{d}{dr} \frac{p(\bar{e})}{r} < 0$$

As $r$ goes up, the probability of success goes down. The intermediaries observe this ex-ante
riskiness and make a decision of whether to structure as Banks or VCs. However, once the
investment project is undertaken by the intermediary, we observe only the ex-post riskiness of
the project. This is the probability of success of the project given the risk class level $r$ and the
optimal effort level exerted by the intermediary. It can be illustrated as follows:

$$\frac{p(e^*)}{r}$$

As shown in the previous section, our model predicts that VCs always exert lower monitoring
effort than banks. For a fixed $\hat{r}$, we can see that the probability of success of the investment
projects is lower under VC finance than under bank finance. This occurs solely because $e_{it}^* <
 e_{st}^*$.

$$\frac{p(e_{it}^*)}{\hat{r}} < \frac{p(e_{st}^*)}{\hat{r}}$$  \hspace{1cm} (1)

However, as mentioned in the beginning of this work, VCs focus exclusively on high-risk
projects and Banks focus on low-risk ones. So when I say that Banks monitor more, I mean that
they monitor more given the same ex-ante risk class $r$. We will discuss this issue below.

5.2 Conflicting Results in the Academic Literature

The above result brings us to another very important topic to be discussed here. Most of
the literature on VCs suggests that VCs exert higher effort when monitoring their portfolio firms
than banks. Gorman and Sahlman (1989) find that VCs visit their portfolio companies an average
of 18.7 times per year and are generally more involved in the management of their portfolio
firms. Blackwell and Winters (1997), on the other hand, find that most bank loans to smaller
firms are monitored once or twice a year. At first glance, this seems to go against the predictions of our model as it predicts that VCs will exert lower monitoring effort than banks. This inconsistency arises primarily because in the real world we only observe the *ex-post* riskiness of the portfolio firms of the two intermediaries (after the optimal effort level has already been chosen). To compare the two we have to keep in mind that the two intermediaries focus on projects with different *ex-ante* risk classes. The average risk class of investment projects that a VC undertakes is much higher than the average risk class of investment projects that a Bank undertakes. But if we fix the risk class of the projects, equation (1) above still holds. The above finding is illustrated in Figure 7 below.

![Figure 7: A closer look at the market division](image).

As seen in Figure 7 above, for any fixed *ex-ante* risk class *r* a Bank’s optimal effort level would exceed that of a VC. However, we have to keep in mind that VCs undertake projects that are, on average, much more risky than the Bank’s portfolio firms. Therefore, the relevant Optimal Effort Level interval for a Bank is the one that occupies the lower levels of *r* (“Bank Finance Market” in Figure 7). And the relevant interval for a VC is the one that lies on the higher levels *r* (“VC Finance Market” in Figure 7). In other words, our work does not conflict with previous research in the field: In our model VCs are also observed to exert higher monitoring level than Banks.
6 Extensions

6.1 Dynamic Extension

In reality bank loans are both risky and illiquid. This is also a standard assumption in the banking literature; see e.g. Diamond and Dybvig (1983). However, in my base model there are two separate projects: one illiquid and one risky. One way to extend this model is to eliminate the dichotomy of the two projects that are present on the balance sheet of the financial intermediary. In order to do so we must come up with a single project that would have the characteristics of both the illiquid project (that can be liquidated should a financial intermediary choose to structure as a bank) and the risky investment project. To accomplish this I chose to make our model dynamic using the Overlapping Generations Model (OLG) framework. The OLG model was first invented by Maurice Allais in 1947 and later popularized by Samuelson (1958) and Diamond (1965). In the model agents start out with a certain endowment that they can lend to an intermediary who can then invest them into a risky project. The agents live for three periods. They invest in the first period and can consume in the second and last period of their lives. The intermediary invests in new projects each period so at least two projects are overlapping.

6.1.1 The Investment Project

Here I completely eliminate the illiquid project and focus on the risky investment project. The investment project is much the same as in the original model. At maturity it pays off \( rR \) with probability \( \frac{p(e)}{r} \) and zero with probability \( 1 - \frac{p(e)}{r} \). Once again, \( r \) is the \textit{ex-ante} risk class of the investment project and \( R \) is some constant. However, to give the investment project a degree of illiquidity to make the model work we have to also introduce the concept of \textit{asset-specificity}. This concept was first developed in Williamson (1988) and Shleifer and Vishny (1992). These authors argue that some assets on a bank’s balance sheet have a high degree of specificity and will yield a lower payoff if managed by outside agents. These assets cannot be easily redeployed by their new owners since they require specific skills that only the original owners possess. In other words, when assets are liquidated or sold off to outside agents, they produce lower payoffs
and, therefore, have a very low liquidation value. An example of such assets would be knowledge intensive assets such as patents or prototypes.

I incorporate asset-specificity into the model by allowing the financial intermediary to sell off her investment project (only when it is clear that the project will be a success) to outside agents that do not possess the same skills as the intermediary. When the relationship lender continues with the project, it pays off $r\tilde{R}$ with probability $\frac{p(e)}{r}$ and zero otherwise. But if the project is sold off to an outside agent, it can only yield $r\tilde{R}$. As before, $R$ is a constant where $\tilde{R} > R$.

6.1.2 Description of the Game

The game begins in period $t=-1$. There are numerous agents in the economy with endowments of $\frac{L_{-1}}{n}$, where $n$ is the number of agents in the economy and $L_{-1}$ is the amount needed to undertake the first investment project (we will refer to this project as P1). We also have an intermediary (in this case she does not have a prior endowment) who is presented with an investment project, but needs outside funding to be able to undertake it. I assume that the investment project has a positive NPV and that the intermediary successfully raises $L_{-1}$ from the agents in the economy to proceed. The depositors are promised to be repaid $D_{-1}^{VC}$ at $t=1$. Here, at $t=-1$, the intermediary can only offer long-term claims to her depositors because she has no assets of her own and will not be able to payoff anything if liquidated. In other words, at $t=-1$, the intermediary has no choice but to structure as a VC. I also assume that the investment project has a liquidation value of zero. This project matures at $t=1$ and pays off $r\tilde{R}$ if it is successful and is managed by the original intermediary. It can also payoff $r\tilde{R}$ if it is successful and is sold off to outsiders before maturity. I assume that $r\tilde{R} \geq D_{-1}^{VC}$, so the depositors will be paid off in full in either case.

Now at $t=0$, a new generation of endowed agents is born and the intermediary is presented with another investment project (we will refer to it as P2) that requires an initial investment of $L_0$. Once again, the intermediary needs to attract outside depositors to be big enough to finance the new projects. But here the intermediary is able to choose the type of claims to offer her depositors. She can offer them a long-term claim as before (once again becoming a
VC). She can also offer them a short-term demand deposit claim (that is becoming a Bank). The intermediary can do so because she is able to pledge the cash flows from the first project (P1) as collateral. If P2 is unsuccessful, its depositors can force liquidation of P1 to outside agents. This strategy will yield them a payoff of $rR - D_{-1}^{VC}$. It is important to note that this will only happen if P1 is a success; the intermediary is allowed to go on only if it is successful in her first endeavour. Otherwise, the game starts over and the intermediary becomes a VC by offering long-term claims. Here I will only consider the case in which P1 is a success.

\[
\begin{array}{cccc}
t = -1 & t = 0 & t = 1 & t = 2 \\
-\bar{L}_1 \text{ (Project 1)} & -\bar{L}_0 \text{ (Project 2)} & r\bar{R} - D_{-1}^{VC} \text{ or } 0 & r\bar{R} - D_0^i \text{ or } 0 \\
& & r\bar{R} - D_{-1}^{VC} \text{ or } 0 & r\bar{R} - D_0^i \text{ or } 0 \\
\end{array}
\]

**Figure 8: Risky Projects in the OLG framework**

As Figure 8 illustrates above, at $t = -1$ Project 1 is financed by the financial intermediary that structures as a VC and promises to repay its depositors $D_{-1}^{VC}$ at $t = 1$. This project will return either $r\bar{R}$ with probability $\frac{p(e)}{r}$ or zero otherwise. If it is successful it can be sold off to outside depositors before maturity. Because of asset specificity, it will only yield a lower outcome $r\bar{R}$.

At $t = 0$ Project 2 is financed by the new generation of endowed agents and the intermediary now has the choice to structure as a Bank or a VC. After making her choice, the intermediary offers to repay the new depositors $D_0^i$ (where $i$ = either B or VC depending on the claim). I examine the case where the intermediary structures as a bank next.

### 6.1.3 Bank Finance Game and Solution, Project 2

First, let us look at the game of Bank finance. This game is illustrated in Figure 9 at the back of this paper. After undertaking P1 at $t = -1$, at $t = 0$ the intermediary is presented with P2 and decides to pursue it. She offers the new generation of endowed agents a short-term demand deposit claim. They either reject or accept it. If the contract is rejected, the intermediary is left with the payoff of $r\bar{R} - D_{-1}^{VC} - e_1$ from P1 and the depositors are left with their endowment of $\frac{\bar{L}_0}{n}$. If the claim is accepted by the depositors, the game goes on and the investment project is
financed. Also at this time the intermediary chooses her monitoring effort level $e_2$. Then, at an intermediate time $t = \frac{1}{2}$, Nature decides the outcome of $P_2$ and the depositors observe this perfect signal. If $P_2$ is successful, the intermediary gets both the payoff from $P_1$ and $P_2$: $r\bar{R} - D^{VC}_{-1} - e_1 + r\bar{R} - D^B_0 - e_2$. The depositors of $P_1$ get $D^{VC}_{-1}$ and depositors of $P_2$ get $D^B_0$. However, if the depositors observe a signal of failure of $P_2$ at $t= 1/2$, they will run the intermediary taking possession of $P_1$ and selling it off to an outsider. This strategy nets them a liquidation payoff of $r\bar{R} - D^{VC}_{-1}$. In other words, the outsider is only able to get a return of $r\bar{R}$ from the project and payoff the original depositors; the difference is left to the new depositors of $P_2$. The Bank will have a payoff of $-e_2$ from $P_2$. The problem of bank finance is presented below:

$$\text{Max}_{e_2} \frac{p(e)}{r} \{r\bar{R} - D^{VC}_{-1} - e_1 + r\bar{R} - D^B_0\} + \left(1 - \frac{p(e)}{r}\right) * 0 - e_2$$

Subject to:

$$\frac{p(e)}{r} * D^B_0 + \left(1 - \frac{p(e)}{r}\right) * \alpha \geq L_0 \quad (PC)$$

Where $\alpha = r\bar{R} - D^{VC}_{-1}$

$$e \in \arg\max_{\tilde{e}_2} \left[\frac{p(\tilde{e})}{r} \{r\bar{R} - D^{VC}_{-1} - e_1 + r\bar{R} - D^B_0\} + \left(1 - \frac{p(\tilde{e})}{r}\right) * 0 - \tilde{e}_2\right] \quad (IC)$$

**Proposition 6:** Bank finance is viable (profitable) only if the above program has a solution. Under this financial structure the optimal monitoring effort level is $e^*_{st}$. The proof is presented in the appendix.

### 6.1.4 VC Finance Game and Solution, Project 2

Now I consider the case of VC finance as illustrated in Figure 10 at the back of this paper. Once again, it looks more favourable to the intermediary than the case of Bank finance. In case of both projects turning out successful, the intermediary will receive $r\bar{R} - D^{VC}_{-1} - e_1 + r\bar{R} - D^B_0 - r_2$. The depositors of $P_2$ will receive their promised payment of $D^B_0$. In case of $P_2$’s failure, however, the depositors will not be able to run the intermediary and liquidate the existing project ($P_1$). The depositors will indeed receive zero if $P_2$ goes under. The VC will
receive the rents from P1 and a disutility of effort associated with P2. Her payoff will be 
\( r \tilde{R} - D^{VC}_0 - e_1 - e_2 \). The intermediary’s problem is presented below.

\[
\max_{e_2} \frac{p(e)}{r} \{r \tilde{R} - D^{VC}_0 \} + \left(1 - \frac{p(e)}{r}\right) \cdot 0 - e_2 + \{r \tilde{R} - D^{VC}_{-1} - e_1 \}
\]

Subject to:
\[
\frac{p(e)}{r} \cdot D^{VC}_0 + \left(1 - \frac{p(e)}{r}\right) \cdot 0 \geq L_0 \quad \text{(PC)}
\]

\( e \in \arg\max_{e_2} \left\{ \frac{p(e)}{r} \{r \tilde{R} - D^{VC}_0 \} + \left(1 - \frac{p(e)}{r}\right) \cdot 0 - \tilde{e}_2 + \{r \tilde{R} - D^{VC}_{-1} - e_1 \} \right\} \quad \text{(IC)}

**Proposition 7:** VC finance is viable (profitable) only if the above program has a solution. Under this financial structure the optimal monitoring effort level is \( e^*_{1t} \). The proof is presented in the appendix.

This also leads us to conclude that the Bank will exert higher monitoring effort than the VC much like in the previous version of the model.

**Proposition 8:** The short-term financed intermediary (Bank) will exert higher monitoring effort than long-term financed intermediary (VC). That is \( e^*_{st} > e^*_{lt} \). The proof is presented in the appendix.

The intuition behind this result is very similar to the previous model. Banks have more at stake if their P2 fails because their depositors will force liquidation on the first successful project P1. This fact causes banks to exert higher effort when monitoring their investment projects. VCs, on the other hand, cannot be run by their depositors and therefore will always appropriate the cash flows flow of P1, even if P2 is a failure. Once again, we see that higher monitoring effort is a clear advantage of bank finance. But this advantage soon becomes overshadowed by the risk of inefficient liquidation as the risk class of the investment projects becomes high enough.

### 6.1.5 The Choice of Financial Structure

Now that we have identified the problems associated with each type of intermediary and found their optimal monitoring efforts, we examine their expected profits and see how the *ex-ante* risk class affects them. First, to simplify the analysis we will identify two variables: \( \bar{x} \) and
\( \alpha \). These variables represent the profits from P1 under the original bank management and when sold off to outsiders.

\[
\bar{\alpha} = r \bar{R} - D_{c1}^{VC} - e_1 \quad \text{and} \quad \underline{\alpha} = r \underline{R} - D_{c1}^{VC} - e_1
\]

Now we can take a look at the expected profit functions of both intermediaries. First, let us consider Bank finance.

\[
E\pi_{st} = \frac{p(e)}{r} * \{r \bar{R} - D_0^B + \bar{\alpha}\} - e_{st}^*
\]

As we can see above, the bank will get the payoff from both P1 and P2 only if P2 is successful. Otherwise, the bank is run and P1 is sold off to outsiders and its return is appropriated by the depositors.

Now we examine the expected function of the VC financed intermediary. We have to keep in mind that her long-term financial structure prevents her depositors from liquidating any existing assets of the intermediary.

\[
E\pi_{tt} = \frac{p(e)}{r} * \{r \bar{R} - D_0^{VC}\} + \bar{\alpha} - e_{tt}^*
\]

As seen above, the VC will get the return from P1 regardless of P2’s outcome as its depositors will not be able to liquidate it.

It is clear from the above analysis that the VC’s profits are less dependent on the risk associated with P2. The Bank, on the other hand, has much more at stake; it can lose the proceeds of both P1 and P2 if P2 turns out to be a failure. This is what forces the Banks to exert higher effort when monitoring their investments. This higher effort is seen as a clear advantage of Bank finance. Unfortunately, this advantage diminishes as the risk of inefficient liquidation increases.

6.2 Future Extensions

In the future, I would like to apply my model to research the difference between Venture Capital markets in the United States and Europe. The European Commission in its 1994 paper states that the lack of a vibrant venture capital market in Europe puts European firms at a
disadvantage when compared to US based firms. I would like to explore the reasons behind this discrepancy and suggest policy options that could encourage further growth and development of a vibrant Venture Capital market in Europe.

7 Conclusion

In this paper I studied the choice of a financial intermediary to structure as a Bank or a VC when presented with a risky investment project. When an intermediary structures as a Bank, she is faced with the threat of a bank run if the investment project is observed to be failing. This, however, makes the intermediary work harder exerting high monitoring effort that results in higher probability of success of the investment project. When an intermediary structures as a VC, she becomes immune to bank runs since the depositors hold long-term debt claims. This, however, is not necessarily good news as the intermediary will not exert high monitoring effort (as opposed to a Bank) since the threat of a bank run is nonexistent. The intermediary’s choice of structure depends solely on the degree of riskiness of the prospective investment project. At low risk levels the advantages of Bank finance outweigh the disadvantages and lead the intermediary to structure as a Bank. However, at high risk levels the advantages of higher monitoring effort are overshadowed by the threat of a bank run and lead the intermediary to structure as a VC. These findings are consistent with stylized facts of VC firms and the related literature.

This model can possibly help answer the question of why Banks are conservative, while VCs are very aggressive when it comes to financing investment projects. This risk taking behaviour of VCs has been observed for many years, but there is very little work done to explain this phenomenon. Hopefully this paper could contribute to the field and serve as a stepping stone to the comprehensive explanation of VC risk taking.
8 Appendix

**Proof of Proposition 1:** Program BF has a solution \((e_{st}^*, D^*)\). To solve for this solution, we need to maximize the expected profit function subject to the constraints. To do so, first we need to take the first derivative of the expected profit function and set it to zero:

\[
\frac{dE(\pi_{st})}{de_{st}} = p'(e) \left( R + \frac{C_2}{r} + \frac{A_2}{r} - \frac{D}{r} \right) - 1 = 0
\]

Then we need to set it equal to \(D\):

\[
D^* = rR + C_2 + A_2 - \frac{r}{p'(e)}
\]

Then we need to set the participation constraint (PC) equal to \(D\):

\[
D^* = \frac{r}{p(e)} \left( L_0 - A_2 \right) + A_2
\]

Now we have two equations and two unknowns: \(D\) and \(e\). Next we need to set the two equations equal to each other and solve for the optimal solution. This is illustrated graphically in Figure 4, where the lines "Effort Level: Bank" and "PC" cross. This gives us the solution \((e_{st}^*, D^*)\).

**Proof of Proposition 2:** Program VC Finance has a solution \((e_{it}^*, D^*)\). To solve for this solution, we need to maximize the expected profit function subject to the constraints. To do so, first we need to take the first derivative of the expected profit function and set it to zero:

\[
\frac{dE(\pi_{it})}{de_{it}} = p'(e) \left( R + \frac{A_2}{r} - \frac{D}{r} \right) - 1 = 0
\]

Then we need to set it equal to \(D\):

\[
D^* = rR + A_2 - \frac{r}{p'(e)}
\]

Then we need to set the participation constraint (PC) equal to \(D\):

\[
D^* = \frac{r}{p(e)} \left( L_0 - A_2 \right) + A_2
\]
Now we once again have two equations and two unknowns: $D$ and $e$. Next we need to set the two equations equal to each other and solve for the optimal solution. This is illustrated graphically in Figure 4 where lines "Effort Level: VC" and "PC" cross. This gives the solution $(e^*_lt, D^*)$.

**Proof of Proposition 3:** To prove this proposition, we need to compare the results of propositions 1 and 2. This is illustrated in Figure 4 where it is seen that $e^*_st > e^*_lt$. This inequality also holds at different values of $r$: As $r$ changes, so do $e^*_st$ and $e^*_lt$, but the inequality remains unchanged. This is so because the $r$ parameter only changes the slope of the above equations, leaving the differing intercepts unchanged.

**Proof of Proposition 6:** Similar to the above, to find the solution $(e^*_st, D^*_0)$ we need to maximize the expected profit function subject to the constraints. To do so, first take the first derivative of the expected profit function and set it to zero:

$$
\frac{dE(\pi_{st})}{de_{st}} = p'(e) \left( \frac{\alpha}{r} + \frac{D^*_0}{r} \right) - 1 = 0
$$

Then we need to set it equal to $D$:

$$
D^* = \alpha + r\bar{R} - \frac{r}{p'(e)}
$$

Then we need to set the participation constraint (PC) equal to $D$:

$$
D^*_0 = \frac{r}{p(e)} \left( L_0 - \alpha \right) + \alpha
$$

Now we have two equations and two unknowns: $D$ and $e$. Next we need to set the two equations equal to each other and solve for the optimal solution.

**Proof of Proposition 7:** Similar to the above, to find the solution $(e^*_st, D^*_0)$ we need to maximize the expected profit function subject to the constraints. To do so, first take the first derivative of the expected profit function and set it to zero:

$$
\frac{dE(\pi_{lt})}{de_{lt}} = p'(e) \left( \frac{\bar{R}}{r} - \frac{D^*_0}{r} \right) + \bar{\alpha} - 1 = 0
$$
Then we need to set it equal to $D$:

$$D_0^{yc^*} = r\bar{R} + \frac{r}{p'(e)} * (\bar{u} - 1)$$

Then we need to set the participation constraint (PC) equal to $D$:

$$D_0^{yc^*} = \frac{r}{p(e)} * L_0$$

Now we have two equations and two unknowns: $D$ and $e$. Next we need to set the two equations equal to each other and solve for the optimal solution.

**Proof of Proposition 8:** To prove this proposition, we need to compare the results of propositions 6 and 7. This inequality also holds at different values of $r$: As $r$ changes, so do $e_{st}^*$ and $e_{it}^*$, but the inequality remains unchanged. This is so because the $r$ parameter only changes the slope of the above equations, leaving the differing intercepts unchanged.
9 References


10 Figures

Figure 2: Bank Finance Game

Figure 3: VC Finance Game

Figure 9: Bank Finance Game, Project 2 (Dynamic Extension)

Figure 10: VC Finance Game, Project 2 (Dynamic Extension)
Figure 2: Bank Finance Game
Figure 3: VC Finance Game
Figure 9: Bank Finance Game, Project 2 (Dynamic Extension)
Figure 10: VC Finance Game, Project 2 (Dynamic Extension)