Risk Parity
– an attractive alternative approach or a temporary craze?

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Executive Summary

Does risk parity outperform traditional portfolios? Is the relative performance different under different market conditions? The expected performance of the different portfolios is a key element for the investor’s choice of portfolio.

The purpose of this paper is to investigate and analyze the performance of the risk parity portfolio optimization approach, and to examine whether or not it can be expected to outperform other portfolio optimization approaches. Performance and risk was measured using Sharpe Ratio, Modified Sharpe Ratio, Maximum Drawdown and the sensitivity to different market conditions was analyzed by running a regression of the change in interest rates and equity market volatility on portfolio returns.

In the empirical study it was found that risk parity has to be leveraged before reaching an acceptable expected return, when analyzing the period from January 2004 to December 2013. The leveraged risk parity and naïve risk parity portfolios were found to significantly outperform a leveraged equally weighted portfolio and an unleveraged mean-variance portfolio for the investment universes with a Danish home bias, but not for the international biased portfolios. These findings correspond well with the findings of Chaves et al (2011) that risk parity can be said to have performed well over the last 30 years but results highly depend on the defined investment universe.

Results showed that the two risk parity portfolios showed a negative sensitivity to the change in interest rates larger than the other portfolios. The risk parity portfolios were also found to have a negative sensitivity to equity market volatility, but less than that of the equally weighted portfolio. The findings indicate that there is a risk that risk parity will underperform if interest rates start to climb relative to the mean-variance portfolio.
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1 Introduction

In recent years the world has seen several wealthdestroying bubbles burst, such as the Dot.com crisis (2000-2001), the financial crisis (2007-2009) and the European sovereign debt crisis (2011). These crises seem to differ from previous ones, because of their shorter intervals and greater magnitude. This seems to spawn new alternative investment approaches, since investors are continuously seeking for new methods to secure superior returns at lower risk.

Risk parity can be perceived as one such new method. Even though risk parity has existed as an implemented investments approach since the mid-nineties, it has recently gained increasing attention by institutional investors as it offers an alternative approach to portfolio construction to the traditional Markowitz optimized portfolio.

In the current low interest rate scenario safe investments are expected to yield a low return and many investors expect that this is not very likely to change over the next couple of years. You might say that risk-averse investors are in a bit of a predicament. If they invest too much in safer assets such as government and mortgage bonds and less in riskier asset such as equities, they risk a return lower than what they will settle for. If they instead act against their risk-aversion they might get the required return, but will also take on too much risk. So what to do? This gives new sentiment to especially low risk investors that, all other things equal, are increasingly prone to seek out alternative measures to ensure continuously attractive absolute returns at reasonable levels of risk.

Solution the problem: risk parity! Several plans and other institutional investors, such as the Wisconsin State Investment Board and The Pennsylvania Public Schools Employees’ Retirement System in the USA, have already adopted or are considering risk parity as their choice of portfolio construction.

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1 It can be argued that the crisis still continues, but here it is only referring to the declining stock prices in 2011 in most of the world
2 http://www.pionline.com/article/20110118/ONLINE/110119919
3 http://www.psers.state.pa.us/content/investments/program_details/overview/20130930MarketValue.pdf
promises of higher returns at a lower risk from risk parity practitioners sound very ensnaring, but is it really the solution to the risk-averse investor?

1.1 Problem statement

The problem statement of the thesis is based upon the above introduction.

*How has the risk parity approach performed relative to naïve risk parity, equally weighted and mean-variance optimized portfolios?*

Each of the approaches is based on capital allocation across asset classes. Therefore it is important to define what an asset class is and what its characteristics are:

1. What is an asset class, and what are the characteristics?

When asset classes have been grouped into investment universes the performance is measured over the entire period using different performance and risk measurements, leading to the next research question:

2. How is the performance when asset classes are combined into forming distinct investable investment universes based on geography and asset class classification?

As a result of specific market conditions, asset classes are expected to perform differently. Therefore the performance of portfolios might also perform different:

3. Does the relative performance of the portfolios change under different market conditions?

The purpose of the thesis is to analyze an array of risk parity portfolios from the following hypothesis:

*Risk parity portfolios offer an attractive alternative to naïve risk parity, equally weighted and mean-variance optimized portfolios for a Danish institutional investor, regardless of the investment universe.*
1.2 Delimitation

The analysis will focus on risk parity with an asset class approach, rather than risk parity with a risk factor approach. The difference between the two approaches will though be described, so that the framework for risk parity is established. Risk factor based risk parity will not be part of the empirical study.

Volatility and the estimation of the volatility play an intricate part of portfolio theory, especially for risk parity, and is the basis for asset allocation across the asset classes. There are several ways of estimating volatility. The different methods will not be analyzed nor discussed in this paper. As it is with the estimation of volatility, so is it the case for the estimation of the other parameters such as expected return, correlation, level of the risk-free rate and borrowing rate.

When investing several other factors have influence on the performance and return of the portfolios. The commission paid or other expenses, will have a negative effect, as well as tax on investment returns. Even though both factors influence the performance, it is worth a paper in itself regarding the optimal allocation when taking into account expenses and tax. It is out of the scope of the paper to take this into account.

There are a vast number of different securities indices that could be used as proxies for the different asset classes. The number of asset classes have been limited to a relative few, but some which is regarded as being representative for a vast majority of investable assets in the world. Private equity and hedge funds have not been included into the investment universes of particular two reasons: First the available data that is available is of dubious quality (Pedersen, 2011), in respect to being a good and reliable representation of the two asset classes. Second the pricing and liquidity is less than that of the other asset classes and indices, and this also plays a part in the reasoning for not including them in the paper.

When leveraging the portfolios the choice of financing is expected to have a profound impact on the result of the empirical study. The investor has all other things equal the possibility to use a vast amount
of different financing options, such which instruments to use for example, futures contracts and bank loans, and which term to finance by. There can be a big difference between financing short term using futures contracts such as on a three month basis or at a much longer term at for example 1 year or 5 years or maybe even longer. It is not the focus of this paper to measure the effect of any specific financing option, as it is to show the extent at which an investor can end up using leverage and the potential risk that follows this. The type of leverage itself will therefore not be included directly in the calculations of the portfolios, but interest rate levels will be discussed and compared to the applied leverage.

The third “Super Asset Class”, the value storing will only be described, but it will not be part of the empirical study.

1.3 Structure

The first four chapters cover the framework and definition of asset classes, the methodology of the empirical study, the data used in the empirical study, before presenting the results of the empirical study. Subsequently a short chapter presents the limitations and implications of the results, leading to the conclusion in the final chapter.

- **Chapter 2** – Introduces and defines the asset classes and discusses these for use later in the paper. Other literature, especially performance evaluations on the risk parity topic, are reviewed and discussed, so they can be used to compare the empirical results later on in the paper.

- **Chapter 3** – Outlines the methodology for construction the portfolios using the four different portfolio optimization models. The advantages and drawbacks of the different models are identified and discussed. Finally the framework for estimating the parameters used by the models to allocate wealth is presented. Finally the combination of asset classes to be used in the empirical study is decided and hereafter defined as investment universes.
• **Chapter 4** – Discusses the financial data used for the empirical analysis and introduces the key figures for the analysis. The descriptive statistics of the data is discussed and unfulfilled assumptions of the data set are identified corrected for before applied in empirical analysis.

• **Chapter 5** – Presents the results of the empirical study. Performance evaluation is first based on simple key figures such as Sharpe Ratio to compare with existing literature. Leverage is applied creating twice the portfolios, and the methodology for the leveraging is presented. More advanced key figures are introduced to test if this adds any value to the performance evaluation and thereby gives a more accurate and descriptive picture of the potential risk factors of the different portfolios, which an investor might miss, when not considering these.

• **Chapter 6** – Discusses the limitations and implications of the empirical results. In connection with this future implications and ideas for further research within analyzing portfolio optimization using risk parity are proposed.

• **Chapter 7** – Summarizes the results and concludes on these.
2 Literature review

2.1 What is Risk Parity?

Risk Parity (RP) is a portfolio strategy where the portfolio is balanced based on equal risk contribution from either asset classes or risk factors to the overall portfolio. Traditional portfolios are often optimized based on more or less fixed allocations between equities and bonds such as the 60/40 portfolio often referred to in an international context by Martel & Ransenberg (2014), Chaves et al (2011) and Ruben & Melas (2011). In terms of risk contributions, 95% of the risk of this portfolio would be coming from equities, and only 5% from bonds (Ruben & Melas, 2011). In comparison a RP portfolio would have a 50/50 risk contribution from equities and bonds, but have a capital allocation of approximately 20% in equities and 80% in bonds. The figures below illustrate the difference between the capital allocations of a 60/40 portfolio compared to the risk contribution.

Figure 2.1 – Capital allocation of a 60/40 portfolio

![Figure 2.1 – Capital allocation of a 60/40 portfolio](source)

Figure 2.2 – Risk contribution of a 60/40 portfolio

![Figure 2.2 – Risk contribution of a 60/40 portfolio](source)

Source: Own creation and Ruban & Melas (2011)

The next figures illustrate the difference between the capital allocations of a RP portfolio compared to the risk contribution.
As previously mentioned there are two approaches to risk parity: An asset class based approach and a risk-factor based approach. The above figures illustrate the capital allocation and risk contribution of RP with an asset class approach. Risk factors can be such factors as equity risk, fixed income risk, inflation risk and credit/currency risk. These factors are unmistakably similar to asset class based risk contribution. Bhansali et al (2012) defines the two factors, which explain the majority of risk, are global growth and global inflation. This risk contribution is illustrated below.
The risk factor approach does all other things equal result in a third capital allocation between equities and bonds. Since asset allocation using RP only emerged as recently as in the mid-nineties, all literature is of a more recent date. Bridgewater’s “All Weather Fund”, is considered to be the first RP fund. The approach aims to equal the risk contribution, from equity, interest rates, inflation and credit.

Unlike Markowitz’s mean-variance portfolio theory, where both definition and origin is clear cut, risk parity is a somewhat different matter. Since there is not a distinct definition of what risk parity entails and the history of it is more unclear, it is necessary to establish the methodology before conducting any empirical study. Even though this uncertainty many scholars and practitioners regard Bridgewater’s “All Weather Fund” starting in 1996 as being the first to apply the risk parity approach. Therefore the president of Bridgewater Associates, Ray Dalio can be seen as the inventor or founder, if you will, but the term “risk parity” didn’t surface until the mid-2000. Up until then there had not been a common concept for the theory of assigning equal amounts of risk into the different asset classes. Since its inclusion into the investment universe, risk parity and risk parity funds have gained increasingly interest by investors.

The increasing interest in risk parity might not be surprising since the “All Weather Fund”, the risk parity flagship of Bridgewater Associates; over the last 18 years have had an average annual performance of 8% before fees.\textsuperscript{4} In the graph below Bridgewater Associates have simulated what the

\textsuperscript{4} CNBC interview with David McCormick of Bridgewater Associates from July 17, 2013.
All Weather Asset Mix, not to be confused with the “All Weather Fund” would have performed if applied over a 40-year period.

**Figure 3.1: The cumulative returns of the All Weather Asset Mix in the period 1970-2010**

![Graph showing cumulative returns of All Weather Asset Mix and U.S. Conventional Asset Mix](image)

The risk of the All Weather Asset Mix has been set to be equal that of an “U.S. Conventional Asset Mix” with 60% equities and 40% bonds. As it is evident when looking at the graph, the All Weather Asset Mix would have heavily outperformed the “U.S. Conventional Asset Mix” and by yielding a higher return at the same risk. In this paper the empirical study is based on asset class based RP approach and the risk factor approach will not be included further on.

### 2.2 Definition of asset classes

Defining asset classes plays an intricate part of the analysis in this paper. Whether or not investor defines US equities as separate from developed market equities makes a difference if one should invested 5% or 10% in that or those asset classes (Mariathasan, 2011). You often hear mentioned asset classes divided into two master categories: traditional and alternative assets. Most commonly equities and bonds are defined as the two overall traditional assets, but also cash is often defined as a traditional
asset class for itself. On the other side is the alternative assets where the most common and acknowledged alternative assets are, real estate, private equity, hedge funds and commodities. Currency is somewhat the odd one out. As soon as an investor crosses the boundaries of his or her home market, they take on a currency exposure, long or short, when trading assets such as equities or bonds. Whereas cash might be defined as home currency exposure, then currency can be defined as foreign currency exposure relative, to one’s home currency.

Anson (2006) has a slightly different take on the definition of asset classes, who divides assets classes into three overall so-called “Super Asset Classes”.

1. Capital assets are defined by their claim on future cash flows of an enterprise. Therefore capital assets may be valued based on the net present value (NPV) of their future cash flows (Anson, 2006). It is in this category in which equities and bond fall.

2. Assets that are considered as economic input, because these can be are part of the production cycle for producing other goods and assets. These assets do not hold any claim on future cash flows of any enterprise.

3. Assets that are considered “value storage”, for example paintings. These assets do not hold a claim to future cash flows, nor can it be used as input to producing other assets. Another, more common value storage asset is currency. A currency does not yield any return in itself, but it can store value, which it normally does in relation to investing in capital assets. By investing in a currency, the value relative to another currency can be preserved and/or stored.

Within the capital assets category, there are traditional asset classes such as equities and bonds. Hereby, equities can be divided into groups such as large cap, mid cap and small cap. Bonds, for example, can be divided into government bonds, mortgage bonds and corporate bonds. These assets classes are defined by their location, the issuer (company or government) and the size of the company or institution.
Hedge funds and private equity funds differentiate themselves based on the investment strategy rather than on the location of the assets. An example a differentiation of a hedge fund which is based solely on the investment strategy and not by the asset classes invested in is a Market-Neutral hedge fund. The hedge fund might be investing in exactly the same securities, but the objective is to yield a return regardless of the market conditions. In this case it includes going short to optimize the expected return or to hedge certain risk for a period of time. But are they then an alternative asset class? The type of super asset class they belong to is depending on how a return is made. Both hedge funds and private equity achieve their return from for example equities and bonds, either cash, or through derivatives. Since they fit the description of a capital asset Anson (2011) argues that they should not be regarded as alternative assets.

Real estate is often described as an alternative asset class. Anson argues that it is not, for as argues:

"First, real estate was an asset class long before equities and bonds became the investment of choice." (Anson, M. J. P.)

Why is it then that it sometimes still is regarded as an alternative asset? A reason for this is the complexity of real estate as an investment object. In “Managing Asset Classes”, James H. Scott Jr. describes the characteristics of institutional real estate and it market as being inefficient, with high transaction costs, wide bid-offer spread and the individual complexity of the buildings themselves, it being retail space, apartment buildings or office building. These factors make real estate less accessible for the private investor, obviously more than for an institutional, but this entails that the private investor might be unfamiliar with real estate as part an investment portfolio. It can therefore be argued, that it is the unfamiliarity and at the same time characteristics such as high transaction costs, which makes real estate an alternative asset class, relative to highly transferable asset classes such as equities and bonds.

This argument Anson transfers to another distinction between traditional asset classes an alternative asset classes. It all comes down to a matter of liquidity and trading place. Hedge funds and private

---

5 There is here referred to real estate as a pure investment object, and not as a means of housing, even though ones house/apartment of course both proves as a home but also an investment object.
equity are often privately held, and has no market place to determine the price, but their value must be derived by book value, and DCF-models.

Since commodities are defined not as capital assets, but as consumable assets, Greer (1994) and Anson (2006), the perception of them as being an alternative asset class more easily understandable, since they do not image the characteristics of the capital assets. Robert J. Greer summarizes the definition of asset classes as follows:

“Asset classes are distinguished by the characteristics of their economic exposure. They are not defined simply by their historical statistical correlation with each other. Debt, equity, and real estate are asset classes within the superclass of capital assets. Because the value of a capital asset can be strongly affected by forces in the asset's regional economy, capital asset classes have a geographic dimension to them as well.” Greer R.J.

When you add the liquidity of the assets as a determining characteristic you end up with overall asset classes such as equities, bonds, currency/cash, commodities, real estate, hedge funds and private equity. These can furthermore be subdivided bases on locality or investment strategy.

2.3 Papers on performance evaluation of Risk Parity

It does not seem surprising that asset managers which offer risk parity portfolios e.g. Bridgewater, AQR and Invesco have a lot of appraisal towards RP as an attractive investment strategy. Also Edward Qian, chief investment officer at Panangora Asset Management, writes in “Risk Parity and Diversification” highly of risk parity as being superior to a traditional mean-variance optimized portfolio. Though one common denominator for all are that they are all investment companies that offer Risk Parity inspired funds, this including Panangora. This of course should entail a sense of skepticism towards the conclusions of particular the latter, since there quite clearly could be an interest in promoting the risk parity strategy, and indirectly their mutual fund.

Chaves et. Al (2011) have several points of criticism towards risk parity in “Risk Parity Portfolio vs. Other Asset Allocation Heuristic Portfolios”. One of these are concerns the definition of asset classes.
They find that the performance of risk parity is highly sensitive to the inclusion of asset classes, and that results vary substantially with the number of asset classes included. They argue that it would benefit the clarification of the attractiveness of risk parity if, the asset classes to be included were clearly defined. In other words how many asset classes are there!

In “the risk parity approach to asset allocation”, from Callan Investments Institute, they find that in order to reach a return of 8.25%, as they deem required, the risk parity portfolio has to be leveraged by approximately 55% (if looking at a period from 1989-2009). Though in this period it would have outperformed a traditional unlevered minimum variance optimized portfolio, but in the same analysis they find that the risk parity portfolio lies below the efficient frontier. They therefore conclude that a levered minimum variance optimized portfolio, lying on the efficient frontier would dominate that of the levered risk parity portfolio. This is due to the higher return at the same risk or same return at a lower risk.

Besides the papers written by investments officers e.g. from different mutual funds most other papers seem to have a rather critical, or skeptical attitude towards risk parity. The criticism varies from definition of risk parity as a whole including Ben Inker – the dangers of risk parity, to more specific points as Callan Institute of investments, about unclear definitions of which asset classes to include.

2.4 The contribution of the paper to existing literature

Since a greater part, if not all, of present analyses’ on risk parity have been written by international based scholars and investments officers, the vantage point of these papers have had an especially particular American angle (Anderson, Bianchi & Goldberg, 2012). This might result in the conclusions to have an American bias, so to speak, and the conclusions they come to might only be valid for American investors, but not analogously applied to Danish investors whether these are private or institutional. This paper intends to take a Danish angle on risk parity, in order to evaluate its attractiveness from the perspective of a Danish private or institutional investor.
Approximately the last 30 years or, market interest rates have continuously dropped over the entire period creating a fixed income bull market from 1980-2010\(^6\). Since risk parity often entails a high allocation to fixed income (Ruban and Melas, 2011), it seems reasonable to assume that portfolios with a high and long exposure to fixed income might have outperformed other portfolios in that period.

Most of the existing research about risk parity, i.e. from “Wellington Management” and the “Callan Investments Institute”, is written from an American perspective. This paper assumes that Danish investors are likely to hold a larger portion of Danish securities, as a result of the knowledge about the domestic relative to foreign securities. Another argument could be that Danish investors are choosing Danish securities to mitigate the currency risk and/or exchange fees. No matter which argument one chooses, analyzing performance with a Danish home bias seems highly relevant.

The paper also expands the performance evaluation to include key figures, which have incorporated skewness and kurtosis. By testing the whether or not the returns of the portfolio is normal distributed, this papers is expected to give a more accurate analysis of the real risk embedded in risk parity and a better evaluation of its’ attractiveness as a portfolio approach.

By including the different aspects into this one paper, it is assumed that it will give valuable new insight into whether or not; it can be attractive for Danish institutional investor to choose RP or NRP over MV or even EW.

2.5 Partial conclusion

This chapter has defined what an asset class is and analyzed and discussed the characteristics. Blurred lines of what constitutes an alternative asset class have been redrawn. Relevant literature has been presented on performance evaluation and criticism of risk parity has been presented, and provides a context for reflecting on the empirical results and discussion later on. Finally the contribution of this paper to the existing literature has been outlined.

\(^6\) Wellington Management, Solutions – Think function, not form. December, 2011
3 Methodology

This chapter presents the methodology used for the empirical study in Chapter 5. The four portfolio construction methodologies for risk parity (RP), naïve risk parity (NRP), equally weighted (EW) and mean-variance (MV) are presented in section 3.1. In section 3.2 the methodology for the estimations of the required parameters will be presented, as well as the advantages and limitations.

3.1 Constructing the four portfolios

The first step before measuring the performance is defining the methodology for constructing the four portfolios, which are to be evaluated based on their performance\(^7\). Each portfolio is applying its own asset allocation model for obtaining the highest return with the lowest risk possible.

All four investment funds differ on two major areas (also see table below):

1. The method in which an optimal asset allocation is sought.

2. The number of parameters necessary to apply this method.

<table>
<thead>
<tr>
<th></th>
<th>RP</th>
<th>NRP</th>
<th>EW</th>
<th>MV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Correlation</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Expected return</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Borrowing rate</td>
<td>(Yes)</td>
<td>(Yes)</td>
<td>(Yes)</td>
<td>(Yes)</td>
</tr>
</tbody>
</table>

*Table. 3.1 - Parameters to be Estimated to Optimize the Portfolios*

\(^7\)There are fact more portfolios, but they are all combinations of both different investment universes and leverage
When the cost of leverage is \( \neq 0 \), then the borrowing rate is needed for all the leveraged portfolios, no matter if the asset allocation is based on a desired expected return or level of volatility.

### 3.1.1 Risk Parity

There are two different directions within the RP approach. The first approach is based on the assumption that the volatility of asset classes is the basis for the allocation of capital.\(^8\) The second approach builds upon the assumption that capital is allocated by the volatility of risk factors instead of asset classes. Examples of risk factors are inflation, equity risk, interest rate risk and credit risk. The two approaches to RP underscore the lack of a clear definition and therefore it is importance to clarify the applied approach in order to make a correct performance evaluation. In this research the first approach is used, where capital is allocated based on equal contribution of risk to the portfolio from the individual assets classes.

In its simplest form RP portfolios do not need any estimates of returns in order to construct portfolios, since only estimates of volatility and correlations are needed. When the individual volatility and correlation of the different asset classes are calculated, you simply allocate capital to the different asset classes so that the individual contribution of volatility by the assets to the overall volatility of the portfolio is identical. Since RP does not have to estimate expected return, this should limit the possibilities of estimation errors, and therefore this should in theory lead to more precise outcomes, than for example for MV.

When constructing a RP portfolio is likely to allocate a majority of capital to safer assets (i.e. bonds) and less into riskier assets (i.e. equities). Even though this should improve the reward-to-volatility, it also reduces the absolute return of the portfolio. In order to obtain the same absolute return as with a more traditional approach, leverage can be used to lever the expected return to the required level. As previously mentioned, the estimated expected return of the portfolio is not required but the use of leverage to boost the portfolio return entails some estimation.

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\(^8\) Parity is measured by variance and not by standard deviation, although the latter is a more widely used risk measure as a result of its convenient statistical properties (volatility is referring to standard deviation), where nothing else is mentioned.
The marginal contribution (MC) of the individual asset classes is the basis for the allocation of capital in RP. MC tells by how much risk the individual asset contributes to the risk of the overall risk of the portfolio. As MC uses variance as risk measure and therefore implicitly assumes normal distribution of returns, it has the same drawbacks as other key figures assuming normal distribution. MC of an asset class depends on the weight allocated to the asset class, the standard deviation and its covariance with the other asset classes. The fraction determines the change in risk of the total portfolio when there is a small change in the weight of the asset class. The marginal contribution of an indefinite number of assets can be described as:

\[ MC_i = (\text{Weight of Asset Class } i) \times \frac{\Delta \text{Total Risk of Portfolio}}{\Delta \text{Weight of Asset Class } i} \]  

(1)

Where,

\( MC_i \) = is the marginal contribution of asset class i

The total risk of the portfolio can also be determined as the sum of the MC of the individual asset classes. When there is N number of assets classes, it can be written as:

\[ \text{Total Risk} = MC_1 + MC_2 + \ldots + MC_N \]  

(2)

More generally when there are more than two assets, the MC of an asset can be determined by:

\[ MC_i = w_i \times \frac{\sum_{j=1}^{N} w_j \text{Cov}[R_i; R_j]}{\sigma^2[R_p]} \]  

(3)

To summarize when constructing a RP portfolio, the formula above is used to calculate the weight of individual asset class to the portfolio, so that the MC of all the individual asset classes are identical.
3.1.2 Naïve Risk Parity

The optimization is referred to as a “simple” risk parity because capital is allocated based solely on volatility, here referred to standard deviation, and not including covariances or correlations. This makes it even simpler to construct the portfolios, RP. This also entails that fewer parameters have to be estimated. The drawback is that allocated weights might be too small to asset classes, with low correlation to other asset classes, and therefore has a high diversification effect, and that allocations will be too high to asset classes with a high correlation to other asset classes, and therefore have less diversification effect.

3.1.3 Equally Weighted

This asset allocation strategy is the simplest one of them all as it solely bases its allocation of equal capital in all asset classes. It is sometimes also referred to as a 1/N portfolio. The clear advantage is that it entails that no parameters have to be estimated in order to allocate assets and rebalance the portfolio. The drawback is, that from a risk-return perspective investor risk allocating too much to asset classes, which have all from higher risk in form of volatility, to lower expected return and that the asset class mix is not optimal, since the correlations of the assets classes is not taken into account.

3.1.4 Mean-Variance

According the traditional portfolio theory of Markowitz (1952), one seeks to optimize the expected return of a portfolio at a certain given level of risk measured by the standard deviation. This also gives, that in order to find the optimal portfolio on the efficient frontier, the investor has to settle for either a certain level of risk or required return. When investing with the risk-free asset and a market portfolio the optimal portfolio is a combination of the risk-free asset and the market portfolio. The line that can be drawn between the two is called the capital allocation line (CAL). It should be noted that there is no unique optimal portfolio, opposite to the other portfolio optimization theories.

The tangent portfolio includes the highest reward-to-volatility. From the point of MV optimization, this is the most optimal type of portfolio and therefore expected to be the best performing portfolio. The
tangent portfolio is used as the chosen portfolio for MV optimization. The tangent portfolio is calculated by maximizing the following expression (Bodie, Kane and Marcus, 2011):

\[
Max_{w_i} S_p = \frac{E(r_p) - r_f}{\sigma_p}
\]  

Where,
\[\sum w_i = 1\]
- \(w_i\) is the weights of the assets
- \(S_p\) is the Sharpe Ratio of the portfolio
- \(E(r_p)\) is the expected return of the portfolio
- \(r_f\) is the risk-free rate
- \(\sigma_p\) is the standard deviation of the portfolio.

3.2 Leveraging the portfolios

The purpose of leveraging the portfolios is to match the volatility of MV for RP, NRP and EW. This implies leveraging those three portfolios to a level such that they equal the volatility of MV at future period of time. Since the volatility of MV is not known, investors would normally have to rely on estimations of this, but would most likely not meet the target volatility. Hereby would a comparison of risk and return across the portfolios be impossible at worst, inaccurate at best. Therefore the approach used by Anderson et al. (2012) is used. With this method leverage is applied ex-post of the period being measured, and hereby is it possible to determine the leverage needed to equal the volatility of RP, NRP and EW to MV. This of course constitutes a look-ahead bias, as information about the volatility of the returns of portfolio for the period analyzed would not be known in at the time of leveraging. It is though regarded as the lesser of evils.
The formula for leveraging the portfolios *ex-post* is:

\[ w_a \times R_p + (1 - w_a) \times (r_{\text{borrowing}}) \]  

(5)

A comparison on a monthly basis is also carried out. For this the leverage factor is calculated *ex post* the month. The purpose is to test the effect of keeping the methodology of levering the same, but changing the period for which is being calculated, on the return of the leveraged portfolios. This is expected to give insight into the risk that is expected to come with a changing level of leverage.

### 3.3 Investment universes and asset classes

Where part 2.3 defined and outlined the characteristics of asset classes, the purpose of this is to set the investable framework for the portfolios construction based on the four previously presented portfolio theories.

The first part of the portfolio construction is determining the asset classes to be included in the investment universes. The analysis consists of two different aspects that need to be accounted for when selecting the assets. All the indices that are representing the asset classes have to have time series with enough historical data for the analysis to have relevance. They also have to be able to be divided into sub portfolios, which are constructed in order to test the sensitivity to the included asset classes on the performance of risk parity. The sub portfolios have been constructed based on geographical basis, and whether or not the asset class can be categorized as traditional or alternative e.g. bonds vs. real estate.\(^9\) The framework on the subdivision of the 4 investment universes is partly based on other papers such as that of J.P. Morgan Chase and the model portfolio concept of Nordea. The papers provide the basis for

\(^9\) It will be the borrowing rate, when \((1-w_a) < 0\), and the placing rate e.g. risk free rate when \((1-w_a) > 0\)

\(^{10}\) As previously mentioned it is not fitting the model of an alternative asset (Anson), but for all means and purposes real estate is difficult at best, to include in a portfolio which is frequently rebalanced due to pricing and illiquidity. REITS can on the other hand easily be rebalanced, but also share many characteristics with the stock market rather than that of real estate.
the International investments universes and The Nordea Model Portfolio Concept provides the basis for Danish home bias\textsuperscript{11} investment universes.

This framework results in setting up four different investment universes, which are the basis for the combination of the asset classes. It is of course possible to combine all asset classes in all possible combinations, but it is found more relevant to focus on these four, which are regarded likely as to be how investors perceive their investment opportunities.

<table>
<thead>
<tr>
<th>Source: Own creation</th>
</tr>
</thead>
</table>

Table 3.2 – Overview of Investment Universes

<table>
<thead>
<tr>
<th>International excluding alternative assets</th>
<th>Home bias excluding alternative assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>International including alternative assets</td>
<td>Home bias including alternative assets</td>
</tr>
</tbody>
</table>

\textbf{3.4 Partial conclusion}

In this chapter the methodology for construction the portfolios and estimating the parameters for the optimization have been presented and discussed. The reasoning behind the choice of investment universes has also been presented.

\textsuperscript{11} Danish home bias will hereafter only be referred to as home bias
4 Data

4.1 Indices

There is no clear definition of what characterizes a good index. The prerequisite for the selection of the indices in this paper is that they objectively reflect the performance of the asset classes that the analysis is targeting. At the same time they need to be transparent so that the composition of the index is known, to ensure that it represents the relevant asset class, four the analysis.

All indices are denominated in local currency. Even though this paper has the perspective of a Danish institutional investor, at the same time the purpose of this paper is also to test the sensitivity to how an investor defines his or hers investment universe, this including which asset classes you include for example if it only consists of your domestic equities and bond markets or/and if it includes real estate, infrastructure and commodities etc.

If the indices were denominated in a hard currency most commonly USD, even though the underlying assets where denominated in for instant EUR, the performance evaluation might be misleading by the changing exchange rates currencies in between. It is without the scope of this paper to take in currency correlation and volatility, and it is therefore left out, and the portfolios construction is based on indices in local currency as mentioned above.

The returns are calculated for all indices. The returns are calculated by taking the first difference of the natural logarithm of the closing price index.

\[ R_t = \frac{\ln(P_t)}{\ln(P_{t-1})} \]  

(6)

All indices are total return in order to capture the performance excluding tax effects. In coherence with the reason for choosing the given period, the chosen indices have also been chosen based on that data was available as daily returns. By using daily returns makes it possible to compare volatility of the
equity market to the performance of the portfolios in the empirical study. This would otherwise not be possible. Using daily returns obviously also increase the amount of data points approximately 20 times, but since the analysis only applies eleven different asset classes, composed of thirteen indices altogether, the amount of data is still limited, so that it does not constitute a computational problem. Indices can be divided into three categories:

- Official
- Index which is extended using another index with and older inception
- Index which is a weighted average of two or more indices.

The official indices have been used in most cases excluding “Developed World Government Bonds” and “Investment Grade Corporate Bonds” These indices are composed of equal weights of, in both cases, two underlying indices. None of the indices used, have been extended using older indices, to limit the amount of sources leading to misinterpretation of the results of the empirical study. All asset classes which are used as building blocks for the investment universe are listed in the table 4.1:
Table 4.1 – Asset Classes and Corresponding Indices

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Equities</strong></td>
<td></td>
</tr>
<tr>
<td>Danish</td>
<td>MSCI Denmark</td>
</tr>
<tr>
<td>Global Developed Markets</td>
<td>MSCI World</td>
</tr>
<tr>
<td>Emerging Markets</td>
<td>MSCI EM</td>
</tr>
<tr>
<td><strong>Bonds</strong></td>
<td></td>
</tr>
<tr>
<td>Danish Government</td>
<td>Nordea Markets Gov. Bond Denmark</td>
</tr>
<tr>
<td>Developed Markets Gov. Bonds</td>
<td>50% EFFAS EU Gov&gt;1Yr &amp; 50% EFFAS US Gov All&gt;1Yr</td>
</tr>
<tr>
<td>Danish Mortgage</td>
<td>Nykredit Danish Mortgage Bond</td>
</tr>
<tr>
<td>Inflation Linked</td>
<td>Barclays World Inflation Linked Bonds</td>
</tr>
<tr>
<td>Investment Grade</td>
<td>50%Barclays US Credit &amp; 50% BOFA ML EMU Corp</td>
</tr>
<tr>
<td>Emerging Markets Bonds</td>
<td>J.P. Morgan EM Global Diversified Composite</td>
</tr>
<tr>
<td><strong>Alternative assets</strong></td>
<td></td>
</tr>
<tr>
<td>Real Estate</td>
<td>EPRA Developed Markets</td>
</tr>
<tr>
<td>Commodities</td>
<td>Dow-Jones-UBS Commodity</td>
</tr>
</tbody>
</table>

Source: Bloomberg, Datastream & own creation

4.2 Periods

It has been a goal to obtain return series back as far as possible, while still attaining a vast variety of true and representative indices of the different asset classes. Since one of the major research questions are based on the sensitivity of the number of asset classes, which has put a strong limitation on the length of the period. Finally the period of 2003-2013 e.g. eleven years was chosen, though the first year is only used for estimating volatility, correlations and mean for the subsequent periods. Even though a longer historical period is desirable, the geographical dispersion of the asset classes included in this paper, and the number of asset classes included in the paper, was regarded as more important, than increasing the length of the back tested period. It is regarded as an appropriate period since RP didn’t come into existence before 1996 as previously mentioned.
The optimal case would be to have been to have a period stretching as far back as for instant 1971. The background for this the increasing interest rates from 1971-1980 (Bloomberg). A back test of the performances of RP would be interesting since one of the major criticisms of the theory is whether or not it is capable to outperform traditional portfolios in a market with increasing interest rates. The period has been subdivided into an out-of-sample period consisting solely of 2003 and an in-sample period consisting of the remaining ten years from 2004 to 2013.

4.3 Key figures

For evaluating the investment funds constructed different key figures and performance measures are used, of which the most relevant are presented in this paragraph. The expected return denominates the return that an investment is expected to yield over a period of time on a yearly basis:

$$ E(R_p) = \sum_{i=1}^{n} w_i \times E(R_i) $$

(7)

Where,

- $E(R_p)$, is the expected return of the portfolio
- $w_i$, is the weight of the asset $i$
- $E(R_i)$, is the expected return on asset $i$.

The standard deviation is a measure for how much a stochastic variable, in this case return, is distributed around the mean. The standard deviation is a popular risk measure and is closely related to the normal distribution. The variance of a portfolio is given as the sum of the weighted variances of the assets added a term describe the covariance between the assets.
The variance of a portfolio consisting of multiple assets can be written as:

\[ \sigma^2(p) = \sum_{i=1}^{n} (w_i) \sigma^2(r_i) + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} w_i w_j \text{Cov}(r_i, r_j) \]  

(8)

Where, 

\( \sigma(r_i) \), is the standard deviation of asset i.

As long as returns are normally distributed, the standard deviation is a good measure of risk. As the name implies the covariance expresses how much two assets co varies. The covariance is written as:

\[ \text{Cov}(r_i, r_j) = \sigma_i \sigma_j \rho_{ij} \]  

(9)

Where, 

\( \rho_{ij} \), is the correlation between asset i and j

The correlation describes to which degree the movement of two assets are associated. The correlation coefficient can be between -1 and +1. -1 < negative correlation < 0 < positive correlation. If the value is 0 the assets are said to be uncorrelated.

The normal distribution is one of the most well-known probability distributions. Also known as a Gaussian distribution it is characterized by return being symmetrical distributed around the mean. If a time series of returns is normal distributed that makes it easier to calculate the risk by the standard deviation. The main drawback is prerequisite or assumption if you will that the returns are normal distributed. Several studies have shown that this is often far from the case.

Skewness describes the degree of asymmetry of the returns from the mean. With a skewness of 0, then the returns are symmetrical distributed around the mean, therefore they are normal distributed. If the skewness is negative then more returns lower than the mean and if the skewness is positive, then more returns are higher than the mean. This is particularly interesting when calculating the probability of obtaining returns higher than the mean (Bodie et al., 2011).
Kurtosis shows to which degrees the returns are distributed around the mean or in the tails. To put it in other words, it tells something about the probability that the returns lies far away from the mean. This is very interesting in order to evaluate the likelihood of extreme events. A normal distribution has a kurtosis of 3 (Bodie et al, 2011).

The Sharpe Ratio (SR) indicates the risk adjusted excess return which an investor receives. SR consists of the realized return deducted by the risk-free rate, divided by the standard deviation of the returns:

\[ SR = \frac{R_p - R_f}{\sigma_p} \]  

Where,
- \( R_p \) is the annualized average return of the portfolio, that is the mean \( \mu \).
- \( R_f \) is the risk-free rate
- \( \sigma_p \) is the annualized standard deviation of the portfolio

The advantage of SR is that it is easily calculated and that the vantage point is the excess return. Therefore the investment is rewarded if the return of it exceeds, what could be obtained by investing in the risk-free rate. The risk of the investment is expressed using by the standard deviation, which relates to its main disadvantage: the assumption of normal distributed returns, implying the standard deviation captures all risk.

Modified VaR is an expansion to the traditional parametric VaR. A drawback of VaR is that it assumes that returns are normal distributed, which is not assumed to be the case neither for the individual asset classes, nor the portfolios, and especially not the leveraged portfolios. The basis of MVaR is as mentioned above parametric VaR. The formula for parametric VaR can be written as:

\[ \text{VaR} = \mu - z_\alpha \sigma \]
Where:
\( \mu \) is the mean
\( \sigma \) is the standard deviation
\( z_\alpha \) is the alpha quintile of a standard normal distribution

As VaR assumes normality of returns it underestimates the potential risk that lies in negative skewness of the return distribution and fat tails.

MVaR makes up for this by adding and expansion to the key figure, and including both skewness and kurtosis in the formula. This is done by adding a Cornish-Fischer expansion. When the Cornish-Fischer expansion is included, the combined formula for MVaR can be written as:

\[
MVaR = \mu - \left( z_c + \frac{1}{6} (z_c^2 - 1)S + \frac{1}{24} (z_c^3 - 3z_c)K - \frac{1}{36} (2z_c^3 - 5z_c)S^2 \right) \sigma \tag{12}
\]

Where:
\( z_c \) is the critical value for the probability
\( S \) is skewness
\( K \) is kurtosis

Hereby there is the deviation of the portfolios from the normal distribution taken into account. MVaR uses an approximation from a starting point and works well as long as skewness and kurtosis are only deviating little to medium from the normal distribution. MVaR is less applicable when deviations are more extreme.

As it is with VaR, nor is MVaR transformed into different time periods This is due to that MVaR is the risk constituent of the calculation of the performance figure Modified Sharpe Ratio (MSR), which is done with the basis in the yearly mean, standard deviation and risk-free rate.
SR assumes normal distribution the risk that can be inherent in non-normal-distributed returns, which is a consequence of diverging skewness and kurtosis. MSR takes this into account, since it instead of using the standard deviation as risk parameter, uses MVaR. The formula can be written as:

$$MSR = \frac{R_p - R_f}{\text{MVaR}}$$

(13)

Maximum drawdown (MDD) is a risk measure, based on the maximum drawdown from a historical peak to a subsequent valley. MDD measures over the entire historical period, and therefore the period over which the MDD occurs can be different. MDD purely measures the magnitude by which and investor can experience a loss, before the investment starts climbing again. Graphically it can be illustrated as bellow:

![Figure 4.1 – Maximum Drawdown](source: Investexcel.net)

There are multiple rates to choose from as measures of the risk-free interest rate. The choices are somewhat more obvious when you only invest across a domestic investment universe, but the choices multiply exponentially when you invest across countries. The reason is that rates are different from country to country and depends on the exact term of the rate. The choice of rate can be widely discussed and can matter substantially. With the short term interest levels in many developed market countries close to zero, while in some emerging markets much higher, the choice of the risk-free rate
can greatly influence the performance. Moreover, it also influences portfolio constructing, since the risk-free rate is used for asset allocation in the tangent portfolio.

Even though the focus of this paper is from a Danish investor perspective, the US Treasury note 3 month has been chosen as the risk-free rate, since most of the asset classes are international, and it is regarded as more appropriate, than the Danish equivalent.

In general terms the formula can be written as follows:

\[
E_p(r) = w \times R_f + (1 - w) \times E(r_m)
\]  

One of the major assumptions in this equation is that one can invest in the risk-free rate, and also borrow at the risk-free rate. The latter is an unreasonable assumption. The lender will ask for a risk premium for lending money to an investor. The required premium for bearing the risk that the money will not be repaid in full. This premium will obviously vary depending on the creditworthiness of the borrower.

The London Interbank offered rate (Libor) is a reference rate often used by banks and other financial institutions, to set the interest rate level when issuing debt, which is when lending money out. The Libor rates are an average of the rates which several banks are willing issue unsecured lending facilities to other financial institutions. Since the borrowing will be done by an institutional investor in this paper, the Libor rate is regarded as a good proxy for the cost of financing when leveraging the portfolios.

It is important to differentiate between the placement rate and borrowing rate as they can differ quite substantially and thereby can affect the results quite much. In figure 4.2 the spread between Libor 3-month USD - US Treasury Note 3-month is shown.
When looking at the graph it clearly stand out that the spread is significant, and might possibly affect the outcome of a leveraged portfolio if it is not taken into account. The spike in October 2008 marks a spread of more than 450 basis points to the risk-free rate, indicating an increase in the risk premium required by lenders for lending money to other financial institutions. This was of course in the midst of the financial crisis and reoccurrence might not be likely in the near or midterm future, but it proves a good point of the risk when using leverage when creating portfolios.

The Jobson & Korkie-Test tests whether or not SR of two different securities or portfolios are statistically identical. The zero hypotheses is that there is no difference between SR of the two portfolios. Jobson & Korkie have noted that the statistical strength of the test becomes weaker the smaller the sample is. The test assumes normal distribution with the implications, which that entails.

Christoph Memmel (2003) has modified the formula in order to improve the comparison of the two Sharpe ratios. The formula can be written as:
$$Z = \frac{\sigma_2 \mu_1 - \sigma_1 \mu_2}{\sqrt{\theta}}$$  \hspace{1cm} (15)$$

Hereby $\theta$ is the asymptotic variance of the expression in the denominator:

$$\theta = \frac{1}{T} \left[ 2\sigma_1^2 \sigma_2^2 - 2\sigma_1 \sigma_2 \sigma_{1,2} + \frac{1}{2} \mu_1^2 \sigma_2^2 + \frac{1}{2} \mu_2^2 \sigma_1^2 - \frac{\mu_1 \mu_2}{\sigma_1 \sigma_2} \sigma_{1,2}^2 \right]$$  \hspace{1cm} (16)$$

Where,

$\sigma_1$ = standard deviation of portfolio 1

$\sigma_2$ = standard deviation of portfolio 2

$T$ = number of observations

$\sigma_{1,2}$ = covariance between portfolio 1 and portfolio 2

$\mu_1$ = mean of portfolio 1

$\mu_2$ = mean of portfolio 2

Positive Z-values indicate that portfolio 1 outperformed portfolio 2. The magnitude of the Z-value defines whether or not the result is significant, and therefore if the result can be assumed to be true or not. The Z-values follows a normal distribution and a 5% significance level equals a Z-value of 1.6449 and with a 1% - significance level a Z-value of 2.3263. Comparing SR of the portfolios is specifically interesting as it is the chosen performance measure in several of the analysis of RP.

One of the assumptions in the discussed portfolio optimizations is that returns are independent from each other. It has been shown that this is not always the case, for instance with financial data such as equity and bond returns.

Autocorrelation is when the return at time $t$ is correlated with its own previous returns. If the data is in fact auto correlated, an econometric model that uses these dependent values/auto correlated values as representative for future estimated values, might end up with incorrect conclusions. Data should first
be tested for autocorrelation, and if present, the data should be corrected for this. It should be noticed that dependency values in-between can be of different orders. The orders refer to the lag in periods of the dependencies, e.g. first order dependency refers to a dependency with the lag of one period \( t \) to \( t+1 \). As second order refers to a dependency which also includes the lag of \( t \) to \( t+2 \). The Ljung-Box test, tests on all lags whether or not there is significant autocorrelation:

\[
LB = n(n + 2) \sum_{j=1}^{h} \frac{\hat{\rho}_j^2}{n - j}
\]  

(17)

Where,

- \( n \), is the number of data points in the sample
- \( \hat{\rho}_j \), is the autocorrelation at lag \( j \)
- \( j \), is the lag being tested
- \( h \), is the number of lags being tested

The Ljung-Box test follows a Chi-square distribution with \( h \) degrees of freedom, where the zero hypotheses is that data is not auto correlated. Therefore the zero hypotheses can be rejected if the LB-Q statistic exceeds the critical value. In generalized terms that can be written as follows.

\[
LB > \chi^2_{1-\alpha, h}
\]  

(18)

Where,

- \( \alpha \) is the confidence level
- \( h \), is the degrees of freedom in the \( \chi^2 \) distribution

If returns show significance of autocorrelation the data needs to be corrected. Brooks and Kat (2002), and “JP Morgan Asset Management” are both applying a method originally proposed by Geltner (1991, 1993) where the smoothed return \( r_t^* \) at time \( t \) can be seen as a weighted average of the true return \( r_t \) at time \( t \) and the smoothed return \( r_{t-1}^* \) at time \( t-1 \).
The above expression can be rewritten to:

$$r_t^* = \rho r_t + r_{t-1}^* - \rho r_{t-1}^*$$  \hspace{1cm} (19)$$

The above expression can be rewritten to:

$$r_t = \frac{r_t^* - \rho r_{t-1}^*}{1 - \rho}$$  \hspace{1cm} (20)$$

Where, 
$$\rho$$, is the autocorrelation at first lag.

By applying the correction to the smoothed data, the goal is eliminate any effects of autocorrelation at the first lag. Since autocorrelation at second lag to some extend can be attributed to autocorrelation of the first lag, this correction is expected to eliminate most autocorrelation. Results will most likely show that the majority of the indices exhibit an increase in the standard deviation after correction. The purpose of correcting for serial correlation is to increase the predictability capabilities of the returns.

A Jarque-Bera Test provides an indication to which degree the distribution of returns deviates from a normal distribution, thereby taking the skewness and kurtosis into account. In order for a return series to be normal distributed it has to have a skewness equal of 0 and a kurtosis of 3. The purpose of the test is to test to which degree the returns of the portfolios are normal distributed and therefore, to which degree the standard deviation can be said to capture the risk of the portfolios in particular risk parity.

The formula is written as follows:

$$JB = n \frac{n}{6} (S^2 + \frac{(K-3)^2}{4})$$  \hspace{1cm} (21)$$

Where, 
$$n$$ is the number of observations in the period 
$$S$$ is skewness 
$$K$$ is kurtosis
The zero hypothesis of Jarque-Bera Test is that the returns are normally distributed. For the zero-hypothesis to be discarded the result of the Jarque-Bera Test has to exceed a critical value. One of the drawbacks of the Jarque-Bera Test is its sensitivity to the length of the analyzed period. The longer the period, the more correct the value is.

4.4 Descriptive statistics

Before constructing the portfolios, it is worth noticing the performance and statistical moments of the individual asset classes as previously defined. The average yearly return is highest for all equity asset classes, both Danish, developed and emerging markets. But also emerging markets High yield bonds have performed well over the period at par with developed markets. REITs have performed slightly worse than the equities and the EM HY Bonds. What is really worth noticing in respect to the return is the risk depicted by the standard deviation in respect to the EM HY Bonds. They have the second lowest standard deviation only slightly higher than DK MTG. Considering that one would expect that EM HY bonds are riskier than DK MTG, this result is quite surprising. With interest rate generally falling over the analyzed period DK MTG has had if not optimal conditions, then good conditions for appreciating over the period. They have at the same time only had an annual mean of 0.04, which is half that of EM HY Bonds. Obviously that also transfers to the magnitude of the SR.

The SR of EM HY bonds are clearly dominating the DK MTG. Actually the EM HY Bonds are dominating all asset classes by far. CMD have the lowest SR with 0.06 compared to the SR of EM HY Bonds of 2.62. DK MTG has the second highest SR with 1.08. Keeping these results in mind, all other things equal one would expect that specifically the mean-variance optimized portfolio will tend to invest a very large proportion of the capital in especially EM HY Bonds but also to some extend in Danish mortgage bonds and IG bonds which with a SR of 0.96 comes third in respect to relative performance based on SR.

One of the major assumptions of the SR is that the standard deviation is an accurate measure of risk. The standard deviation itself assumes that data is normal distributed, if it is to be used as a risk measure. For a return series to be normal distributed the kurtosis must be three and the skewness 0. If we again highlight EM HY Bonds both the skewness and kurtosis departs from the values of a normal
distribution. With a skewness of -0.46, the returns of the EM HY bonds tend to be more negative than if they were normal distributed. In fact the only assets classes which do not exhibit negative skewness are Developed markets government bonds and inflation linked bonds. All other asset classes exhibit negative skewness.

When looking at the kurtosis DK MTG now have the highest figure, but second come EM HY Bonds.

At first sight the asset classes, DK EQT, WLD EQT, EM EQT, DK MTG, EM HY Bonds all clearly have excess kurtosis to such a degree than one could assume that that returns are in fact not normal distributed. By the looks of it they all have fat tailed distributions, meaning that a large proportion of returns more extreme both positively and negatively than if they were normally distributed.

<table>
<thead>
<tr>
<th>Asset Classes</th>
<th>DK EQT</th>
<th>WLD EQT</th>
<th>EM EQT</th>
<th>DK GOV</th>
<th>USEUGOV</th>
<th>DK MTG</th>
<th>INFLLINK</th>
<th>EM HY Bonds</th>
<th>IG CORP</th>
<th>REIT’s</th>
<th>CMD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>11.39%</td>
<td>7.86%</td>
<td>10.07%</td>
<td>4.44%</td>
<td>4.03%</td>
<td>4.31%</td>
<td>5.76%</td>
<td>8.30%</td>
<td>4.93%</td>
<td>7.26%</td>
<td>2.60%</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.20</td>
<td>0.16</td>
<td>0.17</td>
<td>0.04</td>
<td>0.04</td>
<td>0.03</td>
<td>0.07</td>
<td>0.03</td>
<td>0.04</td>
<td>0.20</td>
<td>0.18</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.32</td>
<td>-0.43</td>
<td>-0.39</td>
<td>-0.09</td>
<td>-0.03</td>
<td>-0.18</td>
<td>0.18</td>
<td>-0.46</td>
<td>-0.38</td>
<td>-0.38</td>
<td>-0.26</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>8.00</td>
<td>9.53</td>
<td>8.10</td>
<td>3.62</td>
<td>2.23</td>
<td>18.31</td>
<td>3.68</td>
<td>14.77</td>
<td>2.19</td>
<td>5.19</td>
<td>2.52</td>
</tr>
<tr>
<td>SR</td>
<td>0.49</td>
<td>0.40</td>
<td>0.51</td>
<td>0.68</td>
<td>0.68</td>
<td>1.08</td>
<td>0.58</td>
<td>2.62</td>
<td>0.96</td>
<td>0.28</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Source: Own creation

4.5 Correction of data

The above analysis is based on raw data. As mentioned previously in the section of autocorrelation, if analysis is based on data which exhibits signs of autocorrelation, any conclusions if data is not corrected for autocorrelation might lead to wrong conclusions. All data is therefore tested for autocorrelation. It is expected to find signs autocorrelations especially for the asset classes which is buildup of assets taken from all over the world such as MSCI World. This is partly due to that fact that equity markets influence each other. If the Japanese market is rising, that influences the European and again the American which again influence the Japanese. Therefore the individual return cannot be said
to be independent of the previous days return. It is expected that it is first and foremost the autocorrelation of t to t-1 that is most severe, due to the reason outlined above.

To test for autocorrelation the Ljung-Box Test is applied. It is applied on a lag of up to five which accounts for a week. Of course one could increase the amount of lags tested upon, but a week is expected to capture the appropriate autocorrelation of the assets, since you have all five weekdays included. Below are the statistical properties of the Ljung-Box Test. The zero-hypothesis is that there is no autocorrelation. We reject the zero-hypothesis, if the at a 95% confidence interval if the Qlb-statistic is > 11.071 and at a 99% confidence interval if the Qlb-statistic is > 15.086. In the below table you can see that only developed market government bonds and commodities, the zero hypothesis can’t be rejected, which means that those two assets classes are the only ones that do not show signs of autocorrelation. All other things equal, the Ljung-Box Test indicates that they are in fact autocorrelated, entailing that conclusions made on the basis of those could be faulty. Therefore the data need to be corrected for autocorrelation and the analysis performed again.

To correct the data correction method which only corrects for first order autocorrelation is used. Since first order autocorrelation is the one that is expected to be most severe, this correction is expected to be sufficient to correct. After the correction, the corrected returns are tested again to see whether or not they are depicting signs of autocorrelation.

In the below table are the results of the test for autocorrelation. Nine out of eleven asset classes show signs of autocorrelation both at a 95 and 99% confidence interval. The only two assets not showing signs of autocorrelation are developed markets government bonds and Commodities. It was expected that most asset classes would show signs of autocorrelation. EM HY Bonds is the asset class which shows the highest degree of autocorrelation and after that comes Danish mortgage bonds. Also EM EQT show very high signs of autocorrelation. Some researchers have found signs that the autocorrelation increases with an increased holding by institutional investors. Though it is without the scope of this paper to investigate the source of the autocorrelation, it might explain some of the test results.
Table 4.3 – Ljung-Box Test, Test for Autocorrelation “Smooted” Data

<table>
<thead>
<tr>
<th>Asset classes</th>
<th>DK Eqt</th>
<th>WLD Eqt</th>
<th>EM Eqt</th>
<th>DK GOV</th>
<th>USEU GOV</th>
<th>DK MTG</th>
<th>INFLLINK</th>
<th>EM HY Bonds</th>
<th>IG CORP</th>
<th>REIT’s</th>
<th>CMD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qlb</td>
<td>20,37</td>
<td>39,71</td>
<td>140,39</td>
<td>18,29</td>
<td>8,65</td>
<td>170,43</td>
<td>24,65</td>
<td>413,33</td>
<td>35,37</td>
<td>18,38</td>
<td>7,50</td>
</tr>
<tr>
<td>n</td>
<td>2870</td>
<td>2870</td>
<td>2870</td>
<td>2870</td>
<td>2870</td>
<td>2870</td>
<td>2870</td>
<td>2870</td>
<td>2870</td>
<td>2870</td>
<td>2870</td>
</tr>
<tr>
<td>a1</td>
<td>0.0351</td>
<td>0.0915</td>
<td>0.2184</td>
<td>0.0667</td>
<td>0.0387</td>
<td>0.2296</td>
<td>0.0880</td>
<td>0.3228</td>
<td>0.0881</td>
<td>0.0645</td>
<td>-0.0273</td>
</tr>
<tr>
<td>a2</td>
<td>-0.0300</td>
<td>-0.0595</td>
<td>0.0081</td>
<td>-0.0315</td>
<td>-0.0282</td>
<td>0.0665</td>
<td>-0.0121</td>
<td>0.1363</td>
<td>0.0214</td>
<td>0.0193</td>
<td>0.0179</td>
</tr>
<tr>
<td>a3</td>
<td>-0.0242</td>
<td>-0.0042</td>
<td>-0.0194</td>
<td>-0.0300</td>
<td>0.0004</td>
<td>0.0462</td>
<td>-0.0138</td>
<td>0.1022</td>
<td>0.0346</td>
<td>-0.0122</td>
<td>0.0086</td>
</tr>
<tr>
<td>a4</td>
<td>0.0530</td>
<td>0.0163</td>
<td>-0.0262</td>
<td>0.0053</td>
<td>0.0211</td>
<td>0.0046</td>
<td>0.0198</td>
<td>0.0880</td>
<td>0.0440</td>
<td>-0.0296</td>
<td>0.0343</td>
</tr>
<tr>
<td>a5</td>
<td>-0.0395</td>
<td>-0.0403</td>
<td>-0.0074</td>
<td>0.0008</td>
<td>-0.0165</td>
<td>-0.0049</td>
<td>0.0102</td>
<td>0.0356</td>
<td>0.0308</td>
<td>-0.0289</td>
<td>-0.0170</td>
</tr>
</tbody>
</table>

Autocorrelation YES/NO

<table>
<thead>
<tr>
<th></th>
<th>Conf. 95 %</th>
<th>Yes</th>
<th>Yes</th>
<th>Yes</th>
<th>No</th>
<th>Yes</th>
<th>Yes</th>
<th>Yes</th>
<th>Yes</th>
<th>Yes</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conf. 99 %</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

Source: Own creation

In order to be able to draw more accurate conclusions based on analysis of the data the correction for the first order autocorrelation is carried out. After performing the corrections, then the Ljung-Box test is again applied to the corrected data. The results can be seen in table 4.4.

Table 4.4 – Ljung-Box Test, Test for Autocorrelation “Unsmooted” Data

<table>
<thead>
<tr>
<th>Asset Classes</th>
<th>DK Eqt</th>
<th>WLD Eqt</th>
<th>EM Eqt</th>
<th>DK GOV</th>
<th>USEU GOV</th>
<th>DK MTG</th>
<th>INFLLINK</th>
<th>EM HY Bonds</th>
<th>IG CORP</th>
<th>REIT’s</th>
<th>CMD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qlb</td>
<td>17,80</td>
<td>19,35</td>
<td>6,73</td>
<td>5,89</td>
<td>4,67</td>
<td>4,21</td>
<td>3,09</td>
<td>18,91</td>
<td>9,31</td>
<td>5,07</td>
<td>5,27</td>
</tr>
<tr>
<td>n</td>
<td>2869</td>
<td>2869</td>
<td>2869</td>
<td>2869</td>
<td>2869</td>
<td>2869</td>
<td>2869</td>
<td>2869</td>
<td>2869</td>
<td>2869</td>
<td>2869</td>
</tr>
<tr>
<td>a1</td>
<td>0.0011</td>
<td>0.0063</td>
<td>0.0091</td>
<td>0.0024</td>
<td>0.0011</td>
<td>-0.0033</td>
<td>0.0017</td>
<td>-0.0115</td>
<td>-0.0013</td>
<td>-0.0010</td>
<td>0.0005</td>
</tr>
<tr>
<td>a2</td>
<td>-0.0305</td>
<td>-0.0686</td>
<td>-0.0368</td>
<td>-0.0343</td>
<td>-0.0298</td>
<td>0.0070</td>
<td>-0.0189</td>
<td>0.0148</td>
<td>0.0108</td>
<td>0.0161</td>
<td>0.0174</td>
</tr>
<tr>
<td>a3</td>
<td>-0.0250</td>
<td>-0.0002</td>
<td>-0.0172</td>
<td>-0.0285</td>
<td>0.0006</td>
<td>0.0342</td>
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<td>0.0452</td>
<td>0.0294</td>
<td>-0.0116</td>
<td>0.0100</td>
</tr>
<tr>
<td>a4</td>
<td>0.0554</td>
<td>0.0207</td>
<td>-0.0227</td>
<td>0.0074</td>
<td>0.0218</td>
<td>-0.0049</td>
<td>0.0204</td>
<td>0.0523</td>
<td>0.0390</td>
<td>-0.0271</td>
<td>0.0342</td>
</tr>
<tr>
<td>a5</td>
<td>-0.0396</td>
<td>-0.0395</td>
<td>0.0097</td>
<td>-0.0002</td>
<td>-0.0162</td>
<td>-0.0146</td>
<td>0.0090</td>
<td>0.0381</td>
<td>0.0272</td>
<td>-0.0251</td>
<td>-0.0162</td>
</tr>
</tbody>
</table>

Autocorrelation YES/NO

<table>
<thead>
<tr>
<th></th>
<th>Conf. 95 %</th>
<th>Yes</th>
<th>Yes</th>
<th>No</th>
<th>No</th>
<th>No</th>
<th>No</th>
<th>Yes</th>
<th>No</th>
<th>No</th>
<th>No</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conf. 99 %</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Source: Own creation
After the correction for the first order autocorrelation, only three out of the eleven asset classes exhibit signs of autocorrelation. This means the correction has successfully diminished the autocorrelation in six out of nine asset classes that originally exhibited autocorrelation. Even though it was expected that correcting only for first order autocorrelation, would diminish it on all asset classes it is still an acceptable result. The three asset classes that are still exhibiting autocorrelation are DK EQT, Developed market EQT and EM HY bonds. Of those three DK EQT stands out, as it is a local market and is not comprised out assets situated in different time zones. Another result is slightly puzzling, and that is that fact EM HY bonds are exhibiting signs of autocorrelation, when EM EQT is not. They have similar properties in the fact that the bonds originate from countries all over the world, generally speaking the same as EM EQT.

One of the issues about the Ljung-Box Test is that it test across the different lags and gives a single number to indicate where or not there is autocorrelation on h lags. It does not say anything about which lags are the most or least autocorrelation. In order to investigate what it takes for the corrected data to no longer show significant signs or autocorrelation, the Ljung-Box Test is performed again but with intrinsic fewer lags to find a break even if you will, of where the asset classes no longer show any sign of autocorrelation.

To start off with the fifth lag is ignored, so that the Ljung-Box test is only performed at four lags together. Here we already obtain a desirable effect. The three asset classes, DK EQT, WLD EQT and EM HY Bonds that were previously depicting signs of autocorrelation, are now all rejecting the zero hypotheses at a 99% confidence interval, indicating that at this level they are not auto correlated. This again means that when testing for autocorrelation at four lags and at a 99% confidence interval, all assets rejects the zero-hypothesis, indicating that none of them are auto correlated after the correcting for first order autocorrelation.

The next step is to move down to only testing on three lags. When testing on only three lags, only WLD EQT shows signs of autocorrelation, and only on a 95% confidence interval. When moving from only testing on three and not four lags, both DK EQT and EM HY bonds no longer shows signs of
autocorrelation neither at a 99% confidence interval nor a 95% confidence interval. WLD EQT does not stop showing signs of autocorrelation until the Ljung-Box test is only performed on one lag. And this was of course the lag at which the correction for first order autocorrelation was performed. This indicate that WLD EQT are heavily auto correlated on not only the first lag but also on the second, since it was not until that lag was removed from the test, that there was a significant drop in the Q-statistics. At the fifth lag there was a somewhat significant drop in the Q-statistics. In all it makes sense that WLD EQT is the most auto correlated asset class.

Table 4.5 – Descriptive Statistics for “Unsmoothed” Data

<table>
<thead>
<tr>
<th>Asset Classes</th>
<th>DK EQT</th>
<th>WLD EQT</th>
<th>EM EQT</th>
<th>DK GOV</th>
<th>USEUGOV</th>
<th>DK MTG</th>
<th>ENFLINK</th>
<th>EM HY Bonds</th>
<th>IG CORP</th>
<th>REIT's</th>
<th>CMD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>11.39%</td>
<td>7.87%</td>
<td>10.07%</td>
<td>4.45%</td>
<td>4.03%</td>
<td>4.31%</td>
<td>5.76%</td>
<td>8.30%</td>
<td>4.93%</td>
<td>7.26%</td>
<td>2.60%</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.21</td>
<td>0.17</td>
<td>0.21</td>
<td>0.05</td>
<td>0.04</td>
<td>0.03</td>
<td>0.08</td>
<td>0.04</td>
<td>0.04</td>
<td>0.22</td>
<td>0.17</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.31</td>
<td>-0.32</td>
<td>-0.17</td>
<td>-0.09</td>
<td>0.04</td>
<td>-0.31</td>
<td>0.17</td>
<td>-0.42</td>
<td>-0.36</td>
<td>-0.33</td>
<td>-0.27</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>8.15</td>
<td>10.06</td>
<td>8.11</td>
<td>3.46</td>
<td>2.22</td>
<td>16.73</td>
<td>3.34</td>
<td>14.84</td>
<td>2.17</td>
<td>5.17</td>
<td>2.53</td>
</tr>
<tr>
<td>SR</td>
<td>0.47</td>
<td>0.36</td>
<td>0.41</td>
<td>0.63</td>
<td>0.65</td>
<td>0.86</td>
<td>0.53</td>
<td>1.87</td>
<td>0.88</td>
<td>0.26</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Source: Own creation

After correcting the data for autocorrelation it was expected to see the means of the asset classes more or less unchanged, where as it was expected to see the standard deviation increase. As it can be seen in the table above, all means are unchanged on two decimal places. This confirms the expectation that the correction should at most have a minor effect on the mean.

At the same time it was expected that the standard deviation would increase across the asset classes. This was to some extent also confirmed. All asset classes show an increase in the standard deviation except for CMD, which showed a slight decrease of a factor 0.97. Of those asset classes which exhibited the highest increase were EM HY Bonds and with a factor 1.40 but also DK MTG with a factor 1.26 and EM EQT with a factor 1.25 showed steep increases in the standard deviations. Though in average did the standard deviation increased by a factor 1.13.
When the standard deviation increase, then the SR decrease due to the math inherent in the performance measure. Therefor it is also expected that SR of especially EM HY Bonds, DK MTG and EM EQT will decrease more than the others. Opposite will the SR of CMD increase. When only the SR is used as performance measure, then EM HY bonds is by far the best performing asset class. This is due to both a high end mean but one of the lowest standard deviations.

### 4.6 Partial Conclusion

In chapter 4 it was tested that most asset classes showed sign of autocorrelation at 5 lags. After correcting for this, volatility increased for all but one asset class. The difference in SR between the asset classes, was notable, and is expected to have an influence on which asset classes have allocated most wealth for the MV portfolio, due to the methodology in which the MV tangent portfolio is calculated.
5 Empirical results

5.1 Performance based on SR

It turns out that the performance ranking of the portfolios, if they are calculated based on the uncorrected returns is remarkably different from the performance, when the portfolios are calculated based on the corrected data. This was also expected.

For the “uncorrected return portfolios” MV performs best in three out of four investment universes. Only for “Home Bias excl. AA” does NRP perform better, but only with a SR of 1.69 to 1.65, which cannot be regarded as much. This does not fit well with the expectation that RP should outperform at least MV. It does however outperform EW. EW is that which in methodology mostly resembles that of a 60/40 portfolio which is used as a benchmark portfolio in several of the existing analysis of RP. This is in line with the results of other analysis on the topic, that RP outperforms fixed capital weighted portfolios. In the below table, all investment approaches have been ranked based on SR and for each individual investment universe.

Table 5.1 – Portfolio Performance Ranking

<table>
<thead>
<tr>
<th>Investment Universe</th>
<th>HB incl. AA</th>
<th>HB excl. AA</th>
<th>Int. incl. AA</th>
<th>Int. excl. AA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Parity</td>
<td>3 = 1.50</td>
<td>3 = 1.62</td>
<td>2 = 1.45</td>
<td>3 = 1.55</td>
</tr>
<tr>
<td>Naïve Risk Parity</td>
<td>2 = 1.52</td>
<td>1 = 1.69</td>
<td>3 = 1.35</td>
<td>2 = 1.57</td>
</tr>
<tr>
<td>Equally Weighted</td>
<td>4 = 0.61</td>
<td>4 = 0.86</td>
<td>4 = 0.54</td>
<td>4 = 0.83</td>
</tr>
<tr>
<td>Mean-Variance</td>
<td>1 = 1.64</td>
<td>2 = 1.65</td>
<td>1 = 1.79</td>
<td>1 = 1.71</td>
</tr>
</tbody>
</table>

Source: Own creation
Average points where 4 is the best and 16 the worst.

- Risk parity: 11
- Naïve risk parity: 8
- Equally weighted: 16
- Mean-variance: 5

As previously tested autocorrelation of asset classes tend to make returns look less volatile than they, are if you correct for the serial correlation. Since the historic performance in itself is not interesting for an investor but rather the future performance, it adds value in that perspective to correct the returns for serial correlation in order to get a better estimate of future performance on the investments.

In the below table the same performance ranking has been made for the investment approaches for the within the four different investment universes, but this time calculations are based on the returns which are corrected for serial correlations.

<table>
<thead>
<tr>
<th></th>
<th>HB excl. AA</th>
<th></th>
<th></th>
<th></th>
<th>HB incl. AA</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RP</td>
<td>NRP</td>
<td>EW</td>
<td>MV</td>
<td>RP</td>
<td>NRP</td>
<td>EW</td>
<td>MV</td>
</tr>
<tr>
<td>Mean</td>
<td>5.49%</td>
<td>5.55%</td>
<td>6.19%</td>
<td>6.95%</td>
<td>Mean</td>
<td>5.57%</td>
<td>5.55%</td>
<td>5.72%</td>
</tr>
<tr>
<td>Volatility</td>
<td>2.80%</td>
<td>3.77%</td>
<td>6.04%</td>
<td>4.21%</td>
<td>Volatility</td>
<td>2.95%</td>
<td>3.08%</td>
<td>7.36%</td>
</tr>
<tr>
<td>SR</td>
<td>1.4</td>
<td>1.43</td>
<td>0.76</td>
<td>1.28</td>
<td>SR</td>
<td>1.35</td>
<td>1.29</td>
<td>0.56</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Int. excl. AA</th>
<th></th>
<th></th>
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<th>Int. incl. AA</th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RP</td>
<td>NRP</td>
<td>EW</td>
<td>MV</td>
<td>RP</td>
<td>NRP</td>
<td>EW</td>
<td>MV</td>
</tr>
<tr>
<td>Mean</td>
<td>5.49%</td>
<td>5.55%</td>
<td>6.19%</td>
<td>6.95%</td>
<td>Mean</td>
<td>5.84%</td>
<td>5.96%</td>
<td>5.45%</td>
</tr>
<tr>
<td>Volatility</td>
<td>3.44%</td>
<td>3.42%</td>
<td>6.53%</td>
<td>4.38%</td>
<td>Volatility</td>
<td>3.71%</td>
<td>3.93%</td>
<td>8.27%</td>
</tr>
<tr>
<td>SR</td>
<td>1.26</td>
<td>1.28</td>
<td>0.69</td>
<td>1.35</td>
<td>SR</td>
<td>1.15</td>
<td>1.11</td>
<td>0.47</td>
</tr>
</tbody>
</table>

*Source: Own creation*
Average ranking where 4 is the best and 16 the worst.

- Risk parity: 8
- Naïve risk parity: 8
- Equally weighted: 16
- Mean-variance: 8

The difference in the performance ranking from the calculations made on the uncorrected data to the corrected data is notable. Where MV before was outperforming all other strategies in three out of four cases, the RP and NRP now outperforms MV for the HB incl. AA and HB excl. AA, where MV still outperforms RP and NRP in Int. incl. AA Int. excl. AA

Based on the ranking results are that, performance highly relies on investors definition of the investment universe, and that RP, NRP and MV are equally well performing. EW is simultaneously the worst performing irrespective to investor’s definition of the investment universe. So far the conclusion is: Do not apply the simplistic equal weight approach to your investments!

To determine whether or not the SR’s of the different portfolios are in fact different the Jobson-Korkie test is applied for each of the investment universes. To repeat jobson-Korkie test results in Z-values which follow a normal distribution and a 5% significance level equals a Z-value of 1.6449 and with a 1%-significance level a Z-value of 2.3263. The below table shows whether or not the SR can be said to be statistically different or equal based on the Jobson-Korkie Test both at a 5% confidence level and at a 1% confidence level.
Table 5.3 – Test for Equal SR, Jobson-Korkie Test – Unleveraged Portfolios

**Jobson Korkie at a 5 % confidence level**

<table>
<thead>
<tr>
<th></th>
<th>HB excl. AA</th>
<th></th>
<th></th>
<th>HB incl. AA</th>
<th></th>
<th></th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NRP</td>
<td>RP</td>
<td>Equal</td>
<td>Different</td>
<td>Different</td>
<td>RP</td>
<td>Different</td>
</tr>
<tr>
<td>NRP</td>
<td>NRP</td>
<td>X</td>
<td>Different</td>
<td>Different</td>
<td>NRP</td>
<td>X</td>
</tr>
<tr>
<td>EW</td>
<td>EW</td>
<td>X</td>
<td>X</td>
<td>Different</td>
<td>EW</td>
<td>X</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Int. excl. AA</th>
<th></th>
<th></th>
<th>Int. incl. AA</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>NRP</td>
<td>RP</td>
<td>Equal</td>
<td>Different</td>
<td>Different</td>
<td>RP</td>
<td>Equal</td>
</tr>
<tr>
<td>NRP</td>
<td>NRP</td>
<td>X</td>
<td>Different</td>
<td>Different</td>
<td>NRP</td>
<td>X</td>
</tr>
<tr>
<td>EW</td>
<td>EW</td>
<td>X</td>
<td>X</td>
<td>Different</td>
<td>EW</td>
<td>X</td>
</tr>
</tbody>
</table>

**Jobson Korkie at a 1 % confidence level**

<table>
<thead>
<tr>
<th></th>
<th>HB excl. AA</th>
<th></th>
<th></th>
<th>HB incl. AA</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NRP</td>
<td>RP</td>
<td>Equal</td>
<td>Different</td>
<td>Different</td>
<td>RP</td>
<td>Equal</td>
</tr>
<tr>
<td>NRP</td>
<td>NRP</td>
<td>X</td>
<td>Different</td>
<td>Equal</td>
<td>NRP</td>
<td>X</td>
</tr>
<tr>
<td>EW</td>
<td>EW</td>
<td>X</td>
<td>X</td>
<td>Different</td>
<td>EW</td>
<td>X</td>
</tr>
</tbody>
</table>

Source: Own creation
As it can be seen in table 5.3 all RP and NRP SRs cannot be said to significantly different from each other at a 1% confidence level. At a 5% significance level RP and NRP can be said to be significantly different in the HB incl. AA investment universe.

For NRP and MV in the investment universes HB incl. AA and Int. excl. AA, they are significantly different at a 5% confidence level, both at a 1%-level they are significantly equal.

There seems to be logic in the fact that RP and NRP tend to have the same SR, since both investment approaches intend to equal the risk contribution from all assets classes even though the methodology for achieving this is different. On the other hand there seem to be no immediate logical explanation for the equality of the SR of the NRP and MV.

All the portfolios have been leveraged using the leverage methodology as described in chapter 3. The portfolios RP NRP and EW have been leveraged to that the volatility equal that of MV, which stay unleveraged.

| Table 5.4 - Performance, risk & returns figures of the leveraged portfolios |
|---------------------------|-------------------|
|                            | HB excl. AA       | HB incl. AA       |
|                            | RP    | NRP   | EW    | MV    | RP    | NRP   | EW    | MV    |
| Mean                       | 8.27% | 8.43% | 4.32% | 6.95% | 9.15% | 8.73% | 4.85% | 7.47% |
| Volatility                 | 4.21% | 4.21% | 4.21% | 4.21% | 4.85% | 4.85% | 4.85% | 4.85% |
| SR                         | 1.59  | 1.63  | 0.65  | 1.28  | 1.56  | 1.48  | 0.45  | 1.22  |
|                            | Int. excl. AA    |                  |
|                            | RP    | NRP   | EW    | MV    |
| Mean                       | 7.52% | 7.64% | 4.07% | 6.95% |
| Volatility                 | 4.38% | 4.38% | 4.38% | 4.38% |
| SR                         | 1.36  | 1.39  | 0.59  | 1.35  |
|                            | Int. incl. AA   |                  |
|                            | RP    | NRP   | EW    | MV    |
| Mean                       | 7.92% | 7.61% | 3.31% | 8.47% |
| Volatility                 | 5.03% | 5.03% | 5.03% | 5.03% |
| SR                         | 1.26  | 1.20  | 0.34  | 1.37  |

Source: Own creation
Average ranking where 4 is the best and 16 the worst.
- Risk parity: 7
- Naïve risk parity: 7
- Equally weighted: 16
- Mean-variance: 10

RP performs best of all portfolios for HB incl. AA, but also comes a close second to NRP for HB excl. AA. MV on the other hand performs best for International incl. AA. In the case of Int. excl. AA the RP NRP and MV are performing remarkably similar to each other. Once again the simplistic EW portfolio performs worst of all.

RP NRP and MV are again remarkably similar to one another, but with MV slightly outperforming RP and NRP. Obviously EW is performing really poorly based on the SR. Again it is interesting whether or not the SR of the four investment approaches can be said to be significantly different from one another.

The test shows that RP and NRP are not significantly different from each other, except for HB incl. AA at the 5% confidence level. This indicates that in reality it has none or little effect if the optimization accounts for correlation or not. This result is somewhat surprising. For Int. excl. AA both at a 1% and 5% confidence level SR of RP, NRP and MV are all the same, meaning that it has no significance which of the three portfolios he chooses.

When evaluating the average performance across the four investment universes using the ranking of SR, then RP and NRP have in average outperformed both MV and EW, with the latter being the far worst performing in all cases. This means that if an investor has no clear definition of his investment universe he is better off choosing either RP or NRP. If Investor does have a defined investment universe, then the conclusion depends on which one he chooses.
Table 5.5 - Test for equal SR, Jobson-Korkie Test – Leveraged Portfolios

**Jobson Korkie at a 5 % confidence level**

<table>
<thead>
<tr>
<th></th>
<th>HB excl. AA</th>
<th></th>
<th>HB incl. AA</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRP</td>
<td>RP</td>
<td>NRP</td>
<td>RP</td>
</tr>
<tr>
<td></td>
<td>Equal</td>
<td>X</td>
<td>Equal</td>
</tr>
<tr>
<td></td>
<td>Different</td>
<td>Different</td>
<td>Different</td>
</tr>
<tr>
<td></td>
<td>Different</td>
<td>Different</td>
<td>Different</td>
</tr>
<tr>
<td>EW</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Int. excl. AA</th>
<th></th>
<th>Int. incl. AA</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRP</td>
<td>RP</td>
<td>NRP</td>
<td>RP</td>
</tr>
<tr>
<td></td>
<td>Equal</td>
<td>X</td>
<td>Equal</td>
</tr>
<tr>
<td></td>
<td>Different</td>
<td>Different</td>
<td>Different</td>
</tr>
<tr>
<td></td>
<td>Equal</td>
<td>Equal</td>
<td>Different</td>
</tr>
<tr>
<td>EW</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

**Jobson Korkie at a 1 % confidence level**

<table>
<thead>
<tr>
<th></th>
<th>HB excl. AA</th>
<th></th>
<th>HB incl. AA</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRP</td>
<td>RP</td>
<td>NRP</td>
<td>RP</td>
</tr>
<tr>
<td></td>
<td>Equal</td>
<td>X</td>
<td>Equal</td>
</tr>
<tr>
<td></td>
<td>Different</td>
<td>Different</td>
<td>Different</td>
</tr>
<tr>
<td></td>
<td>Different</td>
<td>Different</td>
<td>Different</td>
</tr>
<tr>
<td>EW</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Int. excl. AA</th>
<th></th>
<th>Int. incl. AA</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRP</td>
<td>RP</td>
<td>NRP</td>
<td>RP</td>
</tr>
<tr>
<td></td>
<td>Equal</td>
<td>X</td>
<td>Equal</td>
</tr>
<tr>
<td></td>
<td>Different</td>
<td>Different</td>
<td>Different</td>
</tr>
<tr>
<td></td>
<td>Equal</td>
<td>Equal</td>
<td>Different</td>
</tr>
<tr>
<td>EW</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Source: Own creation
5.2 Test of portfolios for the underlying assumptions

Assuming that returns are normal distributed makes working with risk easier as one may use standard deviation and VaR as measures of risk and SR as a performance measure. The ease, by which the normal distribution assumption can be utilised into capturing risk, might also be why it is still widely used when comparing risk and performance of for instant mutual funds. This assumption has though been proven several times over, that it is not the case for the returns. When using the JB-Test the zero hypothesis is that returns are in fact normal distributed. The critical value by which one can reject the zero-hypothesis at a 1% confidence level is 9.21. Since the normal distribution assumption is so easy to work with, we here want to be fairly safe, that we are not rejecting the zero hypothesis unless we are fairly sure. Since the analysis is applied on daily returns, the statistical strength of the test should be very high.

<table>
<thead>
<tr>
<th>Investment Universes</th>
<th>HB excl. AA</th>
<th>HB incl. AA</th>
<th>Int. excl. AA</th>
<th>Int. incl. AA</th>
</tr>
</thead>
<tbody>
<tr>
<td>RP</td>
<td>Rejected</td>
<td>Rejected</td>
<td>Rejected</td>
<td>Rejected</td>
</tr>
<tr>
<td>NRP</td>
<td>Rejected</td>
<td>Rejected</td>
<td>Rejected</td>
<td>Rejected</td>
</tr>
<tr>
<td>EW</td>
<td>Rejected</td>
<td>Rejected</td>
<td>Rejected</td>
<td>Rejected</td>
</tr>
<tr>
<td>MV</td>
<td>Rejected</td>
<td>Rejected</td>
<td>Rejected</td>
<td>Rejected</td>
</tr>
</tbody>
</table>

*Source: Own creation*

In the above table one can see that for all portfolios and in all investment universes, the normal distribution assumption is rejected at the 1% confidence level. When leverage is applied it is commonly accepted that one increase the negative skewness of ones portfolios due to the negative effect of the interest rates payable, and the portfolios also tend to exhibit higher kurtosis, meaning “fatter tails”

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12 [http://www.morningstar.dk/](http://www.morningstar.dk/)
In the above table you can see that also for the leveraged portfolios the normal distribution assumption is rejected. Therefore you need to take these parameters into considerations when measuring the risk of the investment approaches, otherwise you will underestimate the risk and overstate the performance as a result. To capture the negative skewness and excess kurtosis in the performance measurement, MVaR and MSR is applied.

### 5.3 Performance based on advanced key figures

By using the MVaR with the Cornish-Fischer expansion the skewness and kurtosis of the returns of the portfolios are taken into consideration, when measuring the risk of the portfolios. Since the risk is a function of standard deviation, skewness and kurtosis, the higher the standard deviation the higher the risk, and the more negative the skewness is, the more the risk increase and the higher the excess kurtosis, the more risky it gets. This also means that the more equal the kurtosis and skewness is, the less effect it has, and the closer it resembles the traditional performance measure SR.
Since all portfolios exhibit both negative skewness and excess kurtosis, the risk will be higher and therefore the MSR lower, but what is interesting is the relative performance in between the funds. MV is outperforming all other funds in Int. excl. AA and Int. incl. AA.

Measured on MSR, RP and NRP are outperforming MV and EW in the HB investment universes, and MV outperforms RP and NRP in the Int. investment universes. These results are in line with our previous findings, that the investment universes with a higher relative amount of fixed income asset classes (HB) favour the RP and NRP portfolios. Including skewness and kurtosis into the equation, tilts the relative performance towards MV. Measured on total return MV outperforms all other unlevered portfolios in all the investment universes, as it can be seen in in table 5.9.
As it can be seen in table 5.10, when measuring MDD across all unleveraged portfolios, we find that MDD is highest for EW on lowest for MV in average. Though is MDD in HB incl. AA slightly higher for MV, than for RP and NRP. When comparing theses result to both the numbers of total return and MSR, it is slightly surprising that MV doesn’t also here outperform RP, NRP and EW. Since the difference is fairly small, it might just be a coincident.

Table 5.10 – MDD, Unleveraged

<table>
<thead>
<tr>
<th>Investment Universes</th>
<th>HB excl. AA</th>
<th>HB incl. AA</th>
<th>Int. excl. AA</th>
<th>Int. incl. AA</th>
</tr>
</thead>
<tbody>
<tr>
<td>RP</td>
<td>-9.91%</td>
<td>-11.13%</td>
<td>-11.28%</td>
<td>-13.06%</td>
</tr>
<tr>
<td>NRP</td>
<td>-9.36%</td>
<td>-11.45%</td>
<td>-10.84%</td>
<td>-13.80%</td>
</tr>
<tr>
<td>EW</td>
<td>-23.23%</td>
<td>-29.69%</td>
<td>-24.79%</td>
<td>-33.30%</td>
</tr>
<tr>
<td>MV</td>
<td>-8.16%</td>
<td>-11.99%</td>
<td>-7.50%</td>
<td>-11.98%</td>
</tr>
</tbody>
</table>

Source: Own creation

Leveraging the portfolios increases the performance of RP and NRP relative to MV, those two portfolios now perform best in three out of four investment universes. MV still performs best in Int. incl. AA.

Table 5.11 – MSR, Leveraged (MV not leveraged)

<table>
<thead>
<tr>
<th>Investment Universes</th>
<th>HB excl. AA</th>
<th>HB incl. AA</th>
<th>Int. excl. AA</th>
<th>Int. incl. AA</th>
</tr>
</thead>
<tbody>
<tr>
<td>RP</td>
<td>2 = 0.94</td>
<td>1 = 0.91</td>
<td>2 = 0.84</td>
<td>2 = 0.75</td>
</tr>
<tr>
<td>NRP</td>
<td>1 = 0.97</td>
<td>2 = 0.87</td>
<td>1 = 0.85</td>
<td>3 = 0.71</td>
</tr>
<tr>
<td>EW</td>
<td>4 = 0.41</td>
<td>4 = 0.27</td>
<td>4 = 0.36</td>
<td>4 = 0.20</td>
</tr>
<tr>
<td>MV</td>
<td>3 = 0.75</td>
<td>3 = 0.73</td>
<td>3 = 0.80</td>
<td>1 = 0.82</td>
</tr>
</tbody>
</table>

Source: Own creation

Applying leverage has increased the total return of both RP and NRP, whereas it has dropped for EW.
Due to the high volatility of EW, it has in fact been deleveraged.

Table 5.12 - Index of Leveraged Portfolios

<table>
<thead>
<tr>
<th>Investment Universes</th>
<th>HB excl. AA</th>
<th>HB incl. AA</th>
<th>Int. excl. AA</th>
<th>Int. incl. AA</th>
</tr>
</thead>
<tbody>
<tr>
<td>RP</td>
<td>235.34</td>
<td>257.85</td>
<td>217.75</td>
<td>226.61</td>
</tr>
<tr>
<td>NRP</td>
<td>239.27</td>
<td>246.90</td>
<td>220.67</td>
<td>219.94</td>
</tr>
<tr>
<td>EW</td>
<td>156.33</td>
<td>147.71</td>
<td>152.37</td>
<td>140.86</td>
</tr>
<tr>
<td>MV</td>
<td>202.38</td>
<td>216.75</td>
<td>217.30</td>
<td>240.45</td>
</tr>
</tbody>
</table>

Source: Own creation

Where applying leverage has increased the performance of RP and NRP when measured on SR, MSR and total return, then it has also increased the risk when measuring timing risk using MDD. The increase in MDD is almost equal to the leverage factor for each portfolio, but not quite.

Figure 5.1 – Index for HB excl. AA with RP, NRP & EW Levered ex post 10 Year

Source: Own creation
Performance is highly sensitive the investors perception of his or hers investment universe. In average RP, NRP and MV to some extent perform equally well. Has the investor a specific investment universe of the four, the choice of portfolio is no longer irrelevant. For all other investment universes than Int. excl. AA, whether or not you choose one of the risk parity portfolios, RP, NRP or you choose the MV portfolio.

Since SR assumes normal distribution MSR was introduces, as it penalizes negative skewness and excess kurtosis. When comparing the results the finding is that even though SR does not capture the overweight of negative returns or the fat tails of the excess kurtosis, it does not change anything regarding the performance evaluation. One reason for this is that all portfolios exhibit negative skewness and have fat tails, so they all are penalized equally.

MDD was the figure which described the maximum drop in return from the highest peak to the lowest valley. This indicates the timing risk of the portfolio. When leveraging the portfolios MDD for RP and NRP increase where as EW decreases. The obvious explanation for the difference is that RP and NRP are increasing their volatility and has a leverage factor >1 where as for EW it is just the opposite. So when volatility increases, so, does also the timing risk measured by MDD. For HB incl. AA with no portfolios leveraged, MDD of MV is slightly higher than that of both RP and NRP. Still in HB incl. AA, but now with the two latter portfolios leveraged MDD increases to approximately 150% of that of the MV portfolio. This tendency prevails across all investment universes with the MDD of RP and NRP being 50% higher at least than that of MV. For Int. excl. AA it is more than a 100% higher. Altogether the return of an investor is more sensitive to when in the period he or she starts investing, when choosing the leveraged RP or NRP portfolios, than when choosing MV.

As it has been shown RP and NRP have to be leveraged for the volatility of it to equal that of the MV portfolio, this making comparison possible. The mean of the unleveraged portfolios of RP and NRP are below the expected required return of an investor. Therefore further on in the paper, the analysis will focus solely on the performance of the three levered portfolios RP, NRP and EW, and the unlevered portfolio MV, which is the base portfolio.
5.4 The effect of market conditions on performance

One major point of criticism is that RP allocates high amounts capital to bonds. Critics argue that good performance is obtained in an interest rate bull market. With the current levels of interest rates, the mid- and long term expectation of many analytics (Nordea, 2014), is that interest rates will increase. Increasing rates will have a negative effect on bonds and since RP and NRP allocate a large proportion of capital here, the performance is expected to worsen.

At the same time one could expect that the portfolio MV will have in average a higher proportion of capital allocated to equities. EW is also expected to have a relatively higher proportion of equities relative to bonds, than RP and NRP. This entails that they would be expected to be sensitive to the volatility of the overall equity market.

To test this, a framework has been adapted from that of Falkof (2014). The 10 year period has been divided into monthly periods resulting in 120 sub periods. The 10-year US Treasury Note benchmark bond (Bloomberg) has been chosen as a benchmark for the interest rate level. This is in coherence with choosing the 3 month US Treasury bill, meaning that the rates are from the same currency, making them comparable. The shifts in interest rate levels are calculated as the percentage change of the interest rate of the final trading day of the month being evaluated minus the interest rate of the final trading day of the previous month.

Figure 5.2 shows the interest rate level of the 10 - Year US Treasury Note.
The volatility of the MSCI World Local Currency Index (MSCI World) is used as a proxy for equity market volatility. One could also have used the volatility of the S&P 500 American Equity index as proxy of the VIX volatility index. Since the focus is on a Danish Home Bias or International Bias, MSCI World is regarded more relevant. The daily volatility of MSCI World is calculated over a rolling monthly period and annualized to make it intuitive comparable to previous figures.

In figure 5.3 the annualized daily volatility calculated on a rolling monthly basis of MSCI World is shown.
When you compare figure 5.2 to figure 5.3 you notice several sharp drops in the level on the interest rate simulations with spikes in volatility, this indicates a relationship between the level of volatility to the changes in the interest rate. This would also be expected as when risk increases the safer alternative becomes more attractive e.g. more investors reallocating to bonds from equities.

As the figure indicates volatility was high throughout most of 2008, but was also high in May 2010, and in August and September 2011. 2008 was the peak of the financial crisis, May 2010 was mostly related to the “Flash Crash”, and August and September 2011 was the pinnacle volatility-vise of the European Debt Crises, where it became apparent that Greece was unable to service or repay its debt, and fear in the market was, that it was going to drag the rest of the euro zone down with them. These periods of crisis seems to correspond well to the drop in interest rates, as can be seen in figure 5.2

5.4.1 The effect of changing interest rates

A regression is also run, to verify the results of the above analysis. Monthly changes in interest rates are regressed against the monthly returns of the portfolios (Appendix A).
For the regression run in HB incl. AA for RP, we find that there is a slight negative relationship between a change in the monthly interest rate and the monthly return of the portfolio. The slope of the fitted line is -0.09 indicating that an increase in the interest rate of 10 % would result negative return of -0.9 % for RP. Both the upper and lower boundary of the 95 % confidence interval is negative and therefore downwards sloping. We are therefore confident that we can reject the zero hypotheses, that that slope of Y is in fact zero, and there therefore should be no relationship. \( R^2=0.20 \) which indicates that the change in interest rates does a very poor job of explaining the returns of RP.

For NRP there is still negative correlation between the change in interest rates and the return of the portfolio. Here the slope of the fitted line is -0.08 and with an \( R^2=0.16 \) changing interest rates do an even worse job of explaining the return of the portfolio than for NRP.

For EW the regression shows more or less no correlation to changing interest rates, what so ever. The slope of the fitted line is 0.009 and with an \( R^2=0.003 \), it just confirms what would be expected, since the portfolio is optimized based on equal weights, and therefore doesn’t allocate extra capital to asset classes neither based on low risk or expected high returns.

When regressing for MV, we find that also the slope of the fitted line -0.06 slightly less than RP and NRP. Just like for RP and NRP the \( R^2 =0.15 \) is fairly low which confirms that interest rates might have a negative correlation to the that of MV, but there is not much explanatory power in it, when determining the monthly return of MV.

For RP in the investment universe HB incl. AA, the correlation to the change in interest rate decreases when AA are included. It drops from -0.51 to -0.45 and thereby confirms that increasing interest rates have a negative effect on the returns of RP. The slope of the regression has the same negative coefficient. For NRP the correlation drops far more than it does for RP. For EW the results are still inconclusive and MV exhibits almost similar numbers to NRP.

For Int. excl. AA, the slope of RP is still negative, but only with -0.07, the same as for NRP.
There is a clear tendency, that when fixed incomes proportion of the investment universe increases, so does the sensitivity to changing interest rate, which are negatively correlated, of both RP, NRP and MV. RP and NRP are almost equally affected by it, whereas MV is slightly to moderate less affected. For EW the regression did, not show significance.

### 5.4.2 The effect of the level of volatility of equity market

The other factor which is expected to influence especially the return of MV is the volatility of the equity market. A regression is therefor run on where the level of the equity market volatility over a one month period is regressed over the monthly return of the portfolios (Appendix B).

For RP in HB incl. AA there is indeed a negative slope of the fitted line, -0.05, but the $R^2 =0.08$, indicates that the explanatory power of the volatility of the equity market is very low.

EW shows a significantly stronger relationship with the increase in equity market volatility. The slope is only -0.08 but the $R^2 =0.31$, which entails that it is far better at describing the risk of EW, than the change in interest rates.

The regression run on the returns on MV is inconclusive. The p-value of the regression is 0.20, far above the accepted maximum of 0.05 at the 95% confidence level.

RP and NRP are slightly negatively correlated to market volatility across investment universes. This tendency is far more prevailing when incl. AA, than excl. AA. An explanation might be that REITs are more positively correlated to the equity market. EW shows clear signs of being negatively correlated to the equity market volatility across all investment universes.

For MV the results are inconclusive across all investment universes, since the p-value far exceeds the threshold value of 0.05.
5.4.3 Stress test of variable leverage factor

As mentioned in chapter 3 the methodology of applying leverage was doing so *ex post* of the period. The result of this is a fixed leverage factor across the entire 10 year period. Assuming that the evaluated period is now a month, leverage is now applied *ex post* of the month, so that the volatility of RP, NRP and EW equal that of MV on a monthly basis. There is still a *look ahead bias*, but it is limited to only being one month forward and not 10 years. Financing is still assumed to be cost free.

The indexed returns of the new three portfolios and MV for HB incl. AA are shown in the figure below.

![Figure 5.4 - Indexed returns for HB incl. AA with monthly recalculation of leverage](source: Own creation)

Since mid. 2011 MV is performing better than RP and NRP, which is illustrated by the decreasing spread between the lines in figure 5.4. This coincides with beginning increasing interest rates. Although both RP and NRP are yielding positive returns, MV is yielding more.

The leverage multiplies the total return of RP and NRP over the entire 10 year period. For HB incl. AA, RP yielded an index value at the end of the 10 year period of 257.85, when the leverage factor was calculated *ex post* the 10 year period. When the leverage factor is calculated *ex post* monthly, then the index value after 10 year is 221.84. For NRP the picture is the same, from 246.90 to 218.28. EW once more yields the lowest return of them all. The index value actually drops from 147.71 to 157.75.
Therefore the monthly rebalancing has a negative effect on the total return of RP and NRP. At the same time is the volatility over the entire 10 year period bigger than that of MV, due to the shifting leverage factor.

When looking at the figure the return increases rapidly from July 2006 to February 2008. Comparing this to the movement in interest rates, figure 5.2 and the equity market in figure 5.1, there seem to be no apparent connection between them. Shifting the focus to the leverage applied, a sharp rise in leverage occurs exactly in July and stays relative high in that incremental period. Since the leverage factor is calculated by dividing the volatility of MV with that of portfolio one intend to lever, then explanation must be found in level of volatility of RP and NRP compared to MV. In the below table the leverage factor of RP, NRP and EW is compared to the level of volatility of MV.

![Figure 5.5 – Leverage Factors of RP, NRP & EW relative to volatility of MV for HB incl. AA](image)

Since both the volatility of MV and the leverage factors co-varies that must mean that the volatility of both RP and NRP stays more or less unchanged on a monthly basis.
5.5 Weights of asset classes

This very high leverage poses a tremendous risk to the investor. With a leverage factor of for example the portfolios is financed with 20% equity and 80% debt. This entails that an incremental drop in the monthly return of the portfolio by 25% all equity is gone, and your money is lost. As equity goes down credit risk goes up. Investor will most likely be required to either put up more collateral to satisfy the lender or even redeem some of the debt. Since the capital for this must come from the portfolio, this will have a negative effect on the return of the portfolio. Since it is not likely that all asset classes will fall simultaneously, intuition entails that diversification is attractive from a risk perspective especially when leverage is applied. In the figure below, the asset weights have been graphed for MV.

Figure 5.5 – Asset Weights of MV for HB incl. AA, 2004

As figure 5.5 shows, MV has major shifts in asset weights over the period. In June 2004 27.7% of capital is allocated to DK MTG and 44.9% to REITs. In fact no capital is allocated to neither, EM EQT, DK GOV, USEUGOV, EM HY Bonds or IG CORP. MV is therefor in June only investing in six out of eleven asset classes and allocating 72.6% of capital to only two of these asset classes.
Since RP is applying leverage the total asset weights can exceed 100%, which they will when leverage factor >1. This increases the risk if the weight allocated to any specific asset class reaches a level where a low to medium negative return of that asset class, results in the equity part of the portfolio to come close or even reaching the level. When the leverage factor is fixed, it will multiply the asset weights of the unleveraged portfolio by the leverage factor. The result of RP with a fixed leverage factor on the asset weights is illustrated in figure 5.6.

![Figure 5.6 - Asset Weights of RP with Fixed Leverage for HB incl. AA (2004)](source: Own creation)

Compared to MV, the asset weights of RP are far less profound, with only minor changes. The leverage does though increase the weight in each asset class by a factor 1.64. When the leverage is taken into account average allocation to equities are in average 22.2%, 16.24 to alternatives and 125.82% to bonds. The highest average asset weights are DK GOV, DK MTG and EM HY Bonds with 26.38%, 26.31% and 27.06% respectively. This confirms other analysis and our expectation that a large proportion of capital is allocated to bonds.
As already discussed the above methodology for leveraging the portfolios implies a look ahead bias. If the period the investor is trying to equal the volatility over is only one month and not 10 years, the leverage factor is expected to deviate from the 10-year average obtained by the above methodology. This new methodology is applied for the RP in the investment universe HB incl. AA. The figure below show the changes in asset weights, when the leverage factor is variable and recalculated based on the forward looking month’s volatility of RP and MV.

![Figure 5.7 - Asset weights of RP with varying leverage for HB incl. AA (2004)](image)

Source: Own creation

In August 2004 the leverage factor is peaking, would therefore be expected to be the time, where the risk from leveraging is highest. August is the month where RP has the highest allocation to any specific asset class in the entire period. 34.54% is allocated to DK GOV and 30.19% allocated to DK MTG. These asset classes must be is regarded as been some of the safest asset classes measured on volatility. Previous researches show, that MV tend to lead to a large percentage of allocations to few or even one asset class(es) (Chaves et. Al, 2011). The reason for this is, that MV seeks to maximise the risk return relationship, by combining all asset, so that the portfolios lies on the efficient frontier, that is the portfolio with the maximum return at the same risk, or in another way if you will, the lowest risk at the

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13 DK GOV has a volatility of 4.72 % and DK MTG 3.30 %. in the period 2004 – 2013, the lowest and fifth lowest of all asset classes
same return. In this case the tangent portfolio is calculated, which has the maximum SR that can be obtained. As the asset classes do not perform equally well and their relative correlations with each other aren’t the same which influences the allocation. So basically MV looks at the SR of the individual asset class, because this is what it seeks to optimise, and the correlation of the asset classes with the others. This favours asset classes with no or even negative correlation to other asset classes. The problem of lack of diversification arises, when one or few asset classes in terms of SR dominates all other asset classes so extensively, that all capital is allocated to this one asset class. This is not what most people would call diversification but rather a lack of this, but in optimization terms this is the optimal allocation.

To compare MV with the other portfolios, all other funds have an inherent constriction, ensuring that at least a minimum of capital is invested in each asset class. Most obviously is EW, as an equal proportion is allocated to each asset at each time. The same type of mechanism is built in in both RP and NRP, even though the allocation methodology is somewhat more dynamic, and based on volatility and correlations. Unless certain assets yield no return at all, or have no risk at all, at least some proportion of capital will be allocated to all asset classes. From a logical perspective this seems a lot more like diversification than for MV.

The above findings are of course noteworthy, but they still do not grasp how extreme allocations for MV can get. Still looking at HB incl. AA – unleveraged, we now focus on the allocation at October 2006. Something worth noticing here is the 100% allocation to REITs for MV. Obviously when you look at the aggregate performance of MV, it hasn’t really been penalised on this extreme allocation to one asset, on the total return, but that might just have been chance. Undoubtedly you expose your investment to sudden steep drops in values of a specific asset class, when allocating as previously mentioned. Especially real estate prices drop severely during the financial crises. In one, if not the worst month on the equity markets, October 2008, being just after the collapse of one of the world’s oldest and biggest investment banks, Lehman Brothers, REITs dropped by -27% from the start of October to the end of October. To comparison DK EQT, WLD EQT and EM EQT dropped by -20%, -18% and -25%, so obviously there were a lot of allocations, which would have yielded sustainably negative returns.
The high allocation to for instance one specific asset classes makes MV specifically vulnerable to quick shifts in the performance of that, or those specific asset classes. This entails that you might be lucky in long periods of time, but if misfortune strikes, the very narrow asset allocation, increases the risk of losses, from which it will take a very long time to recover from. Time you might just not have.

5.6 Cost of leverage

So far leveraging the portfolios have been assumed to be cost free. Financing costs will obviously affect the return of the leveraged portfolios. Therefore the financing cost is incorporated in to indexed returns of the portfolios. The financing cost is set as the Libor 3-month USD rate. It is then averaged and is assumed to be paid at the end of each period that being at the end of the 10-year period and the end of each month. The average rate of the Libor 3 month USD is 1.99%. Since EW with a leverage factor of 0.70, the portfolio is more so deleveraged, than leveraged. It is assumed that the excess capital can be invested at the average rate of the 3-month US Treasury Bill.

For the 10-year period the indexed return after financing is just the index minus the average rate over the 10 year multiplied by ten times the leverage applied which is the leverage factor minus 1, as it has to be paid for each year. The impact on the total return of the leveraged portfolios can be seen in the table below.

<table>
<thead>
<tr>
<th>Portfolios</th>
<th>RP</th>
<th>NRP</th>
<th>EW</th>
<th>MV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before</td>
<td>235.34</td>
<td>239.27</td>
<td>156.33</td>
<td>205.38</td>
</tr>
<tr>
<td>After costs</td>
<td>218.99</td>
<td>222.41</td>
<td>162.48</td>
<td>205.38</td>
</tr>
</tbody>
</table>

*Source: Own Creation*

In table 5.14 the result of the deduction of financing costs are show on the total return of the portfolios. The deduction of the cost of financing for HB excl. AA shows that the performance of RP is highly...
susceptible to the cost of financing. The break even rate is at 3.63%, so at this average rate, RP is no longer out performing MV on total return. For NRP the rate is 3.98%.

5.7 Partial conclusion

It was found that the risk parity portfolios have to use leverage to obtain a return at level with the required long term return. When leverage was used, the risk parity portfolios outperformed MV and EW for the investment universes with Danish home bias measured on total return. Measured on SR, the risk parity portfolios also significantly outperformed MV and EW. MSR confirmed the relative performance of the four portfolios when measured on SR.

Across all investment universes, the risk parity portfolios were found to have medium to strong negative correlation to the change in interest rates, but the explanatory power on the portfolios returns were fairly low. The volatility of the equity market was found to have a small negative correlation to the return of the especially EW, whereas the both the explanatory power of, impact and correlation, was found to be very small, but though negative. The results for MV were inconclusive.

There was found evidence for the cost of financing can affect the performance of the leveraged portfolios, so that narrows to that of MV. The study also indicates that a variable leverage factor could in fact increase risk and lower return of leveraged portfolios.
6 Reflections

6.1 Implications and limitations of the results

The study conducted adds to the growing body of research within low risk portfolio optimization approaches.

The empirical study provides both quantitative and qualitative evidence on the performance of risk parity.

This study adds to the empirical research already been done, by constructing realistic Danish home biased investment universes, and international biased investment universes and studying the performance between them. This gives an insight in how performance might look like from a Danish institutional investor, whom has a tendency to allocate larger proportion of capital to the Danish equity and bond market, than the relative market capitalization of these.

In certain months the base portfolio MV only allocates capital to one asset class. Even though this theoretically is the most optimal portfolio, it seems unlikely that an institutional investor would accept this concentration. To increase the applicability of the study to real life, it would be advised to set up an constriction on the optimization process of MV. One such constriction could be a min. percentage of capital has to be allocated to min for example 4 asset classes.

The methodology of leveraging ex post is an unrealistic assumption, which makes comparison of performance more rigorous. A more realistic methodology of rebalancing leverage based on expected volatility of the included portfolios, for example based on the rolling year forecasting methodology used for the rebalancing of the unleveraged portfolios.

By not including investor’s choice of financing, a maybe vital part of the overall return and risk of the RP approach has also been left out.

6.2 Ideas for further research

There is a potential for further research within the performance evaluation of RP, both theoretically and empirically.
On the theoretical front further research could include testing the risk allocation methodology of asset class based risk parity to that of risk factor based risk parity. Of interest would also be the sensitivity of the asset based risk parity model to errors in the estimation of the parameters, used to optimize the portfolio.

On the empirical front there are several ways the study could be supplemented and extended. First different financing options could be included in the study and these evaluated hereof also the liquidity risk under financial crises such as the financial crisis (2007-2009) and the European sovereign debt crisis in 2011. Second it would be of interest to test, which risk factors explain the returns of the different asset classes, to give a further indebt explanation of the returns of the RP portfolios.
7 Conclusion

There are several reasons why risk parity seems attractive portfolio optimization approach compared to fixed weighted and mean variance optimized. The implied logical reasoning of equal risk contributions from all asset classes seems tantalizing. This paper focused on whether or not a risk parity portfolio would outperform that of other portfolios, and how performance was influenced under different market conditions. The comparison was done between RP, NRP, EW and MV.

The identified asset classes were combined to form four distinct investment universes which, was expected realistically determine the choices a Danish institutional investor has.

The study found evidence for that in average, across the four investment universes, RP, NRP and MV performs equally well when unleveraged, and EW performs worst. RP and NRP outperform MV for the Danish home biased investment universes and MV outperforms the others for the international biased investment universes. We find that the choice of portfolio does matter, when the investor has defined a specific investment universe. Results indicated that when the proportion of fixed income asset classes in the investment universes increased, so did the performance of RP and NRP. The inclusion of alternative assets was found to have little impact of the relative performance of the portfolios.

An unleveraged risk parity portfolio yields a return to low to meet the required return by investor, and leverage has to be applied. When leverage is applied \textit{ex post} the performance of RP and NRP across all investment universes increases and now also outperforms MV and EW in Int. excl. AA.

RP, NRP and MV were all found to have negative correlation to changing interest rates, though the first two slightly more than MV. It was also found that the explanatory power of the change interest rates was fairly poor. A negative correlation to increased equity market volatility was also found, but also that had an even lower explanatory power for RP and NRP, but also MV. It was found that the rebalancing methodology- and cost of leveraging has an impact on the return of RP.
Including cost of financing the leverage does for obvious reasons lower the total return of RP and NRP. The inclusion also showed a high sensitivity of the total return to an increase in the rate at which investor borrows.

When analyzing the period from 2004 to 2013 both RP and NRP seem as attractive approaches for a Danish institutional investor, when he has a Danish home bias. With an expectation of many analysts and investors of increasing interest rates, one should though be cautious, since it will affect the expected performance of risk parity portfolios.


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