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Master Thesis

The Leverage Lifecycle Strategy: A Performance Review

Den gearede livscyklus strategi: En præstationsbedømmelse

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Abstract

Proper savings and investing for retirement matters vitally for retiring safe and pleasant. This thesis challenges conventional investment strategies for retirement savings, by modelling a strategy that employs leverage while being young, in order to reduce risk later. The leverage lifecycle strategy is devised by Ayres and Nalebuff (2010a), and currently only little research exists. This thesis presents an extended version, because it allows asset diversification by including 8 different asset classes (but uses 7). The results are based on 200 cohorts from Monte Carlo simulations. The portfolio initially contains 200% risky assets, and deleverages continuously until the investor has 49% left. The leverage lifecycle strategy outperforms with 5.23% in certainty equivalent compared to a strategy that invests the same percentage amount of risky assets for an investor (with CRRA=23). Also, the leverage lifecycle strategy needs 9.16% less investments in risky assets to obtain the same mean of final wealth. Similarly, the investor can retire 24 months earlier when applying the leverage lifecycle strategy. It does, however, produce a larger uncertainty of the outcome of final wealth. The thesis also examines the resilience of the leverage lifecycle strategy by altering various input variables, which indicates that decreasing the amount of risky investments continuously improves an investor's final wealth.

Contents

1 Introduction

1.1 Background

Retirement savings is a long continuous process, where one saves money until retirement. Having the financial freedom or safety at retirement is what most people desire. However, a study (Lusardi & Mitchell, 2005), which examined financial literacy among americans identified, that many of the age 50+ respondents showed little knowledge about financial literacy: Only half of the respondents answered two simple questions correctly regarding inflation and the compounding effect, and only one third of all respondents could answer these two questions and an additional question about risk diversification correctly. Furthermore, only 19% had tried to devise a retirement plan and succeeded. This financial illiteracy and lackadaisical attitude towards retirement savings can have great negative impact on one's personal finance.

Whether or not one understands basic financial cardinal concepts, yet it is even more difficult to comprehend investment strategies. Two professors from Yale University, Ian Ayres and Barry Nalebuff propose to apply leverage investments during the early phase of retirement savings. This controversial investment strategy confronts the conventional advisement that leveraging amplifies risks. However, the authors advocate for buying stocks on margin to *reduce* risk in their article (Ayres & Nalebuff, 2010a). At present there are almost no literature that investigates their investment strategy. Bear in mind, that currently, the approach is legally constrained, yet its theoretical foundation, on which the hypothesis is constructed, is intact.

The investment strategy is, however, a simplified version of current investment strategies, because it is a two-asset class model only consisting of stocks and government bonds, meanwhile other asset classes are excluded from the strategy. This leads to a new starting point, where one can expand the asset class universe by including new asset classes and identify and analyse the model's new attributes and influence on the leveraged investment strategy.

1.2 Problem Statement

This thesis addresses the following problem statement:

How can employment of leverage with diversified asset classes in the early phase of retirement savings improve an investor's final wealth?

The problem statement will be answered from an analysis of a leveraged lifecycle strategy. In the early phase of the lifecycle an investor will be leveraged to obtain a high expected return, and later on the portfolio will be deleveraged.

The analysis of this thesis is based on Ayres and Nalebuff (2010a), yet extended with multiple asset classes, including an examination of asset allocation under different periods. The strategy will be compared to other different strategies based on Monte Carlo simulations. There will be resilience testing with various factors of the model in order to be able to conclude coherently upon the problem statement.

1.3 Problem Definition

The purpose of this subsection is to confine the problem statement, hence the focus in this thesis will be as accurate and profound as possible. It is therefore important to emphasise that the asset allocation is a mean to an end. Thus, the *construction* of allocation will not be analysed as thoroughly as the *impact* it has on the strategy.

There will be applied a fixed risk free rate and a fixed margin rate. This simplifies the model comparing to floating rates. Also, it is in this thesis anticipated that the investor operates within the US investment environment. Hence, this affects among others not only the legal aspects, but also the basis of the investment decisions.

1.4 Acknowledgement

This thesis applies the leverage lifecycle strategy described in the original article from Ayres and Nalebuff (2010a). Their work is newly published and is to be considered highly specialised and controversial within the field of leverage investing in retirement savings. Currently, there is only very little literature about the subject. The article's approach and connecting literature has been widely used in this thesis, as well as their calculations have been used as an inspirational source. This thesis' mean-variance analysis in the theory section applies the approach from Luis M. Viceira and John Y. Campbell (2001).

1.5 Structure

Initially, the theory is examined in section 2 within the area of utility theory, modern portfolio theory and lifecycle investing. It is expected that the reader has the sufficient comprehension regarding economics, finance and mathematics. In section 3 the model is presented, explained and discussed as well as the relevant benchmarks. New concepts are introduced such as diversification across time and risk measurements. In section 4 the data is presented and the credibility and performance is analysed. The portfolios are also constructed within some conditions by a convenient method, so they can be implemented to reality, and odd theoretical results vanishes in an appropriate manner. The hypothetical investor is also defined with his savings rate and risk appetite as well as the methods of simulating the potential cohorts.

Finally, the investment strategies are analysed in section 5 where the analysis encompasses a performance review based on the same mean of final wealth. Section 6 analyses and discusses the resilience of the model, by implementing different factors and altering various input variables. At the end of the thesis, in section 7, a conclusion is presented with the findings.

1.6 Scientific Theoretical Approach

In this section the scientific theoretical approach is examined. It is divided into three parts; ontology, epistemology and methodology. It clarifies the perspective of and methods to solve the given problem statement. This thesis' scientific theoretical approach is primarily based upon the philosophy of objectivism.

Ontology is the study of reality. It deals with questions related to existence. This thesis is build upon an objective reality. This means, the reality exists and it is independent of human consciousness. Thus, the leverage lifecycle strategy can be constructed, analysed and compared to its benchmarks. It is done by an objective manner, by using different financial risk measurements and economic utility metrics. The empirical results are therefore objective, they are what they are, and wishing otherwise will not make it so. Each strategy's performance can be evaluated objectively and without the consent of subjective opinions or feelings.

Epistemology is the study of validating and acquiring knowledge. Since the mind is neither omniscient nor infallible, the concerns regarding epistemology must be examined. Reason is an absolute i.e. the method of grasping the objective reality can only be done by acquiring knowledge. The theoretical analysis and conclusions derive from logic, by identifying non-contradictory concepts. The empirical analysis is based on concrete historical observations.

The primary methodology that has been applied is the hypothetico-deductive method. The hypothesis 'that employing leverage can improve retirement savings', will be tested by extending the model with the use of multiple assets.
Although there is an empirical analysis where historical asset returns are observed and integrated into the model, by the use of the inductive method, it is not of the primary importance in this thesis.

2 Theory

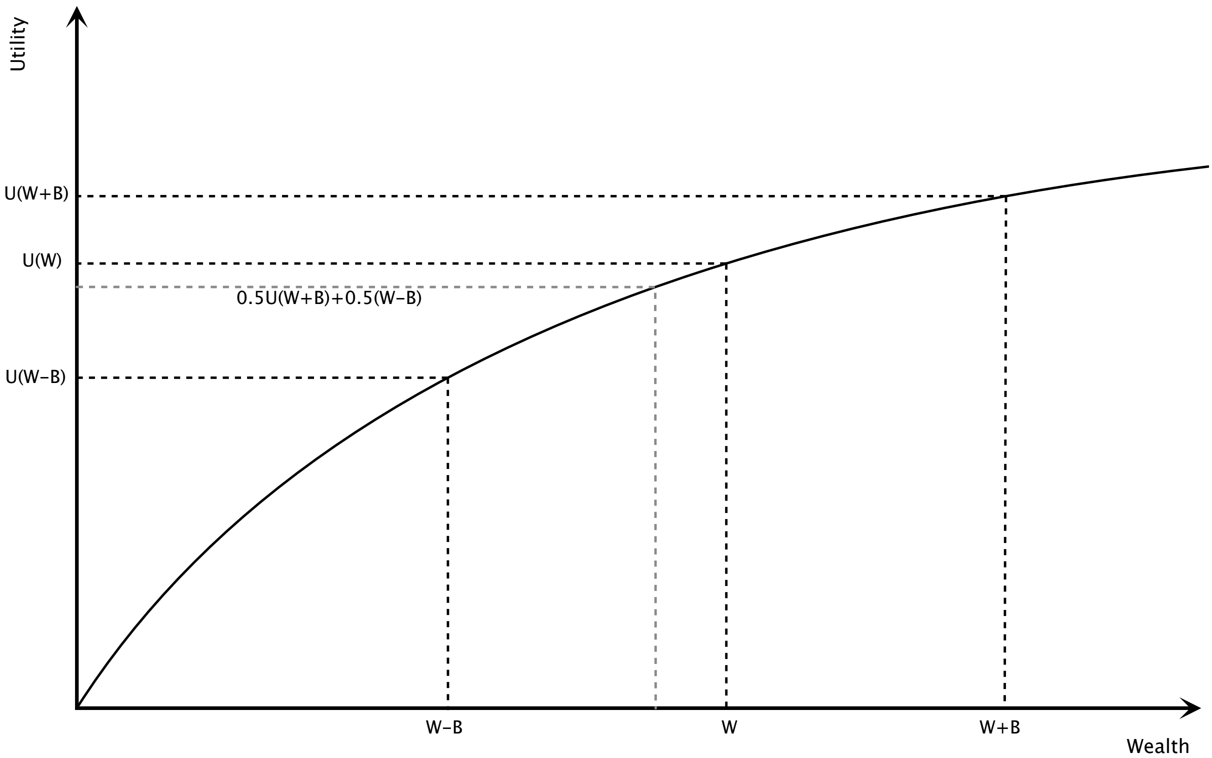
2.1 Utility function

Every individual faces different investment and consumption opportunities and strategies to accommodate his preferences and needs. Obviously, he will prefer to increase his wealth but probably not at any cause. The uncertainty of the outcome involved in his decisions leads to higher risks.
The decision on portfolio choices depends on the individual. Some individuals might find it appealing to purchase a lottery ticket, while others find it unattractive and prefers to save the money in the bank with a certain known interest rate. Utility theory measures these set of preferences, and a theoretical foundation is necessary to understand the individual's attitude regarding the trade-off between risk and expected return. This concept can easily be illustrated by a simple example with the following setup:
An individual is given a choice between two different scenarios, where one of the scenarios has a certain outcome and the other has an uncertain one. In the scenario with a certain outcome, the person will immediately receive \$75. In the other scenario, a coin is being flipped, and depending on the result between toss and tail, the person has equal probability to receive \$100 or \$50. The expected payoff from both scenarios is \$75, but the risk is much greater with the scenario of a gamble. If an individual prefers to avoid the bet he is risk averse, while a person who is insensitive or indifferent to the risk involved is considered risk seeking or risk-neutral respectively. In this thesis it is assumed that the individual is risk averse, meaning in practice that he is disincentive to take unnecessary bets.

The concept of risk aversion means the utility function ($U(W)$) is strictly increasing in wealth $U'(W) > 0$, but decreasing at a marginal rate $U''(W) < 0$. This results in a concave function where the degree of curvature describes the intensity of the risk aversion for the individual.
The interpretation of risk aversion is illustrated in figure 2.1 where the utility function is

shown, and how an individual's risk preference is formed concave. It is illustrated, that the certainty equivalent ($U(W)$) i.e. the amount an individual will accept with certainty, of not taking the bet B, is greater than the expected payoff ($0.5U(W+B)+0.5(U(W-B))$) from taking the bet.

Figure 2.1: Utility and Risk Aversion



Source: By own creation

The higher the degree of curvature is, the higher the risk aversion will be. The expected utility functions are not uniquely defined and therefore not comparable. The economists Arrow (1965) and Pratt (1964) have defined a measurement to make risk aversion comparable. The measurement is known as the coefficient of absolute risk aversion (ARA) and is defined as

$$ARA(W) = -\frac{U''(W)}{U'(W)}$$

and quantifies the absolute dollar amount that an investor is willing to pay to avoid a bet of a given absolute size. Correspondingly, this measurement can be translated to a relative term also known as the coefficient of relative risk aversion (RRA).

$$RRA(W) = -\frac{U''(W)}{U'(W)}W$$

The RRA measurement determines the fraction of wealth that an investor is willing to pay to avoid a bet of a given size relative to wealth. It is to be considered a very useful benchmark tool as the measurement is independent of wealth.

2.1.1 Power Utility Function

Since the utility function is concave the absolute risk aversion should decline. This favours the so-called power utility function (Luis M. Viceira and John Y. Campbell, 2001) that has the following characteristics

$$U(W) = \begin{cases} \frac{W^{1-\gamma}}{1-\gamma} & \gamma \neq 1 \\ \ln(\gamma) & \gamma = 1 \end{cases}$$

This utility function clearly shows, that only the parameter γ affects the utility. A more thorough understanding of the parameter will become clear by computing ARA and RRA:

$$ARA(W) = -\frac{U''(W)}{U'(W)} = \frac{\gamma}{W}$$

Obviously, the power utility shows a decreasing absolute risk aversion. Meaning, as wealth increases, one should hold more dollars in risky assets. Finally, we can calculate the RRA

$$RRA(W) = -\frac{U''(W)}{U'(W)}W = \gamma \tag{2.1}$$

With a power utility we only have one parameter showing that the relative risk aversion only depends on γ. Hence, the result is called Constant Relative Risk Aversion (CRRA). As a consequence of CRRA utility, one will hold the same percentage of risky assets as wealth increases. The interpretation of γ is very clear. It is a parameter that explains an investor's risk aversion and is invariant to the scale of wealth, meaning, we can easily understand the risk preferences for an individual simply by the value of γ no matter the amount of wealth. Furthermore, we can also use γ as a comparison between different risk attitudes for different individuals. The implications of various γ levels will be explained in section 2.2.4 and 4.5.3.

2.2 Modern Portfolio Theory

Modern portfolio theory was developed by Markowitz (1952) who laid the foundation between risk and expected return in his article 'Portfolio Selection'. Before his publication the general perception was that assets with high expected returns were also assets with high variance. However, Markowitz proved by his work with mean variance analysis that the correlation between assets also have an impact on the portfolio variance and thereby the investor could lower his risks by diversifying his assets.

2.2.1 Mean Variance Analysis

Consider an investor having to choose between two different assets at time t. One of the assets are risk free from time t to time $t+1$ noted as $R_{f,t+1}$. The other asset has risk and its return from time t to time $t+1$ is given as R_{t+1} with its mean $E_t R_{t+1}$ and has a variance σ_t^2. Observe that the timing convention that returns have time subscripts on the date at which they are realised; the risk free asset is realised at $t+1$ but is already known one period in advance at time t. Hence, the certainty leaves the risk free asset with no variance. On the other hand, the mean and variance are conditional on the investor's information at time t and therefore they are given t subscripts.

In this simple portfolio choice problem the investor will invest α_t of his portfolio in the risky asset and the remaining in the risk free asset. Then the return of the whole portfolio will be

$$R_{p,t+1} = \alpha_t R_{t+1} + (1-\alpha_t) R_{f,t+1} = R_{f,t+1} + \alpha_t (R_{t+1} - R_{f,t+1}) \tag{2.2}$$

The mean return is $E_t R_{p,t+1} = R_{f,t+1} + \alpha_t (E_t R_{t+1} - R_{f,t+1})$ and the variance of the portfolio return is

$$\sigma_{p,t}^2 = \alpha_t^2 \sigma_t^2 \tag{2.3}$$

Since the investor has CRRA preferences he prefers a high mean and a low variance of portfolio returns (Luis M. Viceira and John Y. Campbell, 2001).

Assuming the tradeoff between mean and variance is linear (Luis M. Viceira and John Y.

Campbell, 2001), the investor maximises a linear combination of mean and variance.

$$\max_{\alpha_t} \left(E_t R_{p,t+1} - \frac{\gamma}{2}\sigma_{p,t}^2 \right) \tag{2.4}$$

γ quantifies the investors risk aversion, which arrived from equation (2.1). A high value of γ leads to an investor with conservative risk attitude meanwhile a low value of γ represents an aggressive investor. By substituting (2.2) and (2.3) into the maximisation problem it can be rewritten as

$$\max_{\alpha_t} \alpha_t \left(E_t R_{t+1} - R_{f,t+1} \right) - \frac{\gamma}{2}\alpha_t^2 \sigma_t^2 \tag{2.5}$$

By solving this maximisation problem one will arrive to the solution

$$\alpha_t = \frac{E_t R_{t+1} - R_{f,t+1}}{\gamma \sigma_t^2} \tag{2.6}$$

This means, that the portfolio's share in the risky asset should be equal to the expected excess return, or risk premium, divided by conditional variance times the coefficient γ that represents aversion to variance. This result derived originally from Merton (1969) and Samuelson (1969) as a solution to an optimal portfolio choice. The publications started the theory of lifecycle investing (Wilson & Droms, 2009).

Modern portfolio theory is a relatively simple nonlinear programming model that maximises wealth over a single period. Lifecycle investing is on the opposite based on multi-period consumption-maximisation. Samuelson and Merton maximises consumption over a lifetime by using dynamic stochastic programming.

2.2.2 Diversification of Assets

Using multiple risky assets it is now possible to look upon the effects of diversification of assets. Diversification simply means to reduce risk by investing in multiple different assets. It is the concept from the famous sentence "*Don't put all your eggs in one basket*". Dropping the basket will break all the eggs. Likewise, only having one risky asset in a portfolio will lead to a high risk, which is in contradiction to the risk preferences that have been presented. By having two risky assets X and Y the relationship movements are defined as the covariance between them.

$$\text{Cov}(X,Y) = \sigma_{X,Y} = E[(X - E(X))(Y - E(Y))] \tag{2.7}$$

A positive covariance means that X expects to rise, when Y rises and vice versa. A negative covariance means that X expect to fall when Y rises. This can be a bit unclear and difficult to grasp since it is values in absolute terms. However, by using the correlation between X and Y it is given in relative terms and is computed as

$$\text{Corr}(X,Y) = \rho_{X,Y} = \frac{\sigma_{X,Y}}{\sqrt{\sigma_X^2 \sigma_Y^2}} \tag{2.8}$$

The correlation is in the interval of $[-1;1]$, and is therefore better to use as a comparison. Now that the concepts of covariance and correlation have been defined, the variance of a portfolio with two risky assets from equation (2.3) can be rewritten. Note that σ_i^2 is the variance of asset i and $\sigma_{i,j}$ is the covariance between the assets i and j.

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j\sigma_{i,j} = \overbrace{\sum_{i=1}^{N}\alpha_i^2\sigma_i^2}^{\text{idiosyncratic}} + \overbrace{\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\alpha_i\alpha_j\sigma_{i,j}}^{\text{Systematic}}$$

The idiosyncratic risks are risks that derive from single assets whereas the systematic risks are risks from the combination of holding different assets. As the amount of assets grow in a portfolio the idiosyncratic risks will be reduced asymptotically, but the systematic risk will remain. In other words, the investor can lower his risks by adding more assets in his portfolio given the new assets are not perfectly correlated.

2.2.3 Mean Variance Analysis With Multiple Assets

Until now, we have looked upon two assets in the mean variance analysis. However, the model can easily be extended to multiple assets. By altering the boldface letters as vectors and matrices we can derive to the same setup. Having $\boldsymbol{R}_{t+1}$ as a vector of returns from risky assets with N elements, we also have a mean vector $E_t\boldsymbol{R}_{t+1}$. $\boldsymbol{\Sigma_t}$, which is a variance-covariance matrix and is positive semidefinite, hence an invertible covariance matrix exists as $\boldsymbol{\Sigma_t}^{-1}$. At last, $\boldsymbol{\alpha_t}$ is now a vector of allocations to the risky assets. The maximisation problem now becomes

$$\max_{\boldsymbol{\alpha}_t} \boldsymbol{\alpha_t^T}(E_t\boldsymbol{R}_{t+1} - R_{f,t+1}\mathbf{1}) - \frac{\gamma}{2}\boldsymbol{\alpha_t^T}\boldsymbol{\Sigma}_t\boldsymbol{\alpha}_t \tag{2.9}$$

where $\mathbf{1}$ is a vector of ones, and $(E_t\boldsymbol{R}_{t+1} - R_{f,t+1}\mathbf{1})$ is the vector of excess returns on the N risky assets over the risk free asset. The variance of the portfolio return is $\boldsymbol{\alpha_t^T}\boldsymbol{\Sigma}_t\boldsymbol{\alpha}_t$. The solution to this maximisation problem is

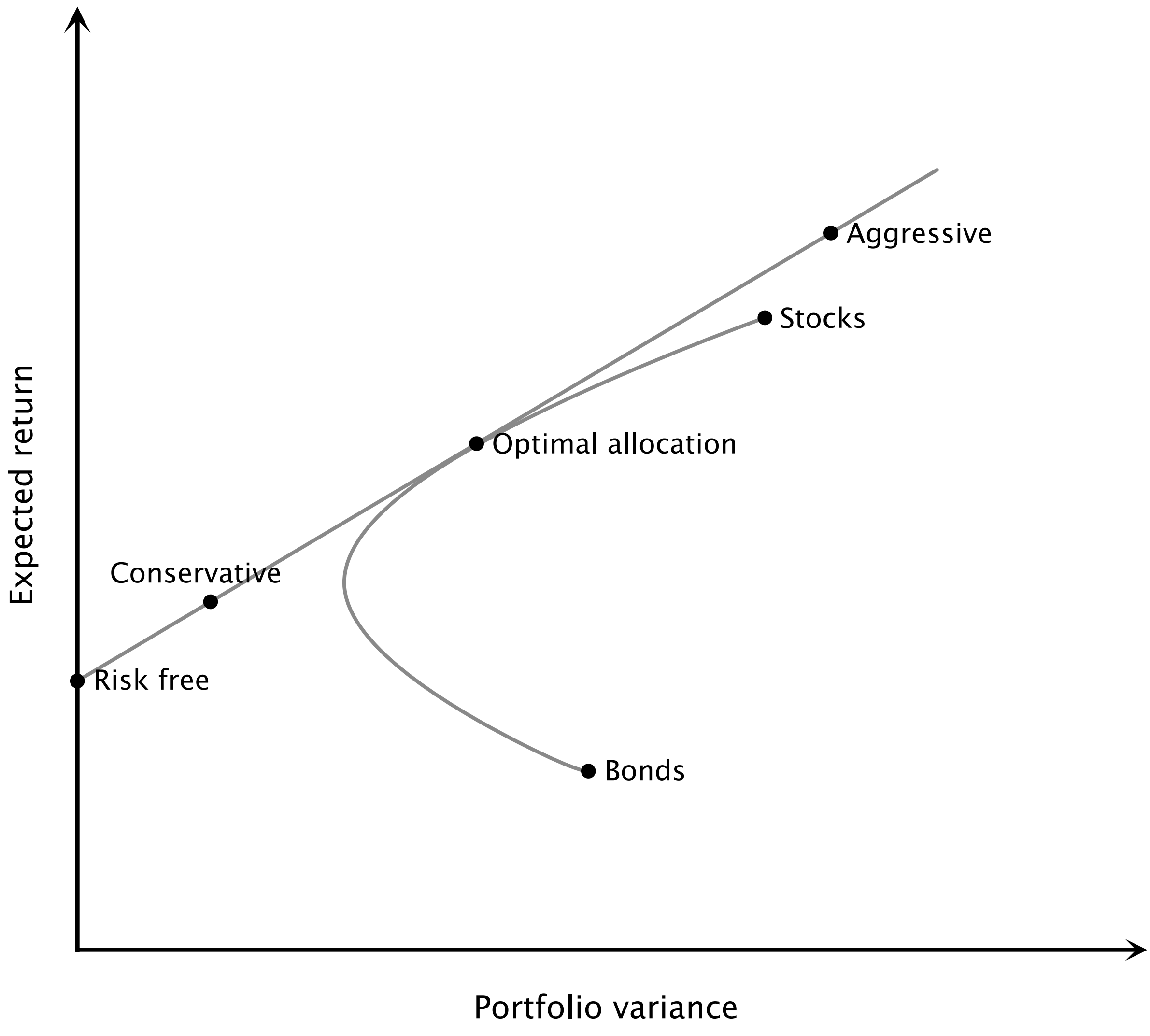
$$\boldsymbol{\alpha}_t = \frac{1}{\gamma}\boldsymbol{\Sigma}_t^{-1}(E_t\boldsymbol{R}_{t+1} - R_{f,t+1}\mathbf{1}) \tag{2.10}$$

Clearly, this result is a simple extension of equation (2.6) where one risky asset has been replaced by multiple assets. Still the optimal asset allocation is depending on the risk preferences $\frac{1}{\gamma}$ and not the composition of assets. In other words, an investor with a high γ will not hold *different* risky assets compared to an investor with a low γ, but will have a lower *fraction* of risky assets.

2.2.4 The Efficient Frontier

The efficient frontier was introduced by Markowitz in 1952 showing that by holding different assets one could obtain different set of portfolios. These different portfolios create a hyperbola, on a expected return - variance plot, as shown on figure 2.2, and the upward sloped part of the hyperbola is called the efficient frontier. If a portfolio lies upon the efficient frontier, the investor cannot increase his expected return without taking more risk, as well as, the investor cannot decrease his risk without achieving less expected return. If a risk free asset is available, the investor can combine this risk free asset with his portfolio.

Figure 2.2: The Efficient Frontier



Source: By own creation

Note that the figure is simply a sketch, thus the placement of stocks and bonds are not precise, although there is a tendency that stocks have higher expected return and variance compared to bonds. The important thing is, by allocating the holdings of a

portfolio between assets (stocks and bonds) the idiosyncratic risks decreases.
The optimal portfolio is the one with the highest Sharpe ratio S_t. A ratio that is defined as the excess return per unit of deviation.

$$\mathrm{S}_t = \frac{E_t R_{p,t+1} - R_{f,t+1}}{\sigma_{p,t}} \tag{2.11}$$

This result is closely related to (2.6), and by rewriting the equation one can see

$$\alpha_t = \frac{S_t}{\gamma \sigma_{p,t}}. \tag{2.12}$$

This solution can provide a better understanding of the risk parameter γ by viewing figure 2.2. The asset allocation that has the highest Sharpe ratio is the optimal allocation found in equation (2.6). The tangent that intersects this point and the risk free rate is a line that an investor can lie upon depending on his risk preferences (γ value). By having a higher γ an investor tends to hold a lower fraction of the risky assets, while an investor who has a low γ will have a higher fraction of the risky assets. Given the investor is allowed to borrow at the risk free rate, he can leverage his portfolio.

3 Model Presentation

Different studies (Viceira, 2001), (Kubler & Paul, 2006) suggest that investing on leverage is not necessarily only for the short term investor, but is also a useful tool for the long term investor who seeks to reduce his risk. Although it is a common piece of advice to buy a home with leverage, the consensus is quite different when it comes to buy other assets on leverage such as stocks. However, one unique study (Ayres & Nalebuff, 2010a) approaches this issue differently: They argue that buying stocks with leverage can reduce risk. Their work is based upon Merton (1969) and Samuelson (1969), which corresponds to equation (2.6), but argues that as long as an investor does not start with all his wealth upfront, he should invest his discounted lifetime savings and not his current savings. They develop a strategy for investing with leverage called the leverage lifecycle strategy. Their model is based upon investing in a risky asset class (stocks) and a risk free asset (government bonds).

However, this thesis will demonstrate what happens when the risky asset universe will expand and new risky asset classes will be added. Therefore, this thesis should be seen as an extension of Ayres and Nalebuff's strategy. In this section, first and foremost a discussion of leveraged markets will be introduced, following the underlying concepts and theory from Ayres and Nalebuff (2010a) as well as a discussion of the premises of their assumptions and limitations.

3.1 Leveraged Markets

It is quite an uncommon piece of advice to invest in stocks on leverage. On the contrary, this is not the case for home mortgage where it is commonly advised to buy a home on leverage. This is clearly seen in the United States' economy where outstanding mortgage debt for residential are approximately 79.8% of the total GDP (Economic Research, 2015). Furthermore, the average loan-to-value for United States for housing was 76% in 2011 and in some instances loan-to-value ratios at 100% were even available before the home mortgage crisis in 2008 (International Monetary Fund, 2011). Historically, the housing market has not always been that highly indebted. A study (Green & Wachter, 2005) identifies the development of leverage in the housing market in the United States and argues it has evolved significantly:

> "The U.S. mortgage before the 1930s would be nearly unrecognisable today: it featured variable interest rates, high down payments and short maturities. Before the Great Depression, homeowners typically renegotiated their loans every year."
> Green and Wachter (2005)

The loan-to-value ratio before the Great Depression was typically 50% or lower. Various regulatory programs[1] have afterwards been created. Until the 1930s, it was imprudent if not preposterous to leverage a housing investment on a loan-to-value ratio on 5:1. However, the following period up to 2000 lead to a massive increasing leverage in the housing market yet no financial apocalypse occurred, quite the contrary, as leveraged investment led millions of families in the 20th century into the housing market and gained a relatively safe investment on a leveraged basis (Ayres & Nalebuff, 2010b). From 2000 to 2006 the housing market increased 80%, but has lost more than a third of the value since then. Clearly, this is not such a stable investment as it has been previously, but the loan-to-value ratio was also tremendously high and the median down payment made by

[1] 1933 FDIS, 1933 HOLC, 1936 FHA, 1938 Fannie Mae, 1970 Freddie Mac are some of the few programs

first time home buyers was 2% in the United States (Trejos, 2007).

The authors Ayres and Nalebuff (2010a) argues that as moderate leveraged (ratios within 5:1 and 20:1) investment in the housing market has empirically led to a stable investment for decades, and so is the case for the stock market. They challenge the consensus of leveraged stock investment belongs to the short-term investor or speculator to gain high expected return. As most people are encouraged to buy a home on leverage, so should the long-term investor replicate such strategy in order to reduce risk.

3.2 Diversification Across Time

To clarify the interpretation of the work from Samuelson (1969) and Merton (1969) it is necessary to understand the concept of compound effect. Consider a two-period allocation problem with a risky and risk free asset. The investor has wealth denoted as W and chooses his weights in the risky assets as α_t and α_{t+1} in order to maximise his expected future wealth.

$$E_t\left(W(\alpha_t R_{p,t} + (1-\alpha_t)(1+R_f) \cdot (\alpha_{t+1} R_{p,t+1} + (1-\alpha_{t+1})(1+R_f))\right) \tag{3.1}$$

In this case the investor is choosing both asset allocations α_t and α_{t+1} before observing the returns, and that implies that the investor takes his second decision only after having observed the outcome of the first one. The return at time t will amplify the return at time $t+1$ as a well known fact. But at the same time the expected return at time $t+1$ will amplify the return at time t. Meaning, if an investor expects a return at period $t+1$ on 10%, it is as if he takes a 10% bigger gamble on the first period. Thus, one can see the investment decisions symmetry, which is the concept that Samuelson (1969) and Merton (1969) argued for. Therefore, an investor should be indifferent towards which period his investments provide the best return. However, not all investors put all their wealth up front, and it is certainly not the case for retirement investing where savings are constantly added to the wealth. Such conditions change the situation e.g. if an individual invest with a constant fraction in a risky and risk free asset, the portfolio will throughout time look like \$100, \$200, \$300 and as a result making the individual more exposed to risks in the last period, i.e the period closest to retirement.

The leverage lifecycle strategy (Ayres & Nalebuff, 2010a) is trying to model a strategy so its conditions are closer to \$200, \$200, \$200 and thereby it reduces the overall risk. This is because the investor does not mostly depend on the return in period 3, but instead he takes advantages of all periods. This is the concept of 'Diversification Across Time', which is also the title from Ayres and Nalebuff (2010a). It allows an investor to diversify his investments through different periods, instead of being dependent arbitrarily on the latest period.

In order to do so, one must differentiate between *current savings* and *future earnings*. Current savings are the savings the investor currently has disposable at his retirement savings account i.e. they are liquid. By only investing current savings with a constant fraction in risky assets as Merton and Samuelson suggests, the investor will achieve the \$100, \$200, \$300 portfolio since he constantly contributes to his savings account. This leads to more vulnerability in the latest period and thereby a higher risk. On the other hand, if he invests his current savings and future earnings he will obtain the \$200, \$200, \$200 portfolio. From a practical perspective it is difficult to invest future earnings. However, by employing leverage while being young and later on to deleverage an investor can, in effect, construct similar conditions for diversifying a portfolio's return across time.

3.3 The Leverage Lifecycle Strategy

The leverage lifecycle strategy is based on the established theoretical foundation from section 2, and with an investor who has CRRA preferences defined as equation (2.10). Since the investor is diversifying across time by using leverage, his optimal weight in risky assets will not be defined as equation (2.6) at each time t. It will instead be translated so that he has leverage in his early period, and in the later period the investor will be deleveraged so he will achieve the same overall risk as equation (2.6). Therefore, his optimal weight in risky assets α_t must change at each time t. In order to determine this α_t we must define some factors to begin with.

The setup of the strategy is defined so an investor starts with no current savings, but contributes to his current savings S on a monthly basis. His future earnings are defined as F. He cannot invest his future earnings directly, but is allowed to buy assets from his current savings on leverage. Naturally, he cannot borrow unlimited amounts, so he is constrained to borrow a maximum up to a certain cap, which is defined as α_{cap}. When he borrows the principal, he must repay at the margin interest rate $R_{m,t}$. It is assumed that the margin interest rate is greater than the risk free rate i.e. $R_{m,t} > R_{f,t}$. As long as the investor is leveraged, his discount rate is the margin rate, which would otherwise be the risk free rate. Thus, while he is leveraged, his future earnings are defined as $F_{m,t}$ and unleveraged $F_{f,t}$. While the investor has α_{cap} as the highest amount of risky assets, he also has a lowest amount of risky assets defined as α_{min}. The investor will achieve this α_{min} at the last period. We can now derive to the optimal allocation in risky assets

$$\alpha_t = \begin{cases} \alpha_{min}\frac{F_{f,t}+S_t}{S_t} & \text{Unleveraged} \\ \alpha_{min}\frac{F_{m,t}+S_t}{S_t} & \text{Leveraged} \end{cases} \tag{3.2}$$

The first solution is when the investor is unleveraged and the other when he is leveraged, and in both cases he may not exceed the cap limit. Consequently, it is quite simple from a mathematical viewpoint, but the intuition behind it is somewhat complex. The variable α_{min} is determined by the investor's CRRA level. It is not directly shown in the formula, but when α_{min} is 0, the investor will only invest in the risk free asset and likewise when α_{min} is 1, the investor will only invest in risky assets. Throughout time $F_{f,t}$ and $F_{m,t}$ will decrease meanwhile S_t increases as the investor will add more contribution to his savings and have less future earnings. However, as S_t is current savings, it can also decrease since the savings are invested in risky assets. Consequently, when risky assets depreciate from time t to $t+1$ the investor must increase his amount in risky assets. This is exactly the concept of an investor with the CRRA preference, thus this requirement has been fulfilled. Furthermore, the formula does not depend on the investor's age or the maturity to retirement.

3.3.1 The Four Phases

Now the basic setup has been developed and we can derive to the investment strategy. The strategy consists of four phases illustrated as an example in figure 3.1. Each phase has different targets depending on the CRRA level for the given investor.

Phase I:

$\alpha_t \geq \alpha_{t,cap}$. All the investor's current savings are invested at maximum leverage.

This is the early period of the investor's retirement savings. Since he has no or only little current savings and a large amount of future earnings, his optimal asset allocation will naturally be high according to equation (3.2). However, the investor is conditioned to buy risky assets up to the cap limit. Since he is leveraged, his future earnings must be discounted with the margin interest rate. Once $\alpha_t = \alpha_{t,cap}$ the investor goes into the next phase.

Phase II:

$\alpha_{m,t} \leq \alpha_t < \alpha_{t,cap}$ The investor deleverages until he achieves $\alpha_{m,t} = \alpha_t$.

In this phase the investor is still leveraged, but has less than the cap limit. He deleverages until he is no longer leveraged and invests 100% of his allocation in risky assets.

Phase III:

$\alpha_{r,t} \leq \alpha_t < \alpha_{m,t}$ The investor puts all wealth into risky assets.

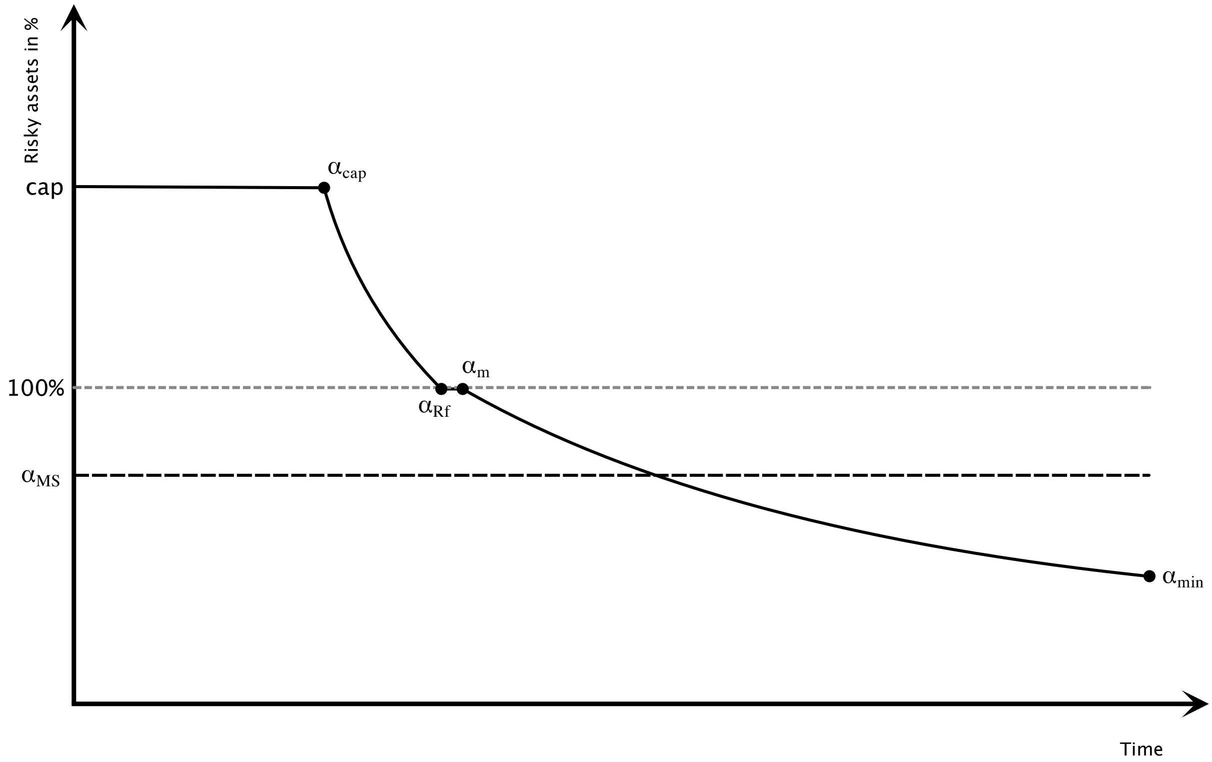
Now the investor is no longer leveraged and his future earnings must therefore be discounted with the risk free interest rate. The spread between the margin and risk free rate determines the length of this phase. If $R_{m,t} = R_{f,t}$ then this phase will not exist, and the investor will automatically skip this phase and move directly from phase II to IV. On the other hand, if the difference between the margin and the risk free rate is large then the investor will be discouraged to leverage, because of the high costs and thus the duration of this phase will last longer.

Phase IV:

$\alpha_{min} \leq \alpha_t < \alpha_{r,t}$ The investor puts his current savings into risky assets and the risk free asset.

This is the last phase where the investor is still unleveraged and now adds the risk free asset into his portfolio. The investor has acquired more risky assets in the prior phases compared to the strategy based on the solution from Merton and Samuelson (denoted as α_{MS}). Therefore, he will in this phase start with a higher fraction in risky assets meaning that $\alpha_t > \alpha_{MS}$, but will throughout time pass α_{MS} and have an even lower fraction in risky assets. Thus he will end this phase with $\alpha_t < \alpha_{MS}$ as illustrated in figure 3.1.

Figure 3.1: The Four Phases



Source: By own creation

Although figure 3.1 seems to have a smooth development it is not always the case. Since risky assets can have negative return during a period, it can decrease an investor's current savings but maintain the future earnings. It implies from equation (3.2) that the investor in such instances will be momentarily set back to the previous phase.

3.4 Benchmark

In section 5 the leverage lifecycle strategy will be analysed and its performance will be used as comparison to different strategies. One of the strategies that will be used as a benchmark tool is the solution from equation (2.6), which is the result from Merton and Samuelson. It will be denoted as the MS-strategy. This strategy is a good benchmark tool since it is the original version of the leverage lifecycle strategy, but uses current savings instead of future earnings. Consequently, a constant fraction of current savings will be invested in risky assets, and therefore the concept of diversification across time can be examined on an empirical basis.

The other strategy is a strategy that is used in target-date funds. This strategy will be denoted as the TDF-strategy. It has a defined target-date that is a maturity date for an investor. It rebalances and thereby the investor will have a high fraction of risky assets in the early period, and decrease through time until he reaches the maturity date. It is a popular investment strategy within retirement investing and since institutions have launched the first fund in 1993 November, the industry has increased to a $185 billion asset industry only 15 years later (Barclays Global Investors, 2008). This strategy is similar to the leverage lifecycle strategy, since it decreases its amount of risky assets through time, but it is not leveraged. There are different approaches to determine the fraction of risky assets. In this thesis, the investor's starting point will be 100% in risky assets and that will decrease each month. The final allocation will depend on his level of CRRA.

3.5 Obtaining Leverage

There are different approaches to gain leverage, which the leverage lifecycle strategy demands and such approaches will be discussed in this subsection. The obvious way to gain leverage is to buy assets on margin, but it is also possible to gain leverage by different derivatives and certificates. Although they may differ in their structure, leverage and legal practicalities, one thing they all have in common is a premium. The costs of gaining leverage can be significant and in some instances be expensive for the investor. Naturally, the investor wishes to gain leverage with lowest possible costs.

3.5.1 Leveraged Exchanged Traded Funds

ETFs are Exchanged Traded Funds that tracks an underlying instrument which could be an index. They also exist as leveraged ETFs (LETF) that delivers the double or triple return of the underlying index. They also exist with inverse properties so they provide a negative return when the underlying index provides a positive return. Obviously, such LETFs will not be used since the strategy is based on long-term positive returns.

LETFs have a daily reset application so that the leverage ratio is the same at the end of each day as they were at their initial public offering. The reset application ensures the return on each day is correctly leveraged. In table 3.1 the impact of resetting is illustrated. During a 5-period timeline the underlying index is volatile, but ends at the original price, whereas the LETF 2x is ending at a lower price than the original. Clearly, such nature of

Table 3.1: LETF volatility effect

Time	Underlying index	Return on index	LETF 2x	Return on LETF 2x
0	100.00		100.00	
1	98.00	-2.00%	96.00	-4.00%
2	100.00	2.04%	99.92	4.09%
3	102.00	2.00%	103.92	4.00%
4	100.00	-1.96%	99.84	-3.92%

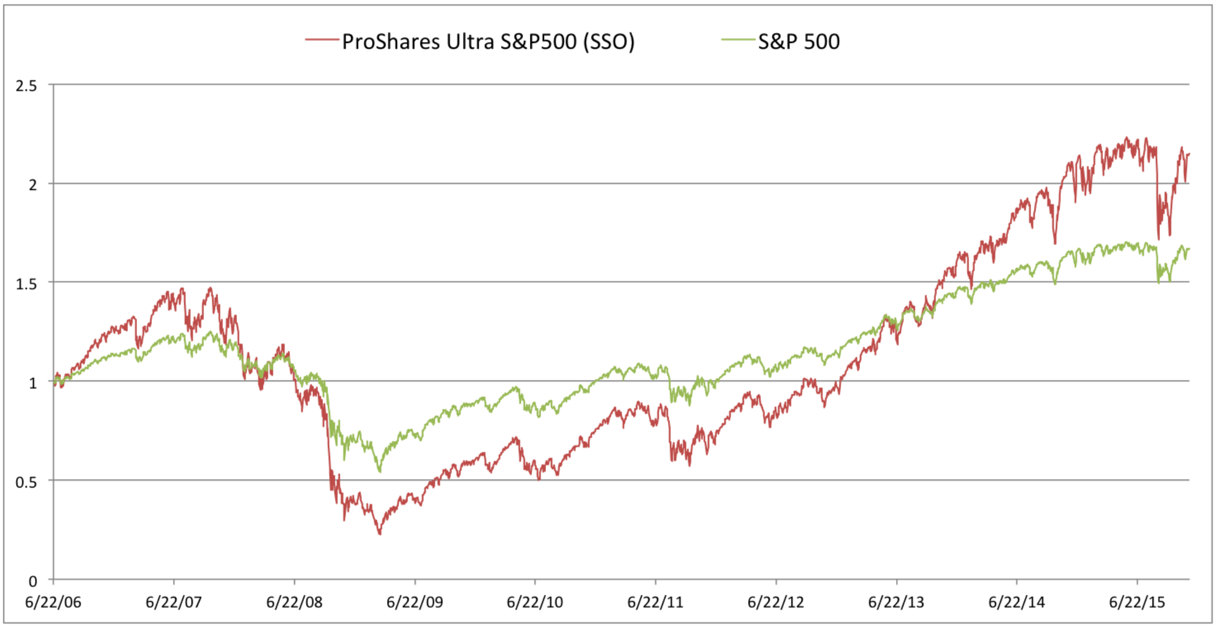
investment is not desirable for the investor since he loses money over time and volatility. A study from Chang and Madhaven (2009) shows the return on LETFs can be defined as

$$(1 + R_{T,LETF}) = (1 + R_{T,index})^x \, e^{\frac{1}{2}\left(x-x^2\right)\sigma^2 T} \tag{3.3}$$

where σ is volatility of the index, x is leverage ratio and T is the time period for which the investment has been held. As $T > 0$ and $\sigma > 0$ then $e^{\frac{1}{2}(x-x^2)\sigma^2 T}$ must be less than 1, which corresponds to the phenomenon 'time-decay' (as time goes by the LETF is losing value).

It should be noted that if the volatility is high enough and the LETF has been held for long enough time, then the constant will be small and the underlying index can have a higher return than the LETF. This situation is illustrated in figure 3.2 where the underlying asset is S&P 500 and the LETF is ProShares Ultra S&P 500 (SSO) and has 2x leverage and is resetting daily. The comparison is based upon the performance for both instruments. Clearly, the impact becomes significant over time since S&P 500 ends at 1.67 and the LETF at 2.15. Also, after the financial crisis in 2008 the S&P 500 falls below 1, and when it recovers then the LETF is still below 1 and takes a long time to recover.

Figure 3.2: LETF With Daily Reset



Source: By own creation

Obviously, one can see that LETFs are not suitable for the long-term investor who seeks to replicate an index with leverage. This is also the conclusion of other studies

(Avellaneda & Zhang, 2010)[2], (Little, 2010), (Ilan Guedj & McCann, 2010). Nevertheless, these LETFs can be used for other purposes such as short-term investing, hedging, trading etc.

3.5.2 Margin

When an investor buys risky assets on margin, it implies that he is borrowing from the broker using other risky assets as collateral in order to gain leverage. There are two requirements that must be fulfilled before one can buy assets on margin:

Initial margin requirement is the amount of collateral the investor must have in order to open the margin account.

Maintenance requirement is the amount of collateral the investor must have in order to keep the account open.

If an investor buys risky assets on margin, and the price of the risky asset falls to a certain level, he will receive a margin call from the broker, which means he must either deposit additional money or risky assets. The margin call occurs when the price (P) drops to

$$P = P_{\text{initial}} \frac{1 - \text{Initial margin requirement}}{1 - \text{Maintenance requirement}} \tag{3.4}$$

An example to illustrate this is, if the investor buys an asset for \$100 and has initial margin requirement on 50% and maintenance requirement on 25%, he will receive a margin call when the asset drops to

$$P = \$100 \frac{1 - 0.5}{1 - 0.25} = \$66.67$$

Under such circumstances, the asset can lose a third of its value before the investor receives a margin call. Since the broker can make a margin call and sell the investor's risky assets if the investor does not increase his collateral, the broker is providing a fairly safe loan

[2]This article gives a greater statistical analysis including a clearer implication of the losses for buy-and-hold strategies with LETFs.

to the investor. Therefore, the loan is low risk and the margin rate should intuitively not be much higher than the risk free rate. That makes the margin premium smaller and the investor can thus buy risky assets on leverage for lower leverage costs.
More importantly, there is no time decay effect for buying risky assets on margin, which makes it more transparent and suitable for the long-term investor. For these reasons margin trade will be used throughout this thesis as the most appropriate solution similar to the choice of Ayres and Nalebuff (2010a).

3.6 Risk Measurements

The investor's objective is to maximise his final wealth i.e. the wealth which has been accumulated at maturity, and at the same time minimise the uncertainty. The strategy that provides the highest final wealth with lowest uncertainty is thereby the most desirable strategy. Given that the investor is investing in risky assets there will be an uncertainty of his final wealth. In section 5 each strategy will be projected with a range of possible final wealth outcome. The standard deviation of the final wealth captures the amount of uncertainty. However, this statistical measurement falsifies the investor's preferences towards uncertainty in regard to his utility. This can be illustrated blatantly with a simple example:

Table 3.2: Example of standard deviation

Strategy	A	B
Final Wealth 1	8	7.35425
Final Wealth 2	8	9
Final Wealth 3	10	10
Final Wealth 4	12	11
Final Wealth 5	12	12.64575
Mean	10	10
Standard Deviation	2	2

Table 3.2 shows that strategy A and B have the exact same mean and standard deviation, but the underlying observations are quite different. The investor's utility preferences are defined as CRRA, thus he prefers strategy A over B. This is because the worst case

scenario from strategy B affects the investor more negatively than the gain benefits him. In section 3.6.1 and 3.6.2 two different risk measures will be examined in order to analyse the risks each investment strategy contains.

3.6.1 Value at Risk

Value at Risk (VaR) is a widely used risk measurement, which estimates the losses on a portfolio that occur with a given probability and duration. It is used by various participants in the financial sector (McDonald, 2013). First, regulators use VaR to assess capital requirements for financial institutions. Second, managers uses VaR as an input in making decisions for risk taking and risk management. Third, managers uses VaR to assess the quality of the bank's financial models. In this thesis, it will be used with the distribution of outcome provided by each investment strategy. Therefore, it captures the question: What is the probability that by following a certain strategy, the investor will fail to achieve a desired minimum level of final wealth?

By defining a few variables, VaR can be examined mathematically. Let L be defined as a stochastic variable that describes a potential loss and α as the confidence level. $VaR_\alpha(L)$ is given by the smallest number c so that the probability that the loss L exceeds c is no larger than $1-\alpha$ (Hult & Lindskog, 2007).

$$VaR_\alpha(L) = inf\{c \in \mathbb{R} : P(L > c) \leq 1 - \alpha\} \tag{3.5}$$

The interpretation of this equation should be understood as: "With an $\alpha\%$ probability the investor will not achieve more than c".

Although VaR is widely used it is not a flawless measurement. Academically, VaR is criticised for not being a subadditive measurement. That means it repudiates the concept of diversification, thus VaR of a combined portfolio can actually be larger than the sums of its components.

Another issue that is more relevant in regard to this thesis is, that VaR only describes the probability that a deficient final wealth can occur, but does not indicate how deficient the final wealth can become. If for instance an investor uses VaR with α of 95% he can determine the 5% lowest final wealth outcomes. This information completely ignores how low all the observations below 5% can be. Will the investor end up penniless if he achieves the minimum outcome or is the minimum outcome close to the 5% lowest outcome? The conditional-VaR measurement also known as the *expected shortfall* quantifies the answer to this question.

3.6.2 Expected Shortfall

The expected shortfall is a risk measure that evaluates the risk of a portfolio by estimating the mean within the VaR interval.
It is expressed in the following equation, where X is the final wealth

$$ES_\alpha = \frac{1}{\alpha}\int_0^\alpha VaR_\gamma(X)d\gamma$$

and it is conditioned that X is within the interval

$$ES_\alpha = E[-X|X \leq -VaR_\alpha(X)]$$

By using expected shortfall as a risk measure, the strategy that leaves the investor penniless will be punished more than the strategy that give a higher minimum outcome.
The expected shortfall is a subadditive risk measurement, which gives it advantages compared to VaR. It also contains more information than VaR because it captures the mean of multiple observations and not a single observation. It is also more sensitive for outliers. On the other hand, the expected shortfall is more difficult to comprehend because the value defines a part of multiple observations.
Due to its advantages it will be used as the risk measurement for the analysis as a sup-

plemental measurement to compare the different strategies.

3.7 The Model Practicalities

3.7.1 Rebalancing

One of the things the investor's asset allocation from equation (3.2) depends on, is the return on his investments. E.g. in phase II the investor may in a period experience that his investments deliver high returns, so his current savings will increase and he will have to rebalance his asset allocation. Rebalancing asset allocation can be done with different intervals. In this thesis it will be done on a monthly basis to make it more convenient, as it is assumed the investor contributes monthly to his portfolio and also to ease up on the calculations. Although it is assumed that there are no transaction costs it will in practice be a very costly strategy to rebalance on a daily basis.

3.7.2 Cap Limit

So far the cap limit has not yet been defined in this thesis nor has its existence been reasoned. If the concept of diversification across time holds true, and it is beneficial to employ leverage in the early period, why should there be any limit? As discussed in section 3.1 leverage with some sort of moderation may be beneficial, meanwhile excessive use of leverage can do more damage than good. In the article from Ayres and Nalebuff (2010a) the authors have made an empirical analysis showing that a high amount of leverage will require high margin rate. Therefore, the margin premium will actually decrease at a certain point, since the cost of borrowing will be too high. Another issue is that with higher leverage the chance of margin call will increase as well as resulting in a higher probability of wipeout, and the investor will have to start over. For these reasons it will be assumed that the cap for leverage will be 2:1 as a moderate leverage level compared to the housing market, and it will be the same cap as in the article of Ayres and Nalebuff (2010a).

3.8 Assumptions and Limitations

The leverage lifecycle strategy is based upon multiple assumptions, which enables the model to work on a theoretical foundation. Such assumptions can have a significant influence on the results as well as they might even prohibit the strategy in the first place. A discussion of these assumptions is necessary to grasp their consequences.

3.8.1 Legal Constraints

It is assumed that an investor's retirement savings are unregulated so that he may invest as he sees fit. Regarding the retirement plans in the United States, the issue of leverage is not straight forward. In the retirement plans 401 (k), 403 (b) and 457 it is prohibited to employ leverage (Internal Revenue Service, 2015). However, in Roth IRA it is a bit more flexible and it is actually allowed to use leverage to buy stock indices on margin. So although there are some constraints in some of the retirement plans in the United States, it is different from country to country and on a worldwide basis one is not completely unconstrained.

3.8.2 Correlation to the Real Estate Market

The leverage lifecycle strategy does not take the implications from the real estate market into consideration. Most people are exposed to housing market through either real estate or rent. If a person is highly leveraged in his real estate and retirement savings, a high correlation between these two markets will increase his risk. A study (Quan & Titman, 1996) suggests that on global scale there is a significant correlation between stock returns and changes in real estate value.

3.8.3 Correlation to Human Capital

The leverage lifecycle strategy has not co-integrated the value of human capital. Depending on the outcome of risky assets that will be discovered in section 4.4 the profession of the investor will naturally have an impact. E.g. if the risky assets contains a large amount of real estate and the same time the investor's profession is exposed to that market by being a real estate agent or similar, then he will face a greater risk towards the real estate market. The investor could also be working in the stock market and if the risky assets contains a large fraction of stocks his labor income risk is also affecting the leveraged lifecycle strategy. One study (Luca Benzoni & Goldstein, 2007) actually shows that when taking human capital into consideration with the portfolio choice problem, a young person should short the market because the value of the claim to labor income is implicitly exposed to the market portfolio.

3.8.4 Intuitions from Economics

One of the basic economic theories is the theory of demand and supply that are the factors, which determines prices of a given object. The theory is applied to almost every aspect of economics and finance and the leverage lifecycle strategy is not an exception. Naturally, if everyone applies the leverage lifecycle strategy, the demand will increase in margin rate and risky assets. The empirical consequences are difficult to predict, but the economic intuition is clear, thus an explanation of the intuitions will be given.

First and foremost, if the demand for margin rate increases, so will the cost of borrowing money as well. As the model incorporates the margin rate it disincentivises the employment of leverage, thus the leverage lifecycle strategy seeks to equilibrium in borrowing costs.

Secondly, a higher demand for risky assets such as stocks leads to a net increase in stocks and thereby artificially the stock prices go up. However, the fundamentals will be unchanged and ratios such as price-to-earnings will be abnormal and suggest that the risky assets will be too expensive. Under these circumstances the return on risky assets will unlikely be as they have been in the past. Some studies (Brunnermeier, 2008), (Chowdhry & Nanda, 1998) show that great amount of leverage can amplify the mechanisms that causes the turmoil of a financial crisis. When the first investors receive a margin call and the broker liquidate their investments it will amplify and continue the domino effect.

3.8.5 A Defence From the Authors

Nalebuff and Ayres acknowledge these limitations in their book (Ayres & Nalebuff, 2010b). They address that the current financial market is not suitable for their strategy if everyone follows it. If this strategy will ever appeal to a broad segment, it will not be done overnight. Furthermore, they consider their strategy as a strategy for the *individual.* The strategy is not applicable to everyone, although, their empirics indicate a lower overall risk. They mention the previous issues (sections 3.8.1, 3.8.2, 3.8.3 and 3.8.4) but also, if for instance, someone has high credit card debt, one should not pursuit return of leveraged investments but rather pay off debt. Finally, the leverage lifecycle strategy demands a high degree of rationality, e.g. if an investor in his young 30s is leveraged he will lose a greater amount of his current savings under a financial crisis compared to unleveraged investments. They respond to this in the following way:

> "Young Investors have so little in the market that even a large percentage loss at the time won't have a crippling impact on their ultimate retirement amount. By investing more when young, you can have less exposure when near retirement age. While losses are never pleasant, they are easier to handle when young, both because they tend to be smaller in magnitude and because you have more time to adjust."
> Ayres and Nalebuff (2010b)

This statement makes it sound like an easy exercise on the rational intellectual level, however, if one loses over half of his current savings in his 30s, it might be easier said than done. The leverage lifecycle strategy requires one to keep investing leveraged, as a 'hair of the dog that bit you' compliance, which is not something everyone has the rationality to adhere to. The authors acknowledge this issue as well, thus they advice investors not to follow their strategy, if the investors cannot stick to the strategy in troubled times.

This argumentation extends their defence in the worries if everyone had leveraged investments. Since the leverage lifecycle strategy is not suitable for everyone, as it is not meant to be suitable for everyone, everyone should not leverage their investments, hence expecting everyone will leverage is a false premise.

4 Data and Methodology

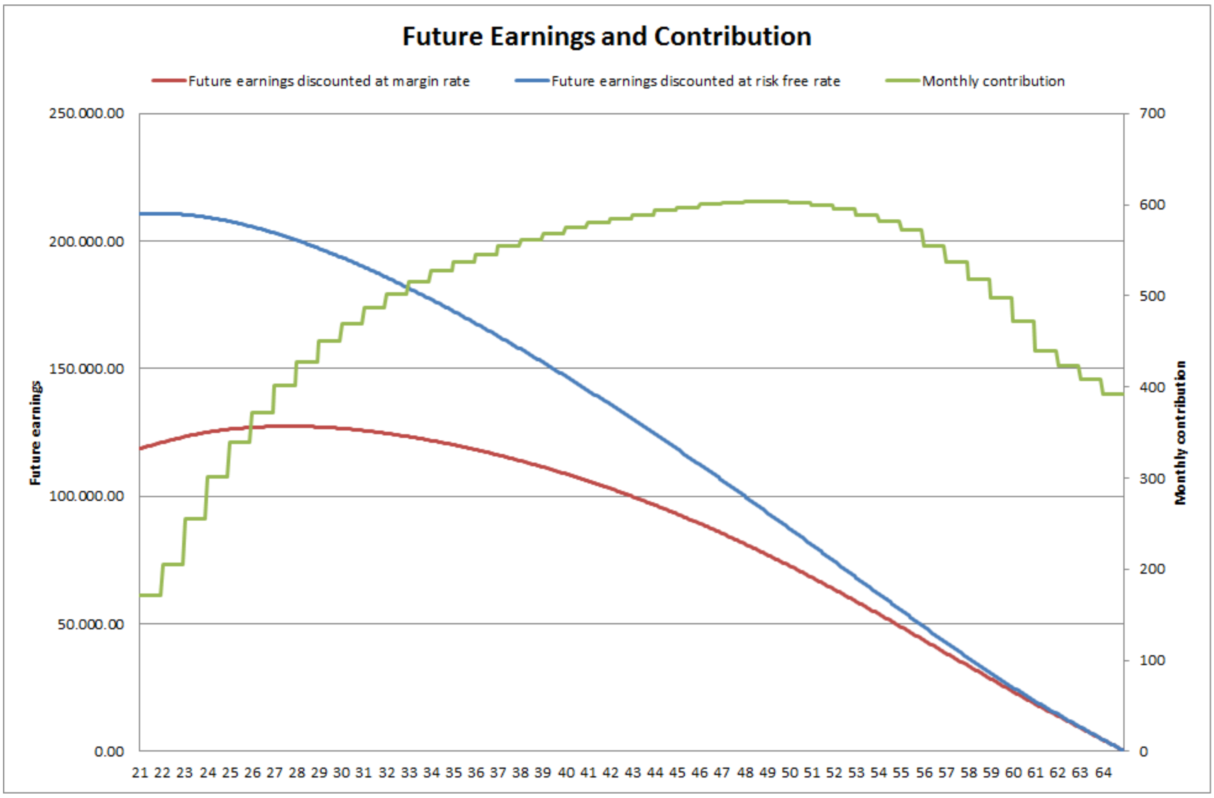
In this section there will be a review of the selected data. There will also be a short discussion of its validity and trustworthiness as well as a brief historical examination for some of the selected data.

4.1 Contribution to Savings

It is assumed that savings are exogenous given where the investor contributes 11% of his income to his retirement savings account. When he is 21 years old he will make his first contribution and his last will be the month before he turns 65 years old. This interval (44 years), as well as the career-average earnings of $59,384, is taken from the Social Security Administration (Clingman & Burkhalter, 2015). The amount or rate of contribution does not provide any difference in the comparison of the strategies. It is the scaled factor income, which is the time-distributed income for the investor, that has an importance for the strategies. The scaled factor income is chosen from the work of Clingman and Burkhalter (2015), that follows a medium-earner investor.

Figure 4.1 shows the monthly contributions of the investor and that illustrates the investor has hump-shaped earnings. The investor's wage is smallest when he enters the job market at the age of 21 and highest when he is 47-50 years old, afterwards his wage will decline. There exist various studies that explain this decline, one of them (Casanova, 2013) justifies the decline because some people in their late years go from full-time to part-time job and therefore reduces their income, but argues that a person who stays employed on full-time does not decline.

Figure 4.1: Future Earnings and Contribution



Source: By own creation

The figure also illustrates the investor's future earnings discounted at margin rate $F_{m,t}$ and risk free rate $F_{r,t}$. They are discounted by using the net present value formula, because the investor's future earnings are in fact future cash flows. The future earnings are used to determine α from equation (3.2).

Clearly, there is a difference whether the investor discounts his future earnings at the margin rate or at the risk free rate. This difference explains phase III from section 3.3.1, where the investor is switching from discounting his future earnings in margin rate to future earnings in risk free rate. During this transition, he is investing 100% in risky assets, and afterwards he can start phase IV by discounting his future earnings at risk free rate.

4.2 Margin Rate

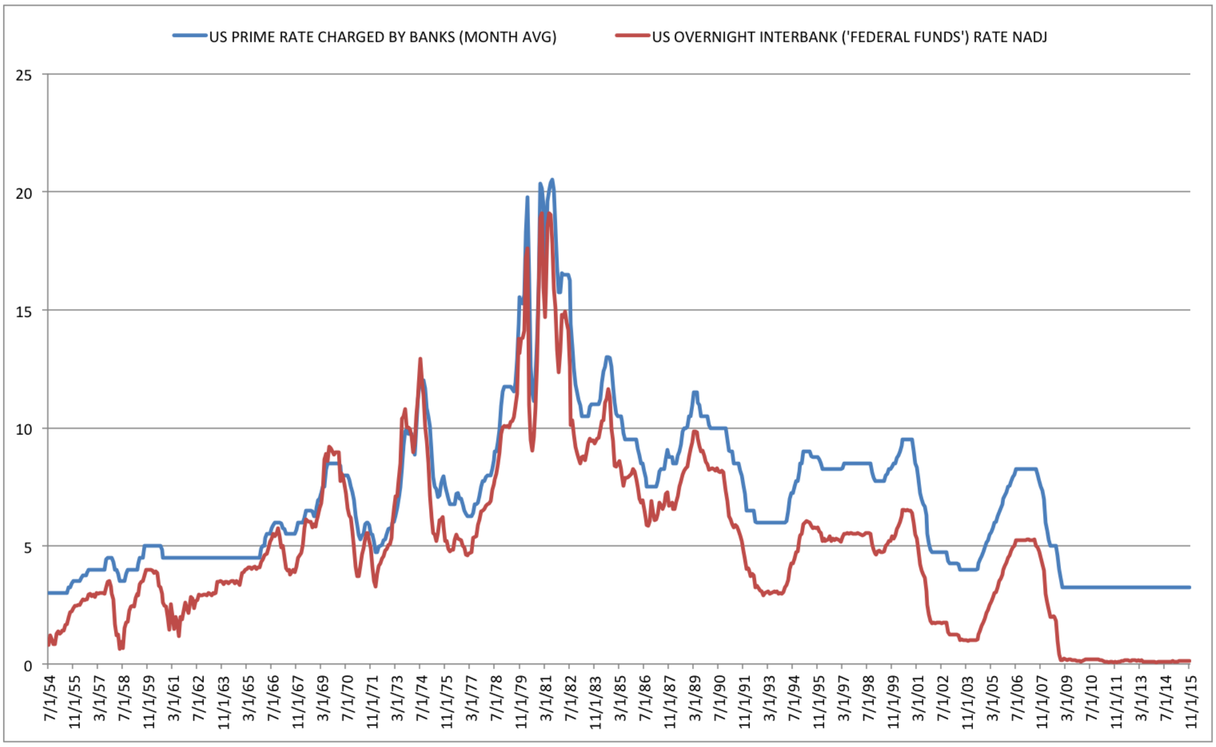
In order to determine the margin rate, an interpretation of its nature and dynamics is required. Each broker charge a different margin rate, depending on various factors where one of them is the loan amount. Typically, the more one borrows the lower the margin rate becomes. Various brokers charge different margin rates as they all offer different bargains. While Merrill Lynch has a margin rate of 8.875% for borrowing less than $25.000 and 5.500% for borrowing between $1,000,000 and $4,999,999[3] then another broker, Interactive Brokers, offers a margin rate of 1.88% for borrowing less than $100,000 and even offer a better margin rate for a higher amount of loan.[4] It is also often seen that brokers offer lower margin rates if one invests in diversified and low risk investments.

In the study from Fortune (2000) the author examines how the brokers obtain fund for margin loans. One way from internal funds, another by using other customers' cash deposits, and thirdly, where 70% comes from, brokers can borrow from commercial banks or other lenders. The rate charged by banks on security loans to brokers is called "the broker call money rate". The author argues that this rate is closely related to federal fund rate as shown in figure 4.2. It illustrates the comparison of the two different rates at the end of each month from July 1954 to December 2015. The author argues that since margin loans allows brokers to liquidate the collateral overnight, margin loans are equivalent to an overnight loan.

[3] Visit `https://www.merrilledge.com/pricing` to see the whole list

[4] Visit `https://www.interactivebrokers.com/en/index.php?f=interest\&p=schedule2` to see all margin rates.

Figure 4.2: Margin Rate



Source: By own creation

The margin rate is usually charged with a premium over the call money rate (Fortune, 2000). The premium covers the broker's cost of recording, monitoring and managing the loan, risk premium for the possibility that the customer's assets might become insufficient to repay the margin loan, as well as a profit for offering the service. It is assumed that this premium will be plus 30 basis points as it has been assumed in the article from Ayres and Nalebuff (2010a). With such an assumption the annualised real margin rate is 3.91%. This is a low margin rate compared to the 1980s peak where the rate was 20%, which would be more than the investor could expect in return from his portfolio. Naturally, he should not leverage his investments during such scenarios. However, this model uses a fixed margin rate, and therefore the expected return of the portfolio will be higher than the margin rate, and the investor should not switch between being leveraged and unleveraged during his early phases.

4.3 The Risk Free Interest Rate

The risk free interest rate is the rate of return of an investment in which no risk is involved. This is only obtainable in theory, meanwhile in practice all investments have some sort of risk. Therefore, a proxy will be used as a measurement of a risk free investment. In this thesis the US Treasury Bill Rate 3 Month will be used as the proxy, as it has generally been regarded as risk free. Table 4.1 shows the annualised real return is 0.98%, which will be used throughout the calculations of excess return.

Table 4.1: Risk Free Rate

Source	Real Return	Data
US Treasury Bill Rate 3 Month	0.98%	1973-2015

4.4 Risky Assets

The risky assets contains 8 different asset classes. They are all denominated in USD and chosen to represent a global index except from the stocks, which are separated in US stocks and foreign stocks. Furthermore, each index has different inception dates. The corporate bonds have only 15 years of observations, meanwhile stocks and commodities have 45 years of observations. All asset classes have been chosen with as long track history as possible from sources with credibility. All data is extracted from DataStream and each index is provided by a trustworthy index provider. Table 4.2 illustrates each asset class' performance and is represented by its real return. The premium risk free performance is the excess annual real return of the risk free asset defined in table 4.1 as well as the premium margin rate is the excess annual real return of the margin rate defined in section 4.2.

Table 4.2: Annualised Real Performance of Risky Assets

Source	Premium Risk Free	Premium Margin Rate	Sharpe Ratio	Std. Deviation	Data
BOFA ML Global Government Bonds	3.35%	0.44%	0.467	7.16%	1986-2015
MSCI US	6.00%	3.09%	0.390	15.39%	1970-2015
MSCI WORLD ex US	5.79%	2.88%	0.337	17.15%	1970-2015
MSCI Global REIT	5.20%	2.30%	0.343	15.16%	1995-2015
Barclays Global High Yield Bonds	6.57%	3.66%	0.670	9.8%	1990-2015
Barclays Global Corporate Bonds	2.73%	-0.18%	0.428	6.37%	2000-2015
Barclays Global Inflation Linked Bonds	3.57%	0.66%	0.490	7.28%	1997-2015
S&P GSCI	3.99%	1.08%	0.201	19.83%	1970-2015

4.4.1 Government Bonds

Bank of America Merrill Lynch Government Bonds Index tracks the return of sovereign debt issued by nations from OECD, thus the index is biased to developed markets. It has a fairly long history back to 1986 and has the second lowest standard deviation and third highest Sharpe ratio of 0.467.

4.4.2 Stocks

The two indices MSCI US and MSCI WORLD ex US enables a split between US and foreign stocks. The US stocks have outperformed foreign stocks with higher return as well as lower volatility within the same period (1970-2015). Both asset classes have lower return than high yield bonds and higher standard deviation. However, bear in mind that the interval is different, and stocks have observations from 1970-1990, where there is no valid observations for high yield bonds. If other asset classes had only observations from 1990, then the picture would look different for premium risk free rate; as US stocks would have 7.41% and foreign stocks 2.90%. On the other hand, commodities would be negative with -0.274% premium risk free rate. This testifies the utmost important subject that historical returns are not an indicator for future returns, nor does each period provide the same return.

4.4.3 Real Estate Investment Trusts

The MSCI global real estate investment trusts index invests in the real estate sector. The index consists approximately of 72% from US, 9% Australia, 6% France, 5% United Kingdom, 4% Japan and the remaining 4% in other countries (MSCI, 2016). Thus, the index is heavily biased to the US market, and furthermore to developed economies. However, the sub-industry is very well diversified from the retail, residential, office, health care, hotel & resort industries and many others with a more equally weighted constituents.

4.4.4 High Yield

The index contains primarily Ba and B credit rated bonds, and exist mostly in the industrial sector (Barclays Capital, 2015b). High Yield bonds are the best performing asset class with a Sharpe ratio of 0.670. The risk adjusted return is very favourable and outperforms by far the other asset classes. Different investment firms (Hans-jörg Naumer & Krings, 2013) (Cincinnati Asset Management, 2012) (Phillips, 2012) advocates for a heavier allocation in high yield bonds in a diversified portfolio, because it has turned into a better established asset class providing less risk than in the past. Thus a large net inflow has occurred lately (Alexeyev, 2014).
However, the authors Niels Bekkers and Lam (2009) analyse return of different asset classes and examine other articles results. They use another index for high yield bonds; Barclays High Yield US, and with another range of data 1984-2008. It has therefore another geographical index and time period, but the return is significantly lower with only 5.35% in excess return to cash.

4.4.5 Corporate Bonds

The index consists of corporate bonds within the industrial, utility and financial sector. The bonds have primarily Aa, A and Baa credit ratings (Barclays Capital, 2015a). It has the fewest observations with only 15 years of all asset classes. It has also the lowest standard deviation of 6.37%, but a fairly low return that results in a Sharpe ratio of 0.428.

It is the only asset class that has a lower return than the margin rate.

4.4.6 Inflation Linked Bonds

The index tracks securities which offer protection against inflation because their cash flows are linked to an underlying inflation index. It consists mostly of securities from the US market (37.5%), U.K (20.3%) and France (16.1%) (Barclays Capital, 2008). The index was created in 1997, and it has thereby only 18 years of data. It has the second highest Sharpe ratio of 0.490.

4.4.7 Commodities

The S&P Goldman Sachs Commodity Index (GSCI) is different to the other indices because it consists of derivatives instead of equities or bonds (Goldman Sachs, 2015). Therefore, it is weighted by the quantity of each commodity in the index by average quantity of production in the last five years. It has no direct counterpart to market capitalisation because it consists of derivatives. The index consists of 24 different commodities and is well diversified in each commodity. The index has the highest standard deviation among the 8 asset classes, a fairly low return and therefore the lowest Sharpe ratio.

4.5 Asset Allocation

Table 4.3 shows the correlation between each asset class. The calculations are based upon first available data observation for each asset class with monthly return. The lower left part of the table shows the correlation of nominal return and the upper right shows correlation of real return, which is why the matrix is not symmetrical. Notice, it is the real return that will be used in the calculations going forward.

Table 4.3: Correlation Matrix

	Bonds	US Stocks	Foreign Stocks	REIT	High Yield	Corporate	Inflation Linked Bonds	Commodoties
Bonds	1	0.066	0.330	0.143	-0.175	0.017	-0.121	0.061
US Stocks	0.056	1	0.649	0.952	0.026	0.107	0.075	0.091
Foreign Stocks	0.326	0.644	1	0.962	0.058	0.112	0.013	0.192
REIT	0.143	0.952	0.962	1	0.053	0.113	0.044	0.320
High Yield	-0.142	0.037	0.069	0.074	1	0.637	0.415	0.101
Corporate	0.033	0.120	0.128	0.128	0.673	1	0.876	0.087
Inflation Linked Bonds	-0.090	0.089	0.035	0.063	0.456	0.888	1	-0.040
Commodities	0.090	0.094	0.195	0.335	0.133	0.150	0.016	1

The correlation matrix indicates that some asset classes are highly correlated as groups. One group is US stocks, foreign stocks and REIT where all three assets are highly correlated, especially REIT with the two other assets.

High yield, corporate and inflation linked bonds are also highly correlated. The real return correlation reduces inflation linked bonds to all assets.

Government bonds have a fairly low correlation to all other assets, and with the second highest Sharpe ratio it will most likely be well weighted in the portfolio. Commodities are also low correlated to most other assets, but compared to government bonds it has a lower Sharpe ratio, thus it is expected to have a lower weight.

Before the leveraged and unleveraged portfolio will be determined, some conditions must be defined to achieve reasonable results. High yield bonds will most likely be heavily weighted because of its high Sharpe ratio, although the high yield bond market is not equally big, liquid or mature as the stock or government bond market. Table 4.4 shows the market capitalisation for each asset class except the commodity index. By dividing each asset class with the total market capitalisation a weight will be given. High yield bonds have only 3.05% of the total market capitalisation, which is very low compared to stocks or government bonds. Therefore, a constraint will be added, so the investor can only invest the double amount of the weight of the market capitalisation, allowing him some overweight with assets that have performed well, yet with limitations.

Table 4.4: Market Capitalisation in 1,000,000 USD

Asset Class	MC	Weights	Constraints
Government bonds	24,442,260	35.58%	71.2%
US stocks	17,917,807	26.08%	52.2%
Foreign stocks	12,401,758	18.05%	36.1%
REIT	761,012	1.11%	2.2%
High yield bonds	2,093,190	3.05%	6.1%
Corporate bonds	7,710,604	11.22%	22.4%
Inflation linked bonds	337,937,6	4.92%	9.8%

Commodities have no market capitalisation, as mentioned earlier, thus this sort of constraint cannot be imposed with the same method. However, commodities have by far the lowest Sharpe ratio and should not be expected with the highest weight. The investor will be allowed to invest 100% in commodities and all constraints will be discussed in the following sections.

Furthermore, the investor is not allowed to short sale any asset. This condition enables the portfolios to achieve convenient allocation by only having non-negative weights, otherwise, the portfolio would be forced to have abnormal composition compared to conventional portfolios for retirement investing.

4.5.1 The Leveraged Portfolio

The leveraged portfolio is the portfolio the investor will use for leverage lifecycle strategy, where he lies within phase I and II, and in these phases he is leveraged and therefore his risk free asset will be the margin rate. Equation (2.10) from section 2.2.3 determines his weights. By calculating the optimal weights with the given data the following portfolio will be constructed:

Table 4.5: Portfolio weights for leveraged portfolio

Asset class	Constrained	Unconstrained
Government bonds	36.1%	23.5%
US stocks	38.2%	17.8%
Foreign stocks	3.5%	0.0%
REIT	0.0%	0.0%
High yield bonds	6.1%	58.7%
Corporate bonds	0.0%	0.0%
Inflation linked bonds	9.8%	0.0%
Commodities	6.3%	0.0%

The impact of the constraints is very clear. High yield bonds only have 6.1% as it is constrained to that limit, and consequently, other assets are added to the portfolio; foreign stocks, inflation linked bonds and commodities, and at the same time the position of government bonds and US stocks increases. Only the high yield bonds and the inflation linked bonds are binding to their constraints. That means, the allocation of 6.1% in high yield bonds and 9.8% in inflation linked bonds are affected by their constraints from table 4.4.

4.5.2 The Unleveraged Portfolio

The unleveraged portfolio will be used for the MS-strategy, the TDF-strategy and the leverage lifecycle strategy when it is no longer leveraged. The weights are calculated the same way as in the leveraged, but this portfolio uses the risk free rate for excess return instead of margin rate.

Table 4.6: Portfolio weights for unleveraged portfolio

Asset class	Constrained	Unconstrained
Government bonds	44.6%	39.3%
US stocks	13.0%	9.7%
Foreign stocks	0.0%	0.0%
REIT	0.0%	0.0%
High yield bonds	6.1%	31.2%
Corporate bonds	22.4%	0.0%
Inflation linked bonds	9.8%	17.9%
Commodities	4.0%	2.0%

Once more, the constraint on high yield bonds have a great impact. Interestingly, when high yield bonds decreases from 31.2% to 6.1% then corporate bonds increases from 0.0% to 22.4%. A plausible reason for this is the high correlation of 63.7%, which disables corporate bonds in an unconstrained portfolio. Meanwhile the weight of corporate bonds increases with constraints. It has no effect for the leveraged portfolio because the premium of margin rate is negative as calculated in table 4.2.

Interestingly, both the unleveraged and leveraged portfolios do not include REIT, due to the correlation degree and Sharpe ratio. Therefore, the portfolios combined will only consist of 7 different risky assets instead of 8.

4.5.3 CRRA

Now that the portfolios have been constructed, the investor's utility preferences can be defined. In section 2.1 the investor was assigned the constant relative risk aversion utility function. By using equation (2.6) and the inputs return, variance and risk free asset of the unleveraged portfolio, the investor's CRRA level (γ value) can be determined. The article by Ayres and Nalebuff (2010a) uses an investor with CRRA level of 4 in their analysis. However, due to that this portfolio has a lower variance, because it contains multiple assets and not only stocks, an investor with CRRA level of 4 should have 408% in risky assets. Having such a high amount invested in risky assets will distort the analysis, because the investor must be unleveraged. Therefore, the investor who follows the MS-strategy will have approximately 71% invested in risky assets, and the investor who follows the leverage lifecycle strategy will have α_{min} approximately 50%. This results in almost similar weights compared to the original work of Ayres and Nalebuff (2010a). It corresponds, however, to a CRRA level of 23, which is considered to be an extremely risk averse investor.

In order to fully grasp the importance of a CRRA level of 23, let us use the example from section 2.1. An investor who has the offer between two scenarios, an uncertain outcome of 50% chance of \$50 and 50% chance of \$100 or the certain outcome of \$75. Both scenarios have an expected wealth of \$75, but the certainty equivalent, i.e. the amount an individual will accept with certainty is not the same. Table 4.7 shows different CRRA levels and the certainty equivalent corresponds to the example. An investor with CRRA of 1 would have a certainty equivalent of \$70.71 and would therefore be willing to pay a risk premium of \$4.29 in exchange of the gamble with expected value of \$75. Going up the CRRA scale, the certainty equivalent drops extremely due to the properties of the logarithmic function. An investor with a CRRA level of 23 is indifferent to take the gamble or receive \$51.60, which indicates how massively risk averse he is.

Table 4.7: Certainty Equivalent for different CRRA levels

CRRA level	1	2	4	8	16	21	23	25	32
Certainty Equivalent in \$	70.71	66.67	60.57	55.14	52.36	51.76	51.60	51.46	51.13

It is questionable whether an investor with CRRA level of 23 is a representative example. However, the high CRRA level only affects the certainty equivalent, i.e. the investor's acceptance towards risk. It does not affect the final wealth that each strategy provides. It is therefore critical to examine the certainty equivalent thoroughly and with a critical stance.

4.5.4 Monte Carlo Simulations

Monte Carlo simulations will be used to examine each strategy's performance based upon the historical return from table 4.2. It is important to have enough simulations to find a good approximation, but not too many to deteriorate the program and the following calculations. Therefore, the simulations will be based upon 200 potential cohorts that each have 528 months retirement savings, which corresponds to 105,600 simulations for each 7 assets. The risk free rate and margin rate will be used as a constant with corresponding values of 0.081% and 0.32% on a monthly basis.

5 Analysis

5.1 Equal Mean

It is important in the analysis that all three strategies (the leverage lifecycle strategy, MS-strategy and TDF-strategy) are comparable. Thus the strategies have been calculated, by achieving the exact same mean of the final wealth. Table 5.1 shows the calculation results of the 200 cohorts. The investor who chooses the leverage lifecycle strategy can expect a higher standard deviation, which quantifies the amount of variation of final wealth. This is interpreted as an underperformance, because the investor is risk averse and therefore seeks as low uncertainty as possible. The standard deviation for the leverage lifecycle strategy is 5.91% higher compared to the TDF-strategy and 3.20% higher compared to the MS-strategy. On the other hand, and perhaps more important, the investor who chooses the leverage lifecycle strategy can expect a higher certainty equivalent of 1.23% and 5.23% compared to the TDF-strategy and the MS-strategy respectively. This is most likely due to the investor's high risk aversion combined with the leverage lifecycle strategy, which has a higher minimum final wealth of $478,628. This corresponds to 3.20% and 8.89% higher than the TDF-strategy and MS-strategy respectively. This implies, the worst performing cohort for each strategy is least bad for the leverage lifecycle strategy. It is, however, not certain whether this outperformance is due to a better strategy or because of the simulations are based on a single cohort. By looking at the lower quartile, the leverage lifecycle strategy does not outperform, hence it is a vague statement that the leverage lifecycle strategy provides higher return for the less good scenarios.

Table 5.1: Final wealth in USD for 200 cohorts with the same mean

	Leverage Lifecycle	Target Date Fund	Merton Samuelson	Lifecycle improvement over Target Date Fund	Lifecycle Improvement over Merton Samuelson
Certainty Equivalent	581,900	574,811	552,969	1.23%	5.23%
Mean	714,786	714,786	714,786	0.00%	0.00%
Standard Dev.	111,803	105,567	108,338	5.91%	3.20%
Min	478,628	463,807	439,553	3.20%	8.89%
Percentile 10th	579,226	585,059	576,949	-1.00%	0.39%
Percentile 25th	640,766	645,175	650,128	-0.68%	-1.44%
Median	703,938	704,599	702,131	-0.09%	0.26%
Percentile 75th	775,731	776,105	773,537	-0.05%	0.28%
Percentile 90th	845,411	842,191	849,488	0.38%	-0.48%
Max	1,231,413	1,170,942	1,180,451	5.16%	4.32%
Sharpe ratio	3.50	3.71	3.61	-5.58%	-3.10%
Average Weight	104%	77%	71%		
Max Weight	200%	100%	71%		
Min Weight	49%	55%	71%		
Present value weighted in risky assets	64.4%	66.7%	70.9%	-3.39%	-9.16%
Expected Shortfall 0.01	486,486	479,815	465,043	1.39%	4.61%
Expected Shortfall 0.05	524,838	526,251	518,284	-0.27%	1.26%
Expected Shortfall 0.1	545,251	546,850	543,245	-0.29%	0.37%

The Sharpe ratio is calculated with excess return of $323,211 which is the equivalent of only investing in the risk free asset. Because of the leverage lifecycle strategy has a higher standard deviation, it has naturally a lower Sharpe ratio.

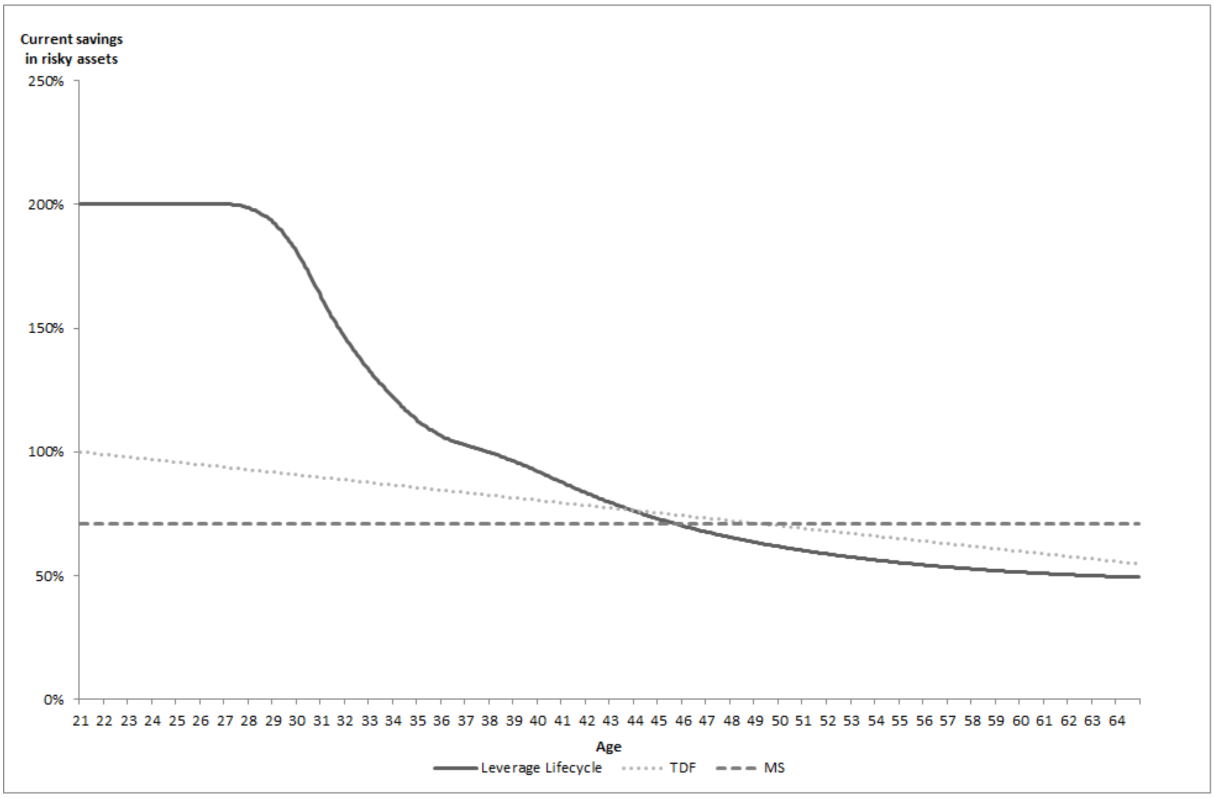
Table 5.1 also shows how the weights are distributed in the portfolio. The leverage lifecycle strategy has a high maximum investment in risky assets with 200% as the cap limit, and the minimum invested in risky assets is 49%. This interval is substantially larger than intervals from the other strategies. Consequently, the average weight of current savings invested in risky assets is also higher. However, this weight is a bit distorted, because it is calculated by its *current savings* invested in risky assets. By using *present value* weighted in risky assets, one will achieve the amount of total dollars invested in risky assets, e.g. if an investor has $1,000,000 at terminate date with 50% of his wealth invested in risky assets, it is heavier weighted than $171 in the very first month invested with 200% in risky assets. Therefore, this measurement is more precise in defining how much of an investor's retirement savings is invested in risky assets. This reveals that the leverage lifecycle strategy obtains the same mean return as its two peers by having invested less in risky assets overall.

The expected shortfall is also calculated for the lower 1%, 5% and 10%. There is only a little improvement for 1% compared to the TDF-strategy and it underperforms for the higher thresholds 5% and 10%. It does perform better than the MS-strategy especially for the lower 1%. This corresponds also to the fact that the investor is very risk averse, thus an outperformance of lower final wealth have high benefits for the certainty equivalent.

It seems like there is an overall small benefit if an investor follows the leverage lifecycle strategy. It provides higher minimum final wealth compared to its benchmarks, which results in an improvement in certainty equivalent. On the other hand, it provides a higher standard deviation.
By comparing the two benchmark strategies, the TDF-strategy seems to have some benefits from the leverage lifecycle strategy compared to the MS-strategy, because it also provides higher minimum final wealth and higher certainty equivalent.

Figure 5.1 illustrates how much each strategy invests in risky assets for each month. This figure is the same theoretical illustration from figure 3.1, but is based upon the average of the 200 cohorts. The leverage lifecycle strategy is heavily leveraged with 200% in risky assets and leaves the average investor to deleverage after approx 85 months (28 years old); where the first cohort must deleverage after 74 months (27 years old), meanwhile the latest deleverages take place after 151 months (33 years old). When he attains 100% in risky assets he switches his portfolio from the leveraged to the unleveraged and from there on, deleverages until he passes the two other strategies, and he ends up with a far more conservative portfolio as projected in the theoretical sections. The TDF-strategy is also decreasing continuously its position of risky assets. It also starts with more risky assets than the MS-strategy and ends up with less risky assets.

Figure 5.1: Weights



Source: By own creation

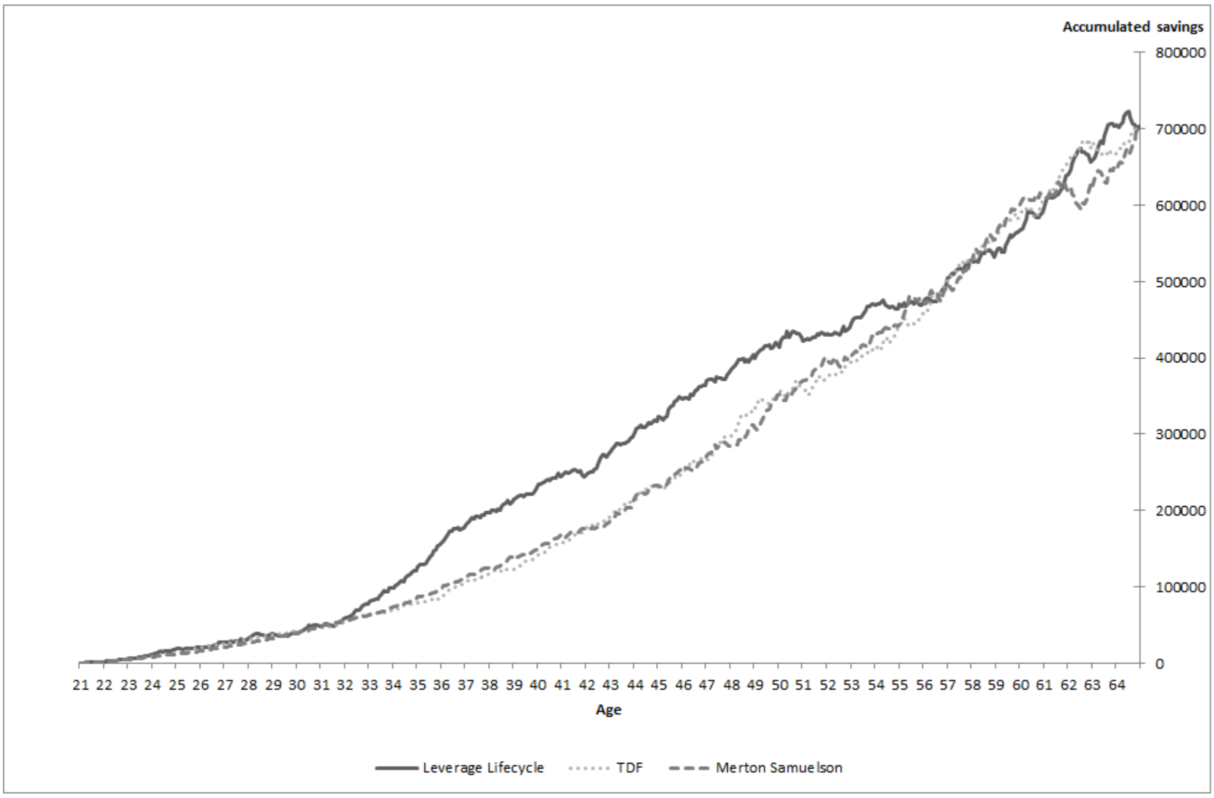
5.1.1 Performance During the Investment Phase

So far, the analysis has only examined the investor's final wealth. However, the strategies are a 44-year long continuous process, which is different in each strategy. Figure 5.2 illustrates the median[5] for each strategy. It should only be used as an example of how an investor's savings *may* potentially develop. They all have the same mean, but the ongoing accumulation of savings are different. The leverage lifecycle strategy develops

[5]The lower median is used given that the cohort is an even number.

distinctively different from the two others. From the age of 32 to 57 in this example the leverage lifecycle strategy gains a higher savings accumulation. This is a consequence of the diversification across time. It strengthens the early savings period by leveraging, which is shown for the median cohort, but bear in mind, the cohort with worst returns during the leverage phase must wait 6 additional years compared to the average investor. Although the investor can expect a higher minimum *final* wealth as known from table 5.1, he can potentially have a long-term underperformance, and only by keeping leveraged investments he can achieve a higher minimum final wealth.

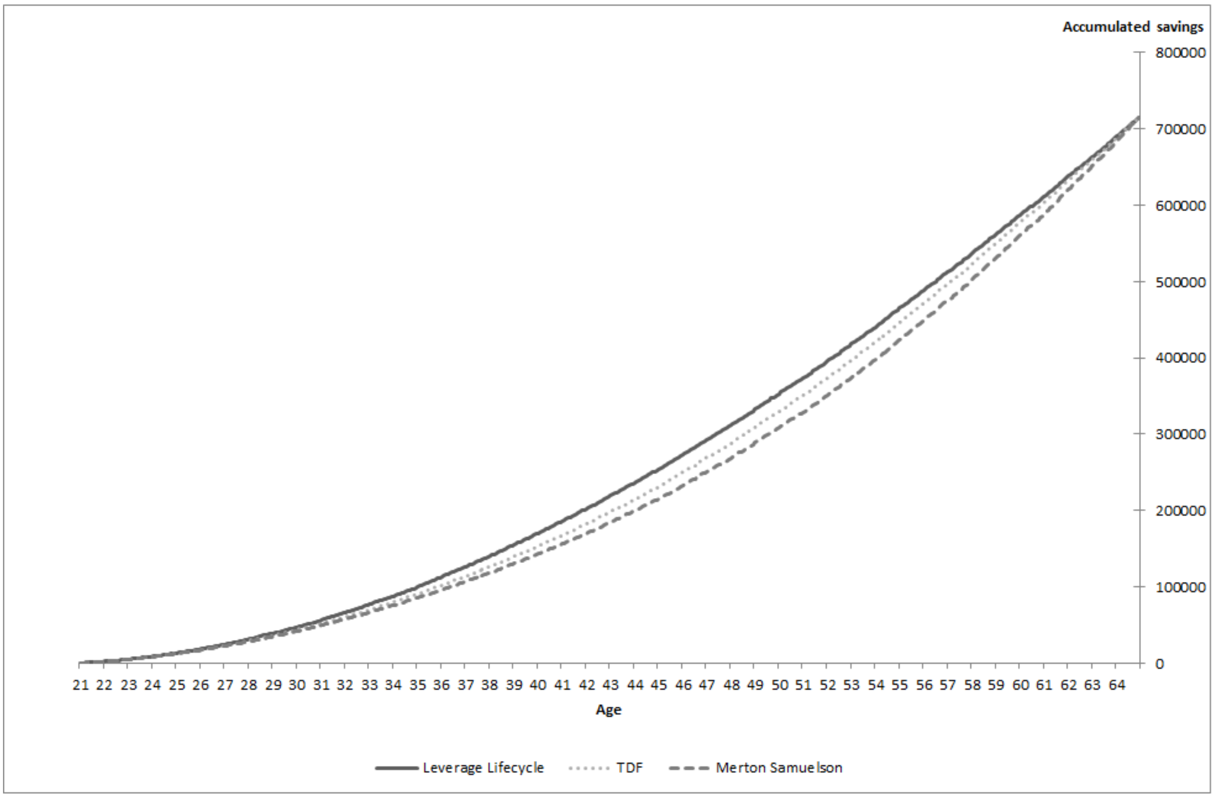
Figure 5.2: Accumulated savings for median cohort with equal mean



Source: By own creation

The development of the median cohort should only be regarded as an example of a single cohort. By comparing all strategies' mean accumulations the overall accumulation can be calculated. Figure 5.3 illustrates this scenario. Clearly, the leverage lifecycle strategy outperforms slightly during the medium-phase. The MS-strategy lacks behind the two other strategies, due to the relative low exposure to risky assets in the early phase, though, the MS-strategy catches up later on due to a higher exposure to risky assets.

Figure 5.3: Accumulated savings for mean cohort with equal mean



Source: By own creation

5.1.2 Drawdown

Table 5.2 shows the maximum drawdown i.e. the highest reduction in current savings on one month, from all 105,600 simulations. The leverage lifecycle strategy has the highest possible drawdown due to the leverage investing. Although -15.75% is a large amount relative to the two other strategies it is still not enough to trigger a margin call or to wipe out all savings.

Table 5.2: Maximum Drawdown

	Leverage Lifecycle	Target Date Fund	Merton Samuelson
Max drawdown	-15.75%	-5.13%	-4.33%

A maximum drawdown of 15.75% from 105,600 simulations seems as a relative small drawdown, given the investor is leveraged with 200% even though the portfolio is well-diversified. This is because the returns are produced from Monte Carlo simulations accordingly to each asset class' normal distribution. The nature of such distribution has empirically not been well proven (Officer, 1972), (Aparicio & Estrada, 1997), but is one of the best representatives. Historical data shows that the distributions of stock returns usually have higher kurtosis thus the distributions have "fat tails". Consequently, the Monte Carlo simulations will not be able to provide scenarios with very high or very low return.

5.2 Equal Weights

Until now, the strategies have been compared to equal mean with different standard deviation and certainty equivalent of final wealth and present value weighted in risky assets. By using such an analogy, the investor can choose the strategy that results in either the lowest standard deviation or the highest certainty equivalent. Moreover, by altering the point of origin, one can settle a fixed amount of investing in risky assets, and see which strategy gives the highest final wealth. This is no paradigm shift but rather an alternative comparability to examine how much savings each strategy can provide with equal amount of risky investments. Table 5.3 summarises this notion where each strategy is set with equal amount of present value weighted in risky assets. This results in a higher mean but also an excessive higher standard deviation. The investor can expect 2.54% higher mean for the leverage lifecycle strategy compared to the TDF-strategy, which corresponds to the investor can retire 7 months earlier. However, the standard deviation is also 12.55% higher and therefore the Sharpe ratio is 6.95% lower. This case demonstrates that the Sharpe ratio is a poor comparison, because the investor prefers higher minimum final wealth even though the standard deviation increases. The leverage lifecycle strategy provides a higher interval of $478,628 to $1,231,413 and the TDF-strategy provides $460,947 to $1,112,972. It provides also a higher mean and certainty equivalent, therefore, the leverage lifecycle strategy gives overall superior results compared to the TDF-strategy.

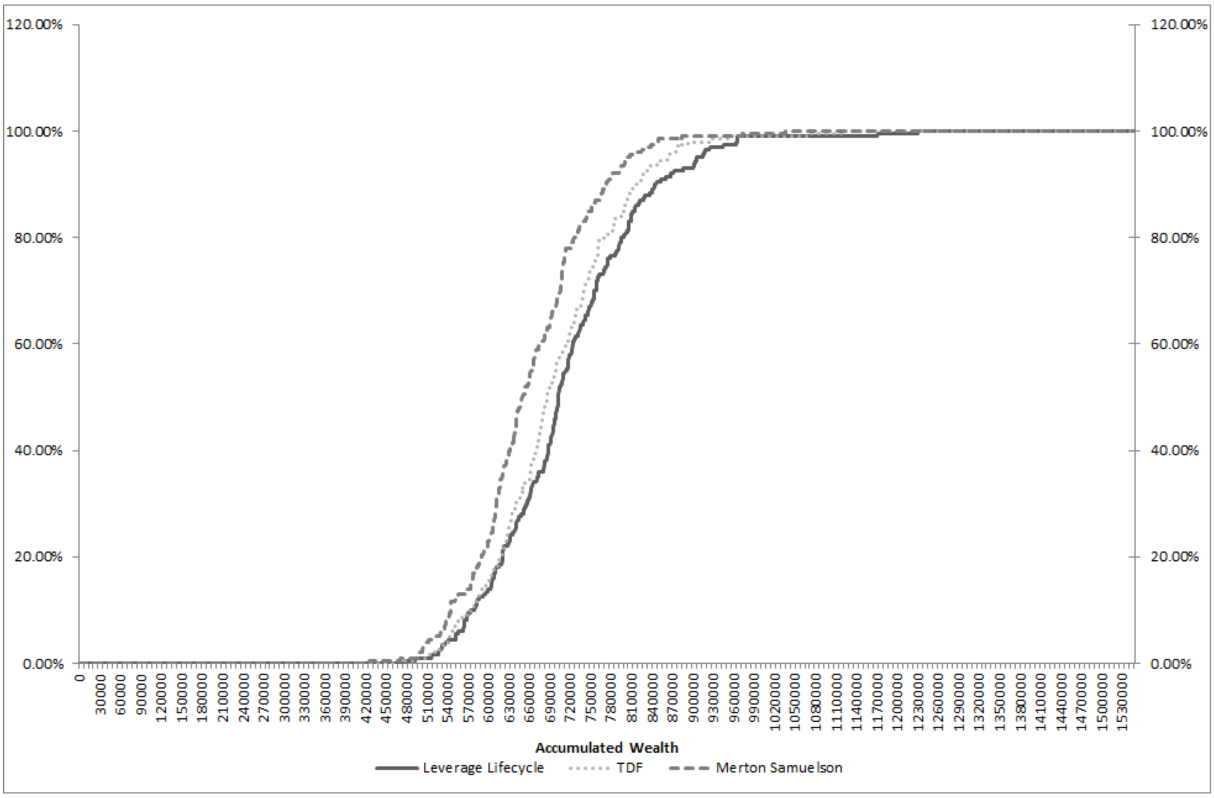
The same characteristics are applicable for the MS-strategy when compared to the leverage lifecycle strategy - and even to a wider extent. The standard deviation is 24.39% higher, but the interval of outcome is also higher as well as the certainty equivalent and mean. The leverage lifecycle strategy provides a 8.15% higher mean, which corresponds to the investor can retire 24 months earlier. This is also seen for the risk measurement, expected shortfall, that indicates the leverage lifecycle strategy provides higher minimum outcome.

Table 5.3: Final wealth in USD for 200 cohorts with the same amount of risky assets

	Leverage Lifecycle	Target Date Fund	Merton Samuelson	Lifecycle improvement over Target Date Fund	Lifecycle Improvement over Merton Samuelson
Certainty Equivalent	581,900	568,980	534,024	2.27%	8.97%
Mean	714,786	697,105	660,903	2.54%	8.15%
Standard Dev.	111,803	99,338	89,879	12.55%	24.39%
Min	478,628	460,947	426,372	3.84%	12.26%
Percentile 10th	579,226	575,233	545,934	0.69%	6.10%
Percentile 25th	640,766	631,084	607,699	1.53%	5.44%
Median	703,938	687,727	650,871	2.36%	8.15%
Percentile 75th	775,731	755,369	710,392	2.70%	9.20%
Percentile 90th	845,411	814,902	773,325	3.74%	9.32%
Max	1,231,413	1,122,972	1,037,861	9.66%	18.65%
Sharpe	3.50	3.76	3.76	-6.95%	-6.78%
Average Weight	104%	76%	64%		
Max Weight	200%	100%	64%		
Min Weight	49%	52%	64%		
Present value weighted in risky assets	64.4%	64.4%	64.4%	0.00%	0.00%
Expected Shortfall 0.01	486,486	475,445	448,923	2.32%	8.37%
Expected Shortfall 0.05	524,838	519,217	495,267	1.08%	5.97%
Expected Shortfall 0.1	545,251	538,450	516,764	1.26%	5.51%

Figure 5.4 illustrates the accumulative probability of final wealth based upon the 200 cohorts. It provides more details than table 5.3 since it shows results for each quartile. Clearly, the leverage lifecycle strategy outperforms the two other strategies, because almost every single cohort can expect higher return with the leverage lifecycle strategy. It is also seen that the TDF-strategy delivers higher return than the MS-strategy.

Figure 5.4: Cumulative Probability of Final Wealth



Source: By own creation

On an overall level the analysis has shown ambiguous results. If an investor follows the leverage lifecycle strategy (with the same mean as the two benchmarks), some parameters indicate outperformance and others the opposite. The investor can obtain higher minimum outcome and higher certainty equivalent, but also a higher standard deviation. However, by comparing the strategies using the same amount invested in risky assets, the leverage lifecycle strategy seems to outperform in almost all areas. The underlying causes are not blatant nor evident, yet a profound and more compelling and resilient examination is needed to determine a clarification of the statement. Section 6 investigates different issues to recapitulate the flaws behind this analysis.

6 Resilience

In this section the resilience of the model will be tested by altering different setups. It will be analysed as in section 5.1 by using equal means and by noting which strategy provides the best outcome.

6.1 Different CRRA Levels

Section 4.5.3 emphasises the importance of different CRRA levels impact on an investor's preferences. To expect the investor has a CRRA level of 23 should be looked upon with a grain of salt. This subsection analyses different levels of CRRA, which corresponds to work with different preferences towards the amounts invested in risky assets as well as the certainty equivalent.

Table 6.1 shows 7 different scenarios with investors who have CRRA level between 17-29. It summarises the certainty equivalent (CE), mean and standard deviation as well as the minimum weight and the present value weighted in risky assets (PVWiRA). Naturally, a lower CRRA level requires a higher amount of risky assets. There is a clear sign of, that the higher amount of risky assets invested, the higher mean can an investor expect.

The leverage lifecycle strategy seems to outperform the two other strategies for all CRRA levels except for an investor with CRRA level of 17. For this exception, the investor is investing heavily in risky assets. Under such circumstances, the gap from maximum to minimum weight in risky assets is reduced. These conditions makes the strategies to have similar behaviour thus there are little room for advantages. The TDF-strategy will decrease from 100% to 94% in the amount of risky assets, meanwhile the MS-strategy will keep constantly 95.9% in risky assets. It is the only scenario where the investor can expect a smaller certainty equivalent, if he chooses the leverage lifecycle strategy.

The rest of the results indicates that the more risk averse the investor is, the more the leverage lifecycle strategy also seems to outperform the MS-strategy in regard to certainty equivalent, standard deviation and necessary investments in risky assets. On the other hand, the TDF-strategy seems to operate in the opposite direction, meaning, the more risk averse the investor is, the less improvement gives the leverage lifecycle strategy compared to TDF-strategy in regard to certainty equivalent and present value weighted in risky assets.

The leverage lifecycle strategy also produce higher standard deviation especially for the less risk averse investor types. That is because the strategy has a relative high minimum weight (α_{min}), and the investor's final wealth variates more. In case of an investor with CRRA level of 29, the investor can gain a lower standard deviation compared to the MS-strategy.

Table 6.1: Final wealth in USD for 200 cohorts with different CRRA levels

CRRA		Leverage Lifecycle	Target Date Fund	Merton Samuelson	Lifecycle improvement over Target Date Fund	Lifecycle Improvement over Merton Samuelson
17	CE	676,822	683,829	679,522	-1.02%	-0.40%
	Mean	976,517	976,517	976,517	0.00%	0.00%
	Standard Dev.	251,180	209,479	210,080	19.91%	19.56%
	Min Weight	76%	94%	96%		
	PVWiRA	91.8%	95.1%	95.9%	-3.52%	-4.28%
19	CE	654,683	637,855	624,681	2.64%	4.80%
	Mean	859,379	859,379	859,379	0.00%	0.00%
	Standard Dev.	176,195	160,455	162,234	9.81%	8.61%
	Min Weight	64%	78%	86%		
	PVWiRA	79.8%	83.4%	85.8%	-4.34%	-7.02%
21	CE	616,538	602,697	584,114	2.30%	5.55%
	Mean	776,366	776,366	776,366	0.00%	0.00%
	Standard Dev.	138,839	128,116	130,548	8.37%	6.35%
	Min Weight	55%	65%	78%		
	PVWiRA	70.9%	74.1%	77.6%	-4.39%	-8.69%
23	CE	581,900	574,811	552,969	1.23%	5.23%
	Mean	714,786	714,786	714,786	0.00%	0.00%
	Standard Dev.	111,803	105,567	108,338	5.91%	3.20%
	Min Weight	49%	55%	71%		
	PVWiRA	64.4%	66.7%	70.9%	-3.39%	-9.16%
25	CE	556,311	552,027	528,344	0.78%	5.29%
	Mean	667,457	667,457	667,457	0.00%	0.00%
	Standard Dev.	93,359	89,158	92,073	4.71%	1.40%
	Min Weight	44%	46%	65%		
	PVWiRA	58.8%	60.5%	65.2%	-2.80%	-9.86%
27	CE	538,373	532,957	508,411	1.02%	5.89%
	Mean	630,037	630,037	630,037	0.00%	0.00%
	Standard Dev.	80,673	76,810	79,738	5.03%	1.17%
	Min Weight	40%	39%	60%		
	PVWiRA	54.3%	55.3%	60.4%	-1.83%	-10.11%
29	CE	520,873	516,678	491,959	0.81%	5.88%
	Mean	599,765	599,765	599,765	0.00%	0.00%
	Standard Dev.	68,985	67,263	70,115	2.56%	-1.61%
	Min Weight	37%	33%	56%		
	PVWiRA	50.3%	50.9%	56.2%	-1.13%	-10.52%

6.2 Only one asset allocation

So far, the leverage lifecycle strategy has been using two different portfolios; one portfolio while it is leveraged and another one while it is unleveraged. This is due to the fact, that when the investor is leveraged, the assets he invests in must provide a higher return than the margin rate, otherwise he will lose money. When the leverage lifecycle strategy contain two different portfolios and the two other benchmark strategies only contains one portfolio, the comparison becomes distorted. In this section, the leverage lifecycle strategy will only have one portfolio, namely the unleveraged portfolio. This is not advisable because the investor would borrow money to buy corporate bonds that gives a lower return than the interest of his principal. However, it does provide a more transparent comparison. Table 6.2 reveals this scenario. The leverage lifecycle strategy now shows different results compared to previously. The standard deviation has now declined significantly compared to its peers, where it is now lower than the MS-strategy and only 2% higher than the TDF-strategy. This is also the case for the maximum outcome, which is now lower than its peers compared to before when the maximum outcome was 4-5% higher for both the other strategies.

Now the leverage lifecycle strategy must invest more in risky assets to achieve the same mean as its benchmarks. Actually, the strategy needs more risky assets than the TDF-strategy. This is most likely because the portfolio the investor has under the leverage-phases delivers lower return, thus he must invest leveraged for a longer time.

Table 6.2: Final wealth in USD for 200 cohorts with the same portfolios

	Leverage Lifecycle	Target Date Fund	Merton Samuelson	Lifecycle improvement over Target Date Fund	Lifecycle Improvement over Merton Samuelson
Certainty Equivalent	567,196	564,519	542,469	0.47%	4.56%
Mean	684,322	684,322	684,322	0.00%	0.00%
Standard Dev.	96,801	94,908	97,786	2.00%	-1.01%
Min	471,396	458,840	432,178	2.74%	9.07%
Percentile 10th	565,899	568,068	559,369	-0.38%	1.17%
Percentile 25th	620,286	621,185	626,215	-0.14%	-0.95%
Median	680,344	675,632	673,073	0.70%	1.08%
Percentile 75th	736,786	740,394	737,687	-0.49%	-0.12%
Percentile 90th	803,251	795,497	806,315	0.97%	-0.38%
Max	1,088,177	1,088,777	1,098,957	-0.06%	-0.98%
Sharpe Ratio	3.73	3.80	3.69	-1.96%	1.02%
Average Weight	105%	75%	67%		
Max Weight	200%	100%	67%		
Min Weight	49%	49%	67%		
Present value weighted in risky assets	65.1%	62.7%	67.3%	3.71%	-3.32%
Expected Shortfall 0.01	478,882	472,236	456,021	1.41%	5.01%
Expected Shortfall 0.05	508,650	514,033	505,378	-1.05%	0.65%
Expected Shortfall 0.1	529,511	532,317	528,392	-0.53%	0.21%

It is obscure whether the new asset allocation rules provides similar results to its benchmarks, because of the fact that the leverage lifecycle strategy is being punished by having assets that provides less return than the margin rate, or because the leverage portfolio creates advantageous benefits that the benchmark strategies do not gain.

6.3 Two-Asset Model

This thesis builds on the original article by Ayres and Nalebuff (2010a), but uses instead 7 different risky assets instead of only 2 (stocks as risky assets and bonds as the risk free asset). It will therefore be examined if the leverage lifecycle strategy provides any significant new results. Table 6.3 summarises each strategy with US stocks as the only risky asset. The table shows an extremely high variation of final wealth. The difference between the minimum and maximum of final wealth have approximately 10 times difference for each strategy, meanwhile with 7 assets the difference is only 3 times. Furthermore, the mean is approximately 40% higher with one risky assets compared to 7 risky assets. However, the certainty equivalent decreases heavily for all strategies, and for the leverage lifecycle strategy it is with 2 assets only \$307,780 compared to 7 assets \$581,900. That means, although the investor can expect a large increase in his mean of final wealth, he would rather prefer the lower final wealth because of his high level of risk aversion.

When comparing the three strategies, the leverage lifecycle strategy has a much smaller minimum final wealth of only \$242,670. This is approximately 25% lower, than if the investor had only invested in the risk free asset, which provides a final wealth of \$323,211. This means, that all three strategies' worst case scenario is worse than only investing in the risk free asset. The expected shortfall for 1% is also 14-15% lower for the investor. This shows the leverage lifecycle strategy produces a higher risk for the very worst scenarios. However, the expected shortfall for 5% and 10% is actually higher compared to the two other strategies. This confirms the importance of CRRA level.

The table shows, that the leverage lifecycle strategy returns -6.21% and -8.54% standard deviation compared to the TDF-strategy and the MS-strategy respectively, which is a greater outperformance compared to 7 assets that correspondingly gives 5.91% and 3.20%.

Table 6.3: Final wealth in USD for 200 cohorts with US stocks only

	Leverage Lifecycle	Target Date Fund	Merton Samuelson	Lifecycle improvement over Target Date Fund	Lifecycle Improvement over Merton Samuelson
Certainty Equivalent	307,780	349,967	346,010	-12.05%	-11.05%
Mean	998,237	998,237	998,237	0.00%	0.00%
Standard Dev.	416,799	444,381	455,697	-6.21%	-8.54%
Min	242,670	275,537	272,346	-11.93%	-10.90%
Percentile 10th	564,455	531,576	506,219	6.19%	11.50%
Percentile 25th	696,977	683,928	657,013	1.91%	6.08%
Median	914,389	887,796	897,577	3.00%	1.87%
Percentile 75th	1,265,781	1,216,772	1,258,055	4.03%	0.61%
Percentile 90th	1,495,064	1,539,499	1,563,844	-2.89%	-4.40%
Max	2,488,591	2,712,892	2,762,784	-8.27%	-9.92%
Sharpe Ratio	1.62	1.52	1.48	6.62%	9.33%
Average Weight	102%	75%	68%		
Max Weight	200%	100%	68%		
Min Weight	49%	50%	68%		
Present value weighted in risky assets	60.9%	62.6%	68.4%	-2.73%	-10.97%
Expected Shortfall 0.01	258,200	300,958	305,289	-14.21%	-15.42%
Expected Shortfall 0.05	380,430	379,502	361,760	0.24%	5.16%
Expected Shortfall 0.1	455,062	434,574	417,223	4.71%	9.07%

In the original work from Ayres and Nalebuff (2010a) they achieve -20.6% in standard deviation compared to their equivalent model of the MS-strategy. However, it has not been possible to replicate as beneficial results for the leverage lifecycle strategy in this thesis due to different input variables.

Table 6.4 shows the maximum drawdown for each strategy. Clearly, the drawdowns are worse compared to section 5.1.2 because it has no diversification of assets.

Table 6.4: Maximum Drawdown under Two-Asset Model

	Leverage Lifecycle	Target Date Fund	Merton Samuelson
Max drawdown	-35.23%	-16.48%	-12.30%

The worst month for the leverage lifecycle strategy is -35.23%. This shows that diversification of assets improves the maximum drawdown. Although -35.23% for a drawdown is a lot, the investor is leveraged 200%. Within the period of 1970-2015 (550 months) the MSCI US had its worst real return of -21.43% in September 1987. This scenario has not been recreated from the 105,600 Monte Carlo simulations while the investor is 200% leveraged in US stocks, otherwise the maximum drawdown would have been -42.86%. It verifies the theory that stocks do not fit perfectly normal distributions because of the "fat tails".

6.4 Different Margin Rates

The margin rate is a cardinal point for the leverage lifecycle strategy. Naturally, a higher margin rate implies higher costs of leveraging and therefore lower return. In section 4.2 the margin rate is examined, and it is shown that it is volatile from a long-term perspective. During some periods in the 1980s, the margin rate was even higher than what the portfolio of risky assets can deliver. Under such circumstances the leverage lifecycle strategy performs poorly.

In today's low-interest environment the leverage lifecycle strategy has an advantage. In section 4.2 there was an example of Interactive Brokers who offers a margin rate of 1.88% for borrowing less than $100,000 and even lower rates if one borrows more. With an inflation rate of 1.02%, the real annual margin rate becomes 0.86%[6] (assuming there are no bargains) meanwhile the model uses 3.91%. That creates incentives for investors today to follow the leverage lifecycle strategy. However, the current margin rate is not a guarantee for the future, and it cannot be expected that the next 44 years (the retirement savings interval) will provide the same margin rate.

Table 6.5 shows different margin rates with an interval of 0.05% on a monthly basis compared to the origin of 0.32%. When the margin rates decreases the leverage lifecycle strategy provides lower standard deviation compared to its benchmarks. If the real annual margin rate is 1.45% the investor can expect not only a higher certainty equivalent, but also a lower standard deviation.

Even if the real annual margin rate increases to 5.16%, then the leverage lifecycle strategy still provides higher certainty equivalent compared to the other strategies.

[6]Visited http://www.inflation.eu/inflation-rates/united-states/inflation-united-states.aspx 21-02-2016

Table 6.5: Final wealth in USD for 200 cohorts with different real margin rates

Margin monthly basis	Margin annual basis		Leverage Lifecycle	Target Date Fund	Merton Samuelson	Lifecycle improvement over Target Date Fund	Lifecycle Improvement over Merton Samuelson
0.12%	1.45%	CE	607,625	582,934	561,773	4.24%	8.16%
		Mean	741,448	741,448	741,448	0.00%	0.00%
		Std. D.	113,916	115,170	117,813	-1.09%	-3.31%
0.17%	2.06%	CE	602,847	580,797	559,409	3.80%	7.76%
		Mean	734,190	734,190	734,190	0.00%	0.00%
		Std. D.	112,685	112,531	115,212	0.14%	-2.19%
0.22%	2.67%	CE	596,309	578,763	557,191	3.03%	7.02%
		Mean	727,447	727,447	727,447	0.00%	0.00%
		Std. D.	112,145	110,096	112,810	1.86%	-0.59%
0.27%	3.29%	CE	589,156	576,682	554,954	2.16%	6.16%
		Mean	720,710	720,710	720,710	0.00%	0.00%
		Std. D.	111,678	107,679	110,425	3.71%	1.13%
0.32%	3.91%	CE	581,900	574,811	552,969	1.23%	5.23%
		Mean	714,786	714,786	714,786	0.00%	0.00%
		Std. D.	111,803	105,567	108,338	5.91%	3.20%
0.37%	4.53%	CE	576,640	572,647	550,704	0.70%	4.71%
		Mean	708,090	708,090	708,090	0.00%	0.00%
		Std. D.	111,350	103,195	105,994	7.90%	5.05%
0.42%	5.16%	CE	570,806	570,582	548,572	0.04%	4.05%
		Mean	701,850	701,850	701,850	0.00%	0.00%
		Std. D.	110,770	100,999	103,822	9.67%	6.69%

Table 6.6 shows the present value of weights in risky assets with different margin rates. It also shows, that lower margin rates improve the leverage lifecycle strategy. If the annual real margin rate is 1.45% the investor only needs to invest 63.9% in risky assets, compared to the TDF-strategy and MS-strategy that requires 70.0% and 73.9% respectively. Even when the annual real margin rate increases to 5.16%, the investor still needs to invest less in risky assets compared to the two other strategies.

Table 6.6: Present value of weights in risky assets for 200 cohorts with different margin rates

Margin monthly basis	Margin annual basis	Leverage Lifecycle	Target Date Fund	Merton Samuelson	Lifecycle improvement over Target Date Fund	Lifecycle Improvement over Merton Samuelson
0.12%	1.45%	63.9%	70.0%	73.9%	-8.64%	-13.49%
0.17%	2.06%	64.1%	69.1%	73.1%	-7.28%	-12.36%
0.22%	2.67%	64.2%	68.2%	72.3%	-5.97%	-11.28%
0.27%	3.29%	64.3%	67.4%	71.6%	-4.62%	-10.17%
0.32%	3.91%	64.5%	65.8%	70.1%	-1.96%	-7.98%
0.37%	4.53%	64.4%	66.7%	70.9%	-3.39%	-9.16%
0.42%	5.16%	64.6%	65.0%	69.4%	-0.58%	-6.85%

If the annual real margin rates increases to 5.16% the investor will still have slight benefits of choosing the leverage lifecycle strategy compared to the TDF-strategy and quite a few benefits compared to the MS-strategy.
If the annual real margin rates decreases the investor who chooses the leverage lifecycle strategy can expect even more advantages compared to the other strategies.

6.5 Different Returns

The Monte Carlo simulations are using historical returns as input variables. However, there is no guarantee that future returns will be either as favourable or unfavourable as it has been in the past. The historical data used in this thesis is mostly from 1970-2015, which is a relative short period compared to the stock market existence. Studies (Jorion & Goetzmann, 1999) (Shiller, 2005) suggests that the US stock market in the 20th century has been an exception rather than a rule. An investor who is about to choose an investment strategy should be more concerned with future risky assets premium rather than historical. Therefore, by creating new simulations with different returns, the strategies can be tested if their performance is due to asset returns. Table 6.7 recapitulates the final wealth with equal mean but in case of each asset class returns 60%, 70%, 80%, 90%, 100%, 110% and 120%. Thereby providing the opportunity to analyse the importance of asset returns. Notice, that each scenario consists of different Monte Carlo simulations, hence one should expect some few variations and the interpretation should not be detailed, but rather show an overall indication if any exists.

When the asset class returns shows 60% it demonstrates a simulation of 200 cohorts where the return is 60% of historical levels for each asset e.g. if US stocks have 10% annual return the simulation of 60% would create 6% annual return for US stocks.

Table 6.7 indicates that the leverage lifecycle strategy will deliver lower standard deviation of final wealth, if future asset returns are projected higher than historical. Likewise, if they are projected lower than historical the leverage lifecycle strategy will deliver higher standard deviation.

Lower returns in the future will also provide lower certainty equivalent for the investor. Interestingly, the scenario with Monte Carlo simulations based on historical return (100%) is the scenario that provides highest certainty equivalent for the investor. This is due to the high risk aversion and that the minimum final wealth has a high influence on the certainty equivalent.

Table 6.7: Final wealth in USD for 200 cohorts with different returns

Percentage of historical return		Leverage Lifecycle	Target Date Fund	Merton Samuelson	Lifecycle improvement over Target Date Fund	Lifecycle Improvement over Merton Samuelson
60%	CE	392,401	420,721	414,053	-6.73%	-5.23%
	Mean	502,826	502,826	502,826	0.00%	0.00%
	Std. D.	86,875	71,043	73,565	22.28%	18.09%
70%	CE	360,857	389,308	379,784	-7.31%	-4.98%
	Mean	544,135	544,135	544,135	0.00%	0.00%
	Std. D.	94,644	80,810	83,751	17.12%	13.01%
80%	CE	429,617	480,023	477,891	-10.50%	-10.10%
	Mean	588,911	588,911	588,911	0.00%	0.00%
	Std. D.	87,742	76,652	78,729	14.47%	11.45%
90%	CE	544,539	564,361	557,044	-3.51%	-2.24%
	Mean	652,508	652,508	652,508	0.00%	0.00%
	Std. D.	94,737	86,807	89,775	9.14%	5.53%
100%	CE	581,900	574,811	552,969	1.23%	5.23%
	Mean	714,786	714,786	714,786	0.00%	0.00%
	Std. D.	111,803	105,567	108,338	5.91%	3.20%
110%	CE	645,519	654,161	649,912	-1.32%	-0.68%
	Mean	792,580	792,580	792,580	0.00%	0.00%
	Std. D.	125,940	116,758	118,925	7.86%	5.90%
120%	CE	717,994	710,940	702,333	0.99%	2.23%
	Mean	860,255	860,255	860,255	0.00%	0.00%
	Std. D.	129,721	135,159	138,794	-4.02%	-6.54%

Table 6.8 shows the present value of weights in risky assets for different future returns. There is a general indication when asset returns increases, the leverage lifecycle strategy needs a lower amount of risky assets to provide the same certainty equivalent. Also, the leverage lifecycle strategy must compensate for more risky assets to deliver the same mean.

Table 6.8: Present value of weights in risky assets for 200 cohorts with different returns

Percentage of historical return	Leverage Lifecycle Strategy	Target Date Fund	Merton Samuelson	Lifecycle improvement over Target Date Fund	Lifecycle Improvement over Merton Samuelson
60%	68.8%	62.9%	66.4%	9.30%	3.58%
70%	67.9%	63.8%	66.8%	6.46%	1.71%
80%	66.7%	66.1%	69.8%	0.86%	-4.49%
90%	65.4%	65.3%	69.3%	0.17%	-5.57%
100%	64.4%	66.7%	70.9%	-3.39%	-9.16%
110%	63.4%	65.9%	70.4%	-3.82%	-9.96%
120%	62.5%	66.7%	71.4%	-6.28%	-12.48%

There are overall trends that indicate if the returns of risky assets are lower then the leverage lifecycle strategy will clearly underperform its benchmarks, meanwhile there are advantages of higher returns for the leverage lifecycle strategy. This is also consistent with the results from Ayres and Nalebuff (2010a); that lower returns provides less benefits for the leverage lifecycle strategy. However, the authors still had 5.7% *higher* certainty equivalent if returns decreased with approximately 30%. Correspondingly to these results, the certainty equivalent is 4.98% *lower* compared to an equivalent benchmark.

Bear in mind, the annualised real margin rate in this subsection is kept constant of 3.91%. Therefore, when the returns of risky assets are lowered, so are the excess returns of margin rate. Consequently, the advantages of investing with leverage will be reduced. Actually, this corresponds in some aspects to the same functionality as increasing the margin rate, which was performed in section 6.4.

6.6 Comparison of results

This subsection examines the results from this thesis compared to the original work from Ayres and Nalebuff (2010a). There are several reasons that the results from section 5 and 6 are different compared to the original article. One of them is that this thesis' model is an extension that captures 7 different risky assets instead of only stocks. Another issue is that there are other input variables such as margin rate, CRRA level, asset returns etc.
Between the two benchmark strategies MS and TDF, it is clear that the TDF-strategy outperforms the MS-strategy in almost every aspect and instances. This is not only the case in the basic model with equal mean, but also under the resilience. Consequently, when the leverage lifecycle strategy is examined, the requirements for outperformance is high. Ayres and Nalebuff compares their leverage lifecycle strategy to a MS-strategy with equal mean as in this thesis, but they fail to make a fair comparison to a TDF-strategy. They use a TDF-strategy that starts with 90% and ends at 50% in stocks, but neither the mean nor present value weighted in risky assets of final wealth is equalised to the leverage lifecycle strategy. This distorts the comparison, because their leverage lifecycle strategy only needs to outperform the MS-strategy.
Their excess return of equity compared to the margin rate is also 3.74%, meanwhile this thesis' leveraged portfolio has 1.78% and in the two-asset model it is 3.08%. This means, their models have inputs that are better suited for leverage investments.
A combination of using a two-asset model, lower margin rate, lower CRRA level and a comparison to the MS-model, should deliver superior results for the leverage lifecycle strategy.
It is debatable whether or not some of this thesis' input variables are more correct compared to Ayres and Nalebuff (2010a) in regard to asset returns and margin rates. Nevertheless, although the leverage lifecycle strategy does not outperform its benchmarks in every aspect at all times, there is a strong tendency that by decreasing the proportion of risky assets by time is beneficial, thus the theory of diversification across time to hold true.

7 Conclusion

The leverage lifecycle strategy is developed by Ayres and Nalebuff. It is based upon the concept of diversification across time, where an investor leverages his investments in his early retirement savings period, in order to deleverage and obtain a low allocation in risky assets before retirement. The leverage lifecycle strategy is considered a challenge compared to conventional retirement savings strategies due to its leverage investing.

The strategy in this thesis is extended by applying 7 risky assets instead of only one into the portfolio, thus it represents a diversified portfolio. The focus on the performance lies primarily on the final wealth. The model is constructed by using 200 cohorts produced by Monte Carlo simulations. The leverage lifecycle strategy is compared with two different strategies, and in the comparison all three strategies are calculated with the same mean of final wealth. The investor's risk appetite is defined as a constant relative risk aversion with a level of 23.

The overall results are ambiguous. On one hand, the leverage lifecycle strategy has a higher uncertain outcome of final wealth, because its standard deviation is 5.91% and 3.20% larger compared to the TDF-strategy and MS-strategy respectively. On the other hand, the strategy seems to have beneficial factors, one of them is a higher certainty equivalent. Also, and perhaps more importantly, the investor needs fewer investments in risky assets to obtain the same mean of final wealth. His present value weighted in risky assets is 3.39% and 9.16% lower compared to the TDF-strategy and MS-strategy respectively.

If the investor equalises the amount of risky investments between each strategy, the leverage lifecycle strategy outperforms the mean of final wealth with 2.54% and 8.15% compared to the TDF-strategy and MS-strategy respectively: the equivalent of retiring 8 and 24 months earlier from the labour market.

To fully grasp the leverage lifecycle strategy's advantages and shortcomings, the resilience has been tested by altering different variables. It is investigated how the outcome of the strategy is affected by amending the investor's risk aversion. If the investor has a CRRA level between 19-29 the leverage lifecycle strategy outperforms in most aspects. However, when the investor has a CRRA level of 17 the leverage lifecycle strategy clearly becomes less beneficial.
The leverage lifecycle strategy benefits, as expected, from lower margin rates, and can even tolerate a small increase in margin rate. This is because the excess return from the portfolio is higher during low margin rates. Correspondingly, when the returns from risky assets are lowered, the excess return decreases and thereby the leverage lifecycle strategy shows disadvantages.

Consequently, the results presented are not fully supporting the superiority of the leverage lifecycle strategy, because it depends highly on the input variables. In general, the concept of diversification across time outperforms strategies that invest a constant fraction in risky assets. Under most circumstances diversified leverage investments can improve an investor's final wealth distribution, but is highly dependent on low margin rate and a high excess return of risky assets.

References

Alexeyev, J. (2014). *State of Global Fund Industry.* http://www.strategicinsightglobal.com/Sample_Reports/State-of-the-Global-Industry-Year-in-Review-and-2014.pdf.

Aparicio, F., & Estrada, J. (1997, April). Empirical Distributions of Stock Returns: Scandinavian Securities Markets, 1990-95. *Carlis III University*.

Arrow, K. (1965). Aspects of the theory of risk-bearing. *Helsinki, Yrjš Jahnsson Foundation*.

Avellaneda, M., & Zhang, S. (2010, May). Path-dependence of Leveraged ETF Returns. *SIAM J. Financial Math*, *1*, 586-603.

Ayres, I., & Nalebuff, B. (2010a, October). Diversification Across Time. *Yale Law and Economics Research Paper*(413).

Ayres, I., & Nalebuff, B. (2010b). *Lifecycle Investing A New, Safe, and Audacious Way to the Performance of Your Retirement Portfolio*. Basic Books.

Barclays Capital. (2008, December). *Global Inflation-Linked Index Fact Sheet.*

Barclays Capital. (2015a, September). *Global Corporate 7-10 Year Very Liquid Index Fact Sheet.*

Barclays Capital. (2015b, September). *Global High Yield Index Fact Sheet.*

Barclays Global Investors. (2008). *Retirement Plan Investments FAQs.* Retrieved 2015-12-13, from {http://www.marketwired.com/press-release/barclays-global-investors-918094.htm}

Brunnermeier, M. K. (2008). Deciphering the Liquidity and Credit Crunch 2007-2008. *Journal of Economic Perspectives*, *23*(1), 77-100.

Casanova, M. (2013, August). *Revisiting the Hump-Shaped Wage Profile.*

Chang, M., & Madhaven, A. (2009, October). *Dynamics of Leveraged and Inverse ETFs.* (Barclays Global Investors)

Chowdhry, B., & Nanda, V. (1998). Leverage and Market Stability: The Role of Margin

Rules and Price Limits. *Journal of Busness*, *71*(2), 179-210.

Cincinnati Asset Management. (2012). *High Yield - An Asset Allocation Perspective.* Retrieved 2016-02-07, from http://www.strategicinsightglobal.com/Sample_Reports/State-of-the-Global-Industry-Year-in-Review-and-2014.pdf

Clingman, M., & Burkhalter, K. (2015, July). Scaled Factors for Hypothetical Earnings Examples Under the 2015 Trustees Report Assumptions. *Social Security Administration*(3).

Economic Research. (2015). *Household Debt to GDP for United States.* Retrieved 2015-12-13, from https://research.stlouisfed.org/fred2/series/HDTGPDUSQ163N

Fortune, P. (2000). Margin Requirements, Margin Loans, and Margin Rates: Practice and Principles. *New England Economic Review*, 19-44.

Goldman Sachs. (2015). *S&P GSCI Commodity Index.* www.goldmansachs.com/what-we-do/securities/products-and-business-groups/products/gsci.

Green, R. K., & Wachter, S. M. (2005). The American Mortgage in Historical and International Context. *Journal of Economic Perspectives*, *19*, 93-114.

Hans-jörg Naumer, S. S., Ann-Katrin Petersen, & Krings, G. (2013). *The Evolution of High-Yield Bonds into a Vital Asset Class.* Allianz Global Investors.

Hult, H., & Lindskog, F. (2007). *Mathematical Modeling and Statistical Methods for Risk Management.*

Ilan Guedj, G. L., & McCann, C. (2010). *Leveraged ETFs, Holding Periods and Investment Shortfalls.* (Securities Litigation and Consulting Group)

Internal Revenue Service. (2015). *Retirement Plan Investments FAQs.* Retrieved 2015-12-02, from https://www.irs.gov/Retirement-Plans/Retirement-Plan-Investments-FAQs

International Monetary Fund. (2011). *Global Financial Stability Report Durable Financial Stability: Getting there From here.*

Jorion, P., & Goetzmann, W. N. (1999, June). Global Stock Markets in the Twentieth Century. *The Journal of Finance*, *54*(3), 953-980.

Kubler, F., & Paul, W. (2006, March). Collaterized Borrowing and Lifecycle Portfolio Choice. *NBER working paper 12309*.

Little, P. K. (2010). Inverse and Leveraged ETFS: Not Your Father's ETF. *The Journal of Index Investing*, *1*(1), 83-89.

Luca Benzoni, P. C.-D., & Goldstein, R. S. (2007, October). Portfolio Choice over the Life-Cycle when the Stock and Labor Markets are Cointegrated. *The Journal of Finance*(5).

Luis M. Viceira and John Y. Campbell. (2001). *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors.* Oxford University Press.

Lusardi, A., & Mitchell, O. S. (2005, December). Financial Literacy and Planning: Implications for Retirement Wellbeing. *Michigan Retirement Research Center*.

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, *7*(1), 77-91.

McDonald, R. L. (2013). *Derivatives Markets* (3rd ed.). Pearson.

Merton, R. C. (1969, August). Lifetime Portfolio Selection Under Uncertainty The Continous-Time Case. *The Review of Economics and Statistics*, *51*(3), 247-257.

MSCI. (2016, February). *MSCI World REIT Index Fact Sheet.*

Niels Bekkers, R. Q. D., & Lam, T. W. (2009, October). Strategic Asset Allocation: Determining the Optimal Portfolio with Ten Asset Classes. *Journal of Economic Perspectives*.

Officer, R. R. (1972, December). The Distribution of Stock Returns. *Journal of the American Statistical Association*, *67*, 807-812.

Phillips, Y. (2012, May). *A Case for High-Yield Bonds.* Russell Research.

Pratt, J. W. (1964). Risk Aversion in the Small and in the Large. *Econometrica*, *32*, 122-136.

Quan, D. C., & Titman, S. (1996). Commercial Real Estate Prices and Stock Market Returns: An International Analysis. *Financial Analysts Journal*, *53*(3), 21-34.

Samuelson, P. A. (1969). Lifetime Portfolio Selection By Dynamic Stochastic Programming. *The Review of Economics and Statistics*, *51*(3), 239-246.

Shiller, R. (2005). *Irrational Exuberance* (2nd ed.).

Trejos, N. (2007). *In pursuit of a down payment.* Washington Post. Retrieved 2015-12-13, from `http://www.washingtonpost.com/wp-dyn/content/article/2007/11/02/AR2007110201240.html`

Viceira, L. M. (2001, April). Optimal Portfolio Choice for Long-Horizon Investors with Nontradeable Labor Income. *The Journal of Finance*, *2*.

Wilson, J. H., & Droms, W. G. (2009, April). Life Cycle Investing Revisiting an Old Planning Approach. *The CPA Journal*, 52-53.