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Executive Summary

As part of the European Union key policy aimed at reducing greenhouse gas emissions (GHG) – European Union Emission Trading Scheme (EU ETS) was launched in January 2005. Following this initiation, a new investment asset was introduced to the market – European Union Allowance (EUA) – a right to emit a ton of CO2. The establishment of the European carbon system created a new type of risk – carbon risk, to which more and more companies and investors are becoming to be exposed.

Despite the growing importance of the carbon market in investment and risk management, the available research of the topic remains limited. Although a handful lot of papers, investigating EU ETS Phase I, have been issued, the empirical research on Phase II remains scarce. The aim of this paper is to shed more light on the new structure of the market and different factors affecting market participants’ carbon risk, as well as provide a thorough analysis of their interdependency, and offer some insight for risk management purposes.

By looking for a presence of a stochastic trend, Hotelling rule and cost-of-carry relationship we establish a methodology to check the efficiency of the EU ETS market. We conclude that current market setting is inefficient and intertemporal arbitrage opportunities are present. By implementing correlation, cointegration and copula methods, we conclude that EUAs futures are a suitable instrument to hedge away carbon risk. We propose implementation of copula approach in the calculation of Value-at-Risk metric as well as optimal hedge ratio for the sound risk management practices.
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Abbreviations and Significant Terms

*Annex I countries*  Industrialised countries that have agreed to reduce their greenhouse gas emissions by signing the UNFCCC.

*Annex II countries*  Developed countries responsible for bearing the costs of climate change mitigation in developing countries. Subset of Annex I countries.

*Annex B countries*  Countries that have ratified the Kyoto Protocol and have committed to reduce their greenhouse gas emissions by a certain percentage (compared to their 1990 levels) by the end of the period 2008-2012.

*ADF*  Augmented Dickey-Fuller stationarity test

*AAU*  Assigned Amount Units, which are emission permits assigned to countries under the Kyoto Protocol. These can be traded between countries, and are also used to offset issued project credits.

*ARIMA*  Autoregressive Integrated Moving Average Model

*BLUE*  Best Linear Unbiased Estimators

*CAPM*  Capital Asset Pricing Model

*CDM*  Clean Development Mechanism

*CER*  Certified Emission Reductions, which are emission permits obtained from the CDM projects, each allowing emission of 1 tonne carbon dioxide equivalents.

*CITL*  Community Independent Transaction Log

*CO2*  Carbon Dioxide, CO2-e – Carbon Dioxide-equivalent

*COP*  Conference of the parties

*D.F.*  Distribution Function

*ECDF*  Empirical Distribution Function

*EG*  Engle and Granger Cointegration Test

*EIT*  Economies in transition

*ERU*  Emission Reduction Units, which are emission permits obtained from the JI projects, each allowing emission of 1 tonne of carbon dioxide equivalents.

*EU ETS*  European Union Emission Trading Scheme
**EUA**  European Union Allowance – emission permit issued and traded within the EU ETS, each allowing emission of 1 tonne of carbon dioxide equivalents.

**EUAA**  European Union Aviation Allowances

**EUTL**  European Union Transaction Log

**GHG**  Greenhouse Gas/Gases

**HFC**  Hydrofluorocarbons

**I.D.D.**  Independently and identically distributed

**IET**  International Emission Trading

**JI**  Joint Implementation

**KME**  Kendall’s τ Moment Estimator

**KP**  Kyoto Protocol

**KPSS**  Kwiatkowski, Phillips, Schmidt and Shin stationarity test

**LULUCF**  Land use, land-use change and forestry

**MPL**  Maximum Pseudo-Likelihood

**N2O**  Nitrous Oxide

**NAP**  National Allocation Plan

**OECD**  Organisation for Economic Co-operation and Development

**OHR**  Optimal Hedge Ratio

**OLS**  Ordinary Least Squares

**PFC**  Perfluorocarbons

**PP**  Phillips-Perron stationarity test

**UNFCCC**  The United Nations Framework Convention on Climate Change.

**VAR**  Vector Autoregressive Model in interdependency analysis or VaR - Value-at-Risk in risk management or

**VECM**  Vector Error Correction Model
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1. Introduction

As part of the European Union key policy aimed at reducing greenhouse gas emissions (GHG) – European Union Emission Trading Scheme (EU ETS) was launched in January 2005. Following this initiation, a new investment asset was introduced to the market – European Union Allowance (EUA) – a right to emit a ton of CO2. The establishment of the European carbon system created a new type of risk – carbon risk, to which more and more companies and investors are becoming to be exposed. Therefore understanding the drivers behind carbon prices movements is essential for sound risk management practices.

The EU ETS is organised into three phases: Phase I (2005-2007), Phase II (2008-2012) and Phase III (2013-2020). Phase I was a learning-by-doing phase, when no emissions reduction targets were set and banking of emissions to succeeding phases was not allowed. Phase I is characterised by over allocation and, hence, failure to trigger internal abatement efforts. Phase II, on the other hand, is targeting to cut EU-15 1990 emissions by 8%, as established by Kyoto Protocol. Furthermore, banking of emission allowances into Phase III is allowed.

Since the launch of the EU ETS carbon prices have followed a rather dynamic price path, sometimes exhibiting dramatic movements. Despite the growing importance of the carbon market in investment and risk management, the available research of the topic remains limited. Although a handful lot of papers, investigating EU ETS Phase I, have been issued, the empirical research on Phase II remains scarce.

The aim of this paper is to shed more light on the new structure of the market and different factors affecting market participants’ carbon risk, as well as provide a thorough analysis of their interdependency, and offer some insight for risk management purposes.

First, by drawing on the presence of stochastic trend, Hotelling rule and cost-of-carry relationship, we establish a methodology to check the efficiency of the EU ETS market. We conclude that current market setting is inefficient and arbitrage opportunities are present. Further,
we provide a theoretical background for the empirical analysis of EUAs interdependency with tradable relevant assets that can be used to hedge away carbon risk.

To our best knowledge, this is the first study that attempts to thoroughly compare EUAs interdependency relationship with energy commodities and financial assets by comparing different interdependency measures – correlation, cointegration and copula – and suggest their practical application to risk management. We find that EUAs futures is a suitable instrument to hedge away the carbon risk.

This paper is organised as follows. Chapter 2 defines and delimits the problem statement and introduces the main methods used in the analysis. In Chapter 3, the background, necessary to understand the functioning of the EU ETS market is presented. In Chapter 4, the theoretical frameworks to test the efficiency of the European carbon market are established. Chapter 5 offers an econometrical specification and a discussion of the concepts to be applied in this paper. In Chapter 6, we present data, as well as main empirical finding of our study, while in Chapter 7, we propose their application to risk management. Chapter 8 concludes and offers insights and potential areas of a future research.
2. Problem Definition and Methodology

This chapter will present the reader with the problem statement and sub-questions of the paper. Delimitations that stem from the formulation of the problem will be also made explicit and a methodological approach to the research process will be given. Finally, the structure of the thesis will be explained along with the relevant literature review and specifications of this paper contributions to the academic research.

2.1. Research Objective and Preliminary Methodological Considerations

The purpose of this paper is to apply correlation, cointegration and copula approaches in order to analyse the EU ETS asset prices development as well as provide some insight for investment and risk management decisions.

In order to address the research objective, the study will combine normative and explanatory approaches. The following Research Questions will guide the analysis and help to select appropriate theoretical frameworks and econometrical models.

1. What are the drivers of the European carbon prices?
In order to understand the environment in which EUAs prices are formed a qualitative study of the EU ETS market will be carried. This will help us to identify relevant drivers of carbon that will be later analysed in a quantitative part of the paper.

2. Is current EU ETS market setting efficient?
An in-depth understanding of the European carbon market setting and its potential implications on CO2 prices and risk management techniques will be build based on the study of the EU ETS efficiency. An empirical analysis will be based on three different conceptual frameworks: stochastic trend, Hotelling rule and cost-of-carry model.

Bredin and Muckley (2009b) argue that the main goal of the EU ETS is to encourage an internal abatement of GHGs, consequently, it is important to set such a legal setting, that would create a scarcity of carbon permits. This, according to Paolella and Taschini (2008), should lead to a
mean reversion around an upward trend of EUAs prices – hence, the presence of a stochastic trend in carbon prices is an indicator of the carbon market setting inefficiency. Not less important, is that the presence of a unit root may lead to unreliable econometrical results.

*Hotelling rule* model builds on the capital asset pricing model theory (CAPM), and investigates if current EU CO2 market structure sends a sufficiently strong signal, for CAPM style spot prices pricing to prevail. In case the model does not hold – intertemporal arbitrage opportunities are available.

*Cost-of-carry model*, in turn, investigates a relationship between EUA spot and futures prices. According to the model, futures price should equal the spot price, adjusted for the opportunity cost of holding a spot position. The hypotheses under the investigation allow to investigate the relationship between spot and futures prices as well as the presence of any risk-free arbitrage opportunities.

3. *Are EUAs spot and futures prices interdependent?*

Building on a qualitative framework of Hotelling and cost-of-carry models, a quantitative research is carried. The econometrical concepts of correlation, cointegration and copula are discussed and applied to investigate the interdependency of carbon spot and future prices.

4. *What is the relationship between the EUAs and energy commodities as well as financial markets?*

The understanding of the EU ETS market and price behaviour of the EUAs is further deepened by the application of the statistical and econometrical concepts to a quantitative investigation of carbon price relationship with those of energy commodities and financial assets.

5. *What are the implications for risk management and investment decisions?*

After the establishment of EUAs price drivers and their interdependence, risk measures to mitigate carbon risk are proposed. First, we discuss Value-at-Risk metric, which helps to evaluate a potential downside of the investment. Then, the application of optimal hedge rations is proposed.
The schematic representation of methodological approach to thesis structure is presented in Figure 2.1.

The data, used in this paper, is collected from Bloomberg platform, and represents the time series of commodities and financial assets. We use SAS 9.2 and R 2.13.1 statistical software for the econometrical analysis.

2.2. Delimitations

The EU ETS is a relatively new and complex market and there are a lot of research areas that require a thorough investigation. Nevertheless, the purpose of this study is to identify interdependency relations between EUAs and energy commodities as well as selected financial assets and apply the results of the carried empirical analysis to suggest sound carbon risk management practices.

The paper will analyse the most liquid carbon asset available for trading – EUAs, and other tradable permits, such as CERs, will not be analysed. Furthermore, the investigation will be limited to the Phase II of EU ETS market. This is due to the fact, that Phase I and Phase II are very different markets (as will be discussed in Section 3.3.), and hence, the conclusions reached during Phase I study would not necessarily hold in Phase II.

In order to gain more insight into the drivers and processes behind EUAs price formation, EU ETS market setting is described and its efficiency is tested. Yet, it is out of scope of this paper to discuss the legal setting and/ or provide any recommendations regarding the market structure, its efficiency and/ or emission allocation methodology.

Another limitation is related to copulas. First, since this thesis is not a detailed study of copula approach, but rather a comparison to correlation and cointegration techniques and its practical application to the EU ETS market analysis and carbon risk management, we will limit the amount of copulas under consideration to the most popular ones: Gaussian, Student-t, Clayton,
Second, limitation stems from the fact, that copula is a relatively complicated concept, and econometrical software that would allow to consider dynamic copulas, is not available for the author. Hence, this study will be limited to static copulas.

Although this paper presents a couple of risk management metrics (Value-at-Risk and Optimal Hedge Ratio), this is done in order to suggest a practical application of different dependence measures and present some insight into potential carbon risk mitigation practices. It is not the purpose of this paper to provide a detailed discussion and analysis of risk management and its practices.

Despite the fact, that during the paper writing process, global economy is about to fall into the second recession in two years, an in-depth analysis of its implications is out of scope of the research objective. Nevertheless, in order to be able to present robust conclusions, the analysed data will be checked for structural breaks and separate results will be reported.

2.3. Structure of the Thesis

This paper consists of eight main sections. In Chapter 3, the carbon market background necessary to understand the functioning of the EU ETS market is introduced. In Chapter 4, the theoretical frameworks to test European carbon market efficiency are established. Chapter 5 offers econometrical specification and discussion of the concepts applied in this paper. In Chaper 6 we present analysed data and discuss empirical findings of our study, while in Chapter 7 we suggest potential risk management metrics and measures, that could be used by the participants of the European carbon market. Chapter 8 concludes and offers potential areas for future research.

Graphical representation of simplified structure and methodological approach of the thesis is given in Figure 2.1.
Since first-differences of analysed data are found to be stationary, implications of stochastic trend on concepts studying interdependency of returns will not be discussed.

Although regression does not form a part of our primary research, however its concepts are applied in empirical analysis.
2.4. Literature Review

Although EU ETS market is relatively new, much has been written on the pilot Phase I of the market. However, few empiric studies were carried on Phase II data.

Classification of Emission Allowances

Based on the fact, that CO2 is an input in a production process, Borak et al. (2006) has classified carbon credits as commodities. Despite of that, a discussion of emission allowances similarities with a financial asset class is carried by Boral et al. (2006) and Benz and Trück (2006).

Carbon Market Drivers

Burtraw (1996) and Gronwald et al. (2011) have categorised carbon market drivers into two main groups: a) policy and regulatory issues and b) market fundamentals that are directly related. Tiits (2009) provides an in-depth qualitative discussion of those factors. Hintermann (2010) and Rickels et al. (2010) find that a renewable energy negatively influences carbon prices, while a positive influence of an unanticipated extremely cold or hot temperature is discussed by Alberola et al. (2007), Chevallier (2008), Mansanet-Bataller et al. (2007) and Rickels et al. (2007). Further, energy prices are found to influence carbon prices by Mansanet-Bataller et al. (2007), Mansanet-Bataller and Pardo (2008) and Hintermann (2010).

Market Efficiency

Three different approaches were adopted in the literature in order to study the efficiency of EU ETS market setting: a stochastic trend, Hotelling rule and cost-of-carry model.

Bredin and Muckley (2009b) find the presence of unit root in EUAs prices in the period from 2005 to July 2009), which signals a EU ETS market failure to create a sustainable cap-and-trade market. Other authors conduct similar analysis indirectly and present contradictory results. Daskalakis et al. (2009) and Alberola et al. (2007, 2008a, 2008b, 2009) confirm the presence of a stochastic trend in European carbon prices. While Bredin and Muckley (2009a) in their early study of Phase II reject a unit root hypothesis. Our investigation found no later literature on the subject, hence an updated study is needed.
Alberola and Chevallier (2007) adapt Hotelling model of spot prices in order to analyse EU ETS efficiency in Phase I. The authors conclude that Hotelling rule does not hold in the pilot phase and that EUA spot prices do not adequately reflect abatement costs at the installation level. To our knowledge, no similar research in Phase II was carried and there is a pending need to investigate the topic.

An alternative approach to study European carbon market efficiency – cost-of-carry model was investigated by various authors: Phase I was studied by Borak et al. (2006), Milunovich and Joyeux (2007), Uhrig-Homburg and Wagner (2008), Chevallier (2008), Daskalakis et al. (2009), while early Phase II was conducted by Tiits (2009). The authors, researching Phase I, draw mixed conclusions: Milunovich and Joyeux (2007) conclude that 2006 and 2007 futures contracts do not demonstrate cost-of-carry relationship with spot prices, while Daskalakis et al. (2009), in the contrast, find the parity between spot and discounted future prices. The research carried by Uhrig-Homburg and Wagner (2008) confirm these findings, but add that this dependence no longer holds in Phase II. Chevallier (2008) reaches the same conclusions and specifies, that Phase I spot price fails to reflect Phase II future prices due to the ban of banking of Phase I allowances to Phase II. Tiits (2009) is the first researcher investigating how Phase II spot prices explain futures that mature in Phase II. However, the author fails to confirm the existence of the cost-of-carry relationship in Phase II due to the lack of evidence that a necessary cointegration condition between spot and futures prices exists. As far as we are aware, this was the only intent to investigate a cost-of-carry relationship in Phase II market and since it was carried in the very early stage of Phase II, an updated investigation is needed.

Interdependency Studies

Mansanet-Bataller et al. (2007) are the first to conduct an econometric analysis between the prices of CO2 and those of energy commodities. Authors apply a simple correlation approach to study the relationship between CO2 prices, energy commodities and the weather. Tiits (2009) complement correlation approach with cointegration methodology in her carbon and energy commodities interdependence study. Cointegration approach was also applied by Milunovich and Joyeux (2007), Uhrig-Homburg and Wagner (2008) in research analysing carbon spot and
future prices interdependence as well as in the work carried by Bunn and Fezzi (2007) in carbon and UK electricity and gas prices analysis.

To our best knowledge, two applications of copula approach in CO2 study were carried. Boerger et al. (2007) compare carbon and energy commodities linear correlation estimates with correlations implied by Student-t and Normal-inverse Gaussian copulas. The authors also calculate a minimum variance portfolio for gas-fired and coal-fired power plants in order to demonstrate a practical application of their study. Gronwald et al. (2011) extend the study to time-varying copulas and add two more copula families to their study – Clayton and Gumbel and apply their findings to compare different Value-at-Risk estimates of theoretical investment portfolios.

Risk Management and Carbon

The literature on carbon risk management is rather sparse. Chevallier (2008) investigates investors risk aversion during Phase I and concludes that in comparison to stock market investors, carbon market investors are more risk averse. Boerger et al. (2007) contribute with their study of risk management from power plant perspective, while Gronwald et al. (2011) focus on the accuracy of the application of Gaussian and Student-t copulas in comparison to a multivariate normal approach versus a univariate GARCH model for Value-at-Risk calculations. Fan and Akimov (2010) calculate optimal hedge ratios of EUAs spot prices in respect to their futures quotes based on OLS regression, VECM and GARCH-VECM.

Carbon Pricing Models

An important strand of literature on EU ETS that will not be covered in this paper is EUA pricing models. For example, Seifert et al. (2006) study carbon pricing under equilibrium conditions, while Çetin (2009) avoid equilibrium approach, but rather focus at no-arbitrage relationship between spot and futures prices in order to come up with an explicit semimartingale representation for the spot prices. Gagliardi (2009), on the other hand, suggest the Heston model combined with jump components for the pricing of the European carbon credits.
2.5. Contributions to the Academic Research

From the literature discussion, presented in the section above, it is clear, that there is a lack of an up-to-date investigation of EU ETS Phase II market. The outright contribution of this paper to existing literature is the extension of analytical approaches, adopted to analyse Phase I market, to Phase II and the extension of time period, studied in early Phase II research. The thesis also offers a unique comparison\(^1\) of dependence measures – correlation, cointegration and copula, which is later extended to suggestions for the practical carbon risk management.

\(^1\) To author’s knowledge, previously this approach was not implemented neither in EU ETS market studies, nor in investigations of other markets.
3. Carbon Market Background

This chapter will present the reader with necessary background information on the EU ETS market. The chapter starts with a historical framework of a world-wide effort to reduce GHGs footprint, then global carbon reduction mechanisms are introduced. The focus is then shifted to European carbon market, with a concentration on the scope, allocation methodology, temporal limits, trading rules and fundamental drivers.


United Nations Framework Convention on Climate Change (UNFCCC), aimed at reducing global warming, was adopted on 9th of May, 1992, following Intergovernmental Negotiating Committee (ING) meeting in New York, United States. After five years, The Kyoto Protocol (KP) to the UNFCCC, was adopted on 11th of December, 1997 in Kyoto, Japan during the third conference of the parties (COP-3) and entered into force on 16th of February, 2005. To date, 191 states have signed a protocol and 190 have ratified it. Although the United States have signed the Protocol, but they did not ratify it, while Afghanistan, Andorra, South Sudan and Vatican City have not expressed their opinion on the protocol.

The UNFCCC divides all participating countries into three main groups:

- **Annex-I** countries include the industrialised countries, that in 1992 were members of the Organisation for Economic Co-operation and Development (OECD) and countries with economies in transition (EIT) including the Russian Federation, the Baltic States and several Central and Eastern European States). The Convention requires Annex I Parties to adopt climate change policy measures with the aim of reducing their GHGs emissions to 1990 levels by the year 2000².

- **Annex-II** Parties consist only of the OECD members of Annex-I (EIT parties are not included), who are required to provide financial support for developing countries to implement emission reduction activities as well as help them to adapt to hostile effects of climate change.

- **Non-Annex-I** Parties are mostly developing countries.

² The convention grants some flexibility for EIT countries, by allowing them to select a base year other than 1990.
After KP came into force in 2005, Annex-I countries have made a legally binding commitment under Annex-B of the KP to reduce six types of greenhouse gases (GHG) listed in Annex-A of the Protocol, namely carbon dioxide (CO2), methane (CH4), nitrous oxide (N2O), sulphur hexafluoride (CF6), hydrofluorocarbons (HFCs) and perfluorocarbons (PFCs) by 5.2% from the 1990 level in the commitment period from 2008 to 2012. Currently there are 38 Annex-B countries that are required to submit annual reports, showing emissions reduction progress. Although six GHGs were defined, a measure unit – CO2-equivalent (CO2-e) was constructed in order to facilitate understanding of the contribution of each type of gas to global warming and make monitoring and reporting more commodious. It should be noted, that neither international aviation, nor shipping emissions are included in KP targets. Annex-A also excludes emissions and removals from the land use, land-use change and forestry (LULUCF) sector, which are treated differently than emissions from the other sectors.

It is important to highlight, that KP is due to expire at the end of 2012. High level talks are being held in order to come up to a “post Kyoto” agreement, with the latest taking place in Copenhagen, Denmark (COP-15) and Cancun, Mexico (COP-16). However, all of them failed to reach an agreement. During last year’s COP-16 Japan, Russia and US hinted that they would not agree to an extension of the KP beyond 2012, demanding that the emission targets are imposed on all nations, while developing countries are interested to see industrialised economies to sticking to their KP obligations and agreeing on tougher commitments from 2013. The inability to reach the global agreement has fuelled talks that the legal framework for the United Nations backed carbon trading schemes (which we will discuss in the following section) will be removed once current KP ceases to exist. One of the last chances for all the parties to reach a consensus and extend the regulation beyond 2012 will be COP-17 which is to be held in December, 2011 in Durban, South Africa.

3.2. Flexible Mechanisms
In order to aid Annex-I countries to reach their commitment targets, KP allows for several so called “flexible mechanisms” which permit industrialised nations to fund the emission reduction

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3 It is important to note, that in Annex-I, EU is treated as one entity, as it opted for a bubble solution, where a Kyoto Protocol commitment of 8% is distributed among its member states.
projects in non-Annex-I countries. The flexible mechanisms included into the Kyoto Framework are as follows:

- **Clean Development Mechanism (CDM)** – “is a mechanism by which Annex-I Parties can invest in emission reduction projects or afforestation or reforestation projects in developing countries and receive a credit for the emission reductions or removals achieved” (UNFCCC, 2007, pg.29). The goal of this mechanism is to generate investments in developing countries and promote their sustainable development. Credits generated via CDM are called Certified Emission Reductions (CERs) and can be used by the Annex-I country in meeting its emissions targets. In order to be able to generate CERs, all CDM projects have to be approved by the Executive Committee of the CDM Board for Projects.

- **Joint Implementation (JI)** – is another project based mechanism, that “allows Annex-I Parties to implement projects that reduce emissions, or increase removals using sinks\(^4\), in other Annex-I countries” (UNFCCC, 2007, pg.31). Credits generated while implementing JI projects are called Emission Reduction Units (ERUs) and can be used by investing in the other Annex-I country to help to meet their emission reduction efforts. It should be emphasised, that in order to avoid double counting of credits, a corresponding amount of emission reductions credits is subtracted from the host party. JI mechanism allows for a possibility of implementing emissions reduction projects in Annex-I countries with lower emissions cutting costs – most likely countries with economy in transition. ERUs can be generated only by JI projects approved by Joint Implementation Supervisory Board.

- **International Emission Trading (IET)** – “enables Annex-I Parties to acquire Assigned Amount Units (AAUs) from other Annex-I parties that are able to more easily reduce emissions” (UNFCCC, 2007, pg.31). The mechanism also allows Annex-I countries to acquire all types of units – CERs, ERUs and Removal Units (RMUs)\(^5\) and thus benefit from the possibility to incur lower costs of a mitigating climate change. Under this regulation “Annex-I Parties may choose to implement domestic or regional systems under which legal entities, such as industrial or power plants that are subject to GHG controls, can trade emission allowances and credits” (UNFCCC, 2007, pg.31). The EU

\(^4\) Sinks – activities carried by LULUCF that absorb CO2.

\(^5\) RMUs are generated in Annex-I countries by LULUCF activities that absorb carbon dioxide
ETS, which will be described in detail in a further section, is an example of regional trading system, operating under the KP umbrella.

3.3. The EU ETS scheme

The EU ETS is the first and the biggest international carbon trading scheme and is a cornerstone of the EU’s effort to reduce GHGs emissions. Although, it is created under the framework of the IET of the KP, its setting is totally independent. The main regulatory body of the system is European Commission (EC), which sets limits of emissions and other rules, while member state governments implement and enforce regulations. Allowances, traded under the EU ETS are called *European Union Allowances* (EUAs) and are equal to one tonne of CO2.

The EU ETS is a cap-and-trade system, where the total amount of EUAs is fixed for a certain period of time. Thus, the total amount of credits in the system is limited to a certain amount that cannot be exceeded. It has to be noted, that the use of emission reduction units, generated through flexible mechanisms under the KP (such as CERs and ERUs), is limited to a certain percentage of total cap, so that the system balance is not disrupted.

The EU ETS is the EC initiative that became legally binding after the Directive 2003/87/EC of 13th of October, 2003 was approved by the European Parliament (EP) and by the EU member states. The Directive 2004/101/EC, linking EU ETS scheme with flexibility mechanisms specified under the KP was approved in 2004. The EU ETS was launched on 1st of January, 2005 and is currently divided into three phases:

- Phase I (2005-2007) – was a *pilot* or learning-by-doing phase, when no emissions reduction targets were set. Member countries had to submit National Allocation Plans (NAPs) with their allocations, which were later reviewed by the EC. The main aim of the trial Phase I was to come up with a sustainable market structure and establish a comprehensive and harmonised approach to carbon emissions registry and place permits trading infrastructure. It was also important to gain knowledge on the behaviour and expectations of market players. Due to the poor data on the emissions of previous
periods, Phase I was over-allocated by 2.4% or 156 million tons and failed to send an appropriate price signal to trigger internal abatement efforts.

- Phase II (2008-2012) is targeting to cut EU-15 1990 emissions by 8% as established by the KP. Although allocations of allowances in Phase II were much stricter than in Phase I and some countries were auctioning up to 10% of allowances, due to a financial crisis emission levels in 2009 decreased 12% y/y and it is expected that Phase II will be over-allocated too.

- Phase III (2013-2020) currently targets a cut of 20% from 1990 emission levels, however there are on-going discussions to increase a reduction effort to up to 30% compared to 1990 levels.

### 3.3.1. Scope of EU ETS

The main aim of EU ETS is to pose an emissions cap on the most energy intensive sectors. Under the Directive 2003/87/EC only producers whose power consumption is bigger than 20MW are included in the system. In 2009 10,888 installations were covered by the EU ETS and represented approximately 42% of the unions GHGs emissions. Included installations are comprised by the industrials from the following economic activities: combustion plants, iron and steel plants, oil refineries, coke ovens as well as pulp and paper, glass, cement, lime, brick and ceramics factories. Other important industrial polluters, such as chemical and aluminium sectors were not included into the system due to heavy lobbying.

It should be mentioned, that during Phase I only CO2 gasses were covered by the EU ETS, while N2O gases were added to Phase II. Furthermore the size of the EU ETS increased in Phase II as new EU members – Bulgaria and Romania – were included into the EU ETS, while Lichtenstein and Norway joined it voluntarily.

Starting from 2012 under new Directive 2008/101/ES\(^6\) one new sector will be included into the system – aviation sector. New emission allowances – *European Union Aviation Allowances (EUAAs)* – that can only be used for emissions of aircraft operators will be introduced. What is

\(^6\) Directive 2008/101/ES approved 19th of November, 2008 amended the pervious Directive 2003/87/EC so as to include aviation activities into EU ETS.
more, in Phase III, when new Directive 2009/29/EC\textsuperscript{7} will come to force, emissions of bulk organic chemicals, non-ferrous metals and gypsum related activities as well as primary aluminium sector and production of nitric, adipic, glyoxal and glyoxylic acid will be included under the extended scheme. Inclusion of shipping sector is also widely debated and its inclusion appears to be feasible in the near future. Under Phase III PFCs from primary aluminium sector will also be included into the EU carbon market.

### 3.3.2. Allocation and Penalties

Kruger (2007) has defined EU ETS as a hybrid system in respect to the allocations of the EUAs due to its unique combination of centralised and decentralised regulation systems. Indeed, as already discussed, the EC is the main regulatory body of the system, which determines the scope and the main rules of the market, however, implementation and control of the application are in the hands of the individual member countries. What is more, “each government is in charge of deciding the amount of quotas available for trading, after negotiating with industrials and after the validation by the EC.” (Chevallier, 2008, pg.33) Each member state is required to submit their NAP 18 months before the start of the Phase. “The elaboration of the NAP requires, that each Member State must decide ex-ante how many allowances to allocate in total for a trading period. It has also to decide how many allowances each plant covered by the ETS will receive per year of the compliance period” (Mansanet-Bataller and Pardo, 2008b, p.16). In their NAPs countries are also allowed to decide on the amount of the allowances to be auctioned\textsuperscript{8}. However, the total amount that is being auctioned in Phase II is not more than 4% of total allowances. After the individual country NAPs are submitted, the EC has to accept or reject the plan within 3 months.

Under the Directive 2009/29/EC, in Phase III Member States will no longer be able to decide on the amount of allocations. In Phase III installations will receive allocations based on the new harmonised allocation methodology. Under the new approach permits will be distributed based

\textsuperscript{7} Directive 2009/29/EC was approved 23\textsuperscript{rd} of April, 2009 and amended previous Directive to extend EU ETS setting.

\textsuperscript{8} In Phase II the following countries chose to include auctioning into their NAPS (the average annual amount in the brackets): Germany (40 Mt, 9%), United Kingdom (17 Mt, 7%), The Netherlands (3.2 Mt, 3.7%), Austria (0.4 Mt, 1.3%), Ireland (0.56 Mt, 0.5%), Hungary (2.7 Mt, 2%)
on the benchmarks that will reflect the 10% of the best performing installations in the EU that are producing the same product. This should guarantee an efficient allocation, which should trigger the significant abatement actions from industrials.

The start of the Phase III will also introduce increased auctioning. The new EU ETS aims to “progressively replace all free allocations as the main method for allocating allowances to all EU ETS sectors except aviation.” This will help to ensure efficiency of allocation and elimination of wind free profits, arising from operators charging their customers the costs of allowances, despite the fact that those allowances were distributed free of charge. In Phase III at least 50% (i.e. ~1Gt) of the allowances will be auctioned, as power producers will no longer receive free allocations. Contrary to Phase I and II, where countries carried country based auctions, auctioning in Phase III will be centralised and carried in a single EU-wide auction platform.

The EU ETS follows a precise time table, where each installation falling under the EU ETS regulation is supposed to receive allocations for the current year on February 28th. By 31st of March regulated industrials are required to submit the report of the verified emissions realised in the previous year. Then, by 30th of April operators are required to surrender the allowances for the previous year emissions. While by the 15th of May, EC is obliged to publish an official report of verified emissions during the previous year. This time table is not only very important for industrial market players if not for carbon traders, as it can be demonstrated by compliance event in April’06, when after the verified emissions CO2 prices crashed almost to zero.

Under the event, that operator fails to surrender an adequate number of allowances a fine has to be paid. During Phase I, penalty was set at 40 €/ tonne and was raised to 100 €/ tonne in Phase II and Phase III. Additional sanctions on the national level may be also applied in the event of failure to comply with the regulation. What is more, the payment of the penalty does not relieve from the obligation to surrender the missing allowances in the following year.

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10 This is typically true for the power production sector, where power prices are driven by flexible marginal power plants. Thus, CO2 costs are added to the costs of marginal production unit. As an evidence of this, Reinaud (2007) research should be highlighted, were author concludes that abrupt fall in May 2006 of EUAs prices of 10€/tonne was immediately followed by a 5-10€/MWh fall of wholesale electricity prices in main EU power markets.
11 Germany, Poland and the UK opted out from this unified auctioning platform.
12 Surrendered allowances are cancelled on the 30th of June.
3.3.3. Temporal Limits

Temporal limits is a very important allowance market setting and as argued by Haites (2006, p.7) “affect basically environmental performance, economic efficiency and market participants behaviour.” Thus, market setting that is flexible in time may help to cost-effectively achieve desired reductions of emissions. What is more as argued by Ellerman and Montero (2007), banking and borrowing may improve price stability and lead to less volatility at the end of each compliance period.

Under the EU ETS there are no restrictions on permits banking or borrowing within the same phase and continuously traded spot price within the period can be observed. What is more, as can be noticed by analysing timing schedule of the system, detailed in the previous section, there is a time period which effectively leads to year-ahead borrowing, as installations have at their disposition allowances from the two periods. This happens because current year allowances are distributed at the end of February, while last year’s emissions have to be covered just by the end of April. Indeed, to support Haites (2006) argument quoted above – in the light of the global economic crisis, some industrials chose to sell their current years allowances and in such a way improve the liquidity situation on their balance sheets.

On the other hand, borrowing between phases 13 is not permitted. This stems from the EC unwillingness to deal with compliance failures spilling from one phase into the other. This setting was especially important before the trial Phase I took off, as there were fears of under-allocation. However, Phase-I turned out to be over-allocated and initially allowed banking 14 from Phase I to Phase II was prohibited. Decision to ban banking send a strong price signal to the market, and EUAs prices crashed to virtually 0 €/ tonne at the end of Phase I. Under the new market setting of Phase II and Phase III, inter-phase banking is allowed. And although Phase II, due to current financial crisis, is also expected to be over-allocated, at the end of Phase II EUAs from the old phase will be cancelled out and replaced with a corresponding number of EUAs

13 This is holds for all Phases – Phase I, Phase II and Phase III.
14 Note, that each country was allowed to freely choose to permit or ban banking. Initially after over-allocation became clear, two countries – France and Poland planned to allow banking from Phase I into Phase II. However, after strong political pressure, they finally banned allowances banking into Phase II.
from the new phase. Recent uncertainty of the Phase III allocation methodology caused some manufacturers\textsuperscript{15} to hold to their Phase II surplus allowances, as a way to fight regulatory changes. In such way a long-feared price collapse, due to over-allocation in Phase II, which became clear during the announcement of 2009 emissions, was postponed until recently.

3.3.4. Trading Carbon Permissions

Although EUAs are allocated to installations covered in the Directive 2003/87/EC, trading of the allowances is not limited to those industrials – every legal and natural body who registers a transaction account is authorised to participate in EU GHG reduction effort by trading in the EU ETS market\textsuperscript{16}. However, the legal framework of the EU carbon trading system does not regulate how and where the trading of permits takes place. And as discussed by Convery and Redmond (2007) market players may trade directly with each other, buy or sell via intermediary or use established exchanges to trade assets.

Each member state is obliged to record electronic transactions of EUAs in the national registry. Union wide accounts are centralised by the EC in the registry, called the Community Independent Transaction Log (CITL), and its role is to monitor and verify every transaction. Under the new Directive 2009/29/EC, a centralised registry of EU ETS transactions will be established in 2012. European Union Transaction Log (EUTL) will be operated directly by the EC and will replace all individual member states registries.

As the end of Phase II is approaching, market participants can choose to trade through a numerous amount of carbon exchanges. According to Point Carbon (2011), the leading platforms in terms of volumes are the Bluenext (BNXT, Paris), Noordpool (NP, Oslo) and European Climate Exchange (ECX, London), with the latter accounting for almost 98% of trades.

\textsuperscript{15} Mainly inefficient steel and iron producers, as it was feared that they will be the ones mostly punished due to the new allocation methodology based on benchmarking to the best state-of-the-art technology available in EU.

\textsuperscript{16} Installations covered by the EU ETS system are called trading system in the Directive, while other participates of the market are referred to as non-trading sector.
3.3.5. Fundamental Price Drivers of EUAs

In order to be able to effectively manage carbon risks it is important to know the factors, driving the EU ETS market. Burtraw (1996) and Gronwald et al. (2011) categorised the factors, that drive carbon market into two main groups:

- policy and regulatory issues;
- market fundamentals that are directly related to the production of emissions.

We have extensively covered the first factor, while discussing the EU ETS market setting, its regulatory issues and policy implications in previous sections. The second factor – market fundamentals – directly depends on carbon supply and demand balance, which in turn is influenced by economic growth, technological developments, linking to other carbon schemes and UN mechanisms, as well as weather and energy prices.

Given, that thermal power sector accounts for around 60% of emissions covered by EU ETS system, all the factors that might cause changes in production are of an immense importance. Below a short summary by factor is given:

- Renewable Energy - Hintermann (2010) has found that Phase I carbon prices were negatively affected by hydropower availability in Nordic markets, those results were also found to hold in Phase II by Rickels et al. (2010). Furthermore, authors discovered the negative effect of wind power availability in German markets.
- Temperature – Alberola et al. (2007), Chevallier (2008), Mansanet-Bataller et al. (2007) and Rickels et al. (2007) all confirm a positive effect of unanticipated extremely cold or hot temperatures on EUA prices in Phase I.
- Energy prices – in independent Phase I studies, Mansanet-Bataller et al. (2007) and Hintermann (2010) find that forward Brent and natural gas prices positively influence forward EUA prices, however no significant influence of coal prices was found by the two studies. However, Rickels et al. (2007) and Alberola et al. (2008) claim that a negative effect of coal prices on EUA prices exist. Mansanet-Bataller and Pardo (2008) discover that while Phase I prices are influenced by all energy commodities in Phase I, future Phase II EUA prices depend only on Brent prices.
• Switching prices – is a theoretical value of carbon price, required to trigger switching of electricity production from coal power plants to more environmentally friendly gas power plants.

This chapter introduced the reader to the basic information on EU ETS market, necessary for European carbon price development analysis. Next chapter will discuss theoretical background of empirical investigation implemented in this paper.
4. Theoretical Framework

In this chapter, we will establish theoretical context that will serve as a base for our research. First, we provide a debate and motivation for carbon classification as a commodity. This is relevant in order to form prior expectations about its price path. Then, we introduce conceptual frameworks – unit root, Hotelling rule and cost-of-carry model – that are used to study market efficiency and presence of risk-free arbitrage opportunities.

4.1. Classification of Emission Allowances

In order to form prior expectations about carbon price movements, we should know what type of asset we are dealing with - whether it is a financial asset, a new type of commodity or a completely new asset class. For example, stock prices tend to demonstrate an upward trend over the long time periods, since investors have to be remunerated for both: time value of money, as well as for the risk they take. On the contrast, commodities, which are driven by supply and demand balance, tend to exhibit mean reversion in the long run. Any temporary price spikes are caused by market equilibrium disturbances.

Although from a first glance, one may think of EUAs as a traditional stock or commodity, a closer look at its properties yields some important differences (Benz and Trück, 2006). Authors argue that while the value of an equity price is based on profit expectations of the underlying company, the price of carbon permits is determined directly by the expected market scarcity induced by the current demand and supply, and emphasize, that the fact that companies by themselves are able to control market scarcity by their abatement decisions has a significant impact on market liquidity and price dynamics.

Borak et al. (2006) and Benz and Trück (2006) point out that utilization of industrial plants is ultimately determined by a company’s stock of emission allowances. A shortage of permits requires company to either upgrade its operations, cut down production or purchase allowances from the market. This classifies EUAs as commodities. However, differently than trading traditional commodities, traders of emission allowances are effectively selling an absence of emission. Hence, industrials, that sell carbon, expect to produce less than they are allowed, and
may sell their unused rights to emit to other market players who expect to emit more than allowed (Point Carbon, 2004).

Mansanet-Bataller and Pardo (2008a) discuss the presence of another difference - from the storability point of view EUAs are more similar to the traditional financial assets, as all transactions are conducted electronically and there are no storage costs associated. Supply and Demand problems of commodities are caused by storability issue that often leads to backwardation\textsuperscript{17}. Contrary to the traditional commodities, carbon equilibrium prices are determined by the level of real emissions, as supply of each period is fixed in advance by the EC.

4.2. Stochastic Trend

Under the design of the EU ETS, the GHG emissions reduction is set to be achieved by creating a scarcity of allowances available to the market participants. This should send a strong price signal to the market and combined with supply/ demand driven commodity pattern should lead, as highlighted by Paolella and Taschini (2008), to mean reversion around an upward trend. Hence, as discussed by Bredin and Muckley (2009b), the discovery of a stochastic trend (a unit root) in EUA prices would be a sign of EU ETS system failure to send a sufficiently strong price signal and create a sustainable cap-and-trade market that encourage a transition to a low carbon economy.

Early studies that are available have shown contradictory results. Daskalakis \textit{et al}. (2009) in Phase I analysis that adopted Augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and Kwiatkowski, Phillips, Schmidt and Shin (KPSS) tests find a stochastic trend in EUA spot prices. Consistent results are presented in Alberola \textit{et al}. (2007, 2008a, 2008b, 2009) studies, where Phase I EUA spot price series are analysed employing ADF, PP, KPSS and Lee-Strazicich tests. On the opposite, Bredin and Muckley (2009a) in their Phase I investigation reject unit root hypothesis using PP test. However, their later study (2009b) that extends analysed period to

\textsuperscript{17} Normal backwardation (also backwardation) – refers to the market condition, where the price of futures contract is trading below the expected spot price at futures contract maturity. This is said to occur due to the \textit{convenience yield} being higher than the prevailing risk free rate. The term was introduced by John Maynard Keynes. Convenience yield are benefits related to holding a stock of storable commodity in inventory. These benefits arise from the use of inventories to reduce production and marketing costs, and to avoid stock-outs.
Phase II (July 1, 2005 to July 29, 2009) and adopts a more extensive set of stochastic trend testing methodologies (ADF, PP, KPSS, LS as well as Elliot et al. and Zivot and Andrews) finds a unit root in both spot and futures prices.

Presence of unit root in carbon price series has important implications for both: further market regulator decisions as well as EUA price modelling. Since EU ETS market is still in its early stages and Phase II market setting is different from Phase I\(^{18}\), there is a pending need to investigate further if carbon prices exhibit a stochastic trend. The corresponding analysis will be carried in section 6.3

### 4.3. Spot Prices – Hotelling-CAPM Model

Further building on carbon attribution to commodities class, current EU ETS cap-and-trade setting efficiency can be analysed using Hotelling (1931) non-renewable resources rule. Although there are couple of remarkable differences between exhaustible resources and carbon credits\(^{19}\), different authors have studied the applicability of the rule to the GHGs permits market. Kling and Rubin (1997) have applied Hotelling rule to the intertemporal market and have demonstrated that under certainty and given that borrowing is allowed the permits real price should rise smoothly over the time at a percentage rate that is equal to the discount rate, however in the case that borrowing is forbidden, the price should rise at a percentage rate inferior to the interest rate. Following this approach, Schennach (2000) has extended a theoretical model of the intertemporal SO2 allowance market to uncertainty setting. With uncertainty prevailing, permits can generate two additional returns. One is a risk premium in the spirit of the capital asset pricing model (CAPM)\(^{20}\) and the other is convenience yield.

Schennach’s model was first applied in practice by Helfand et al. (2006) in his US SO2 market study. While Alberola and Chevallier (2009) use it to analyse the EU ETS market in Phase I.

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\(^{18}\) In terms that banking from Phase II to Phase III is allowed as well as inclusion of new sectors and gasses discussed in section 3.3.1

\(^{19}\) Liski and Montero (2005) mention the following differences: 1) depletable resource market will cease to exist, after its exhaustion, while permits market would still continue to exist; 2) contrary to exhaustible resources, permits extraction and storage costs are equal to zero.

\(^{20}\) Gaudet and Khadr (1991) and Slade and Thille (1997) integrated CAPM approach to risk premium with Hotelling rule, while Slade and Thille (1997) have also applied it to empirical investigation.
With storability of permits as well as banking allowed Helfand et al. (2006) adapt Schennach’s (2000) continuous time model to discrete time:

$$E_t p_{t+1} = (1 + r_t^f + \rho_t)p_t - \psi_t$$

(4.1)

where $E$ is the expectations operator, $t$ and $t + 1$ denote time, $p_t$ is a natural logarithm of EUA allowance price, $r_t^f$ is the risk-free interest rate, $\rho_t$ – CO2 asset risk premium in the spirit of CAPM and $\psi_t$ is convenience yield for holding permits in euros per ton.

We then rearrange equation (4.2) to isolate first-difference prices on the left hand side:

$$E_t p_{t+1} - p_t = r_t^f p_t + \rho_t p_t - \psi_t$$

(4.2)

Following standard practice of CAPM we rewrite $\rho_t$, as $\frac{\sigma_{am}}{\sigma_{mm}} (r_t^m - r_t^f)$:

$$E_t p_{t+1} - p_t = r_t^f p_t + \frac{\sigma_{am}}{\sigma_{mm}} (r_t^m - r_t^f)p_t - \psi_t$$

(4.3)

where $r_t^m$ is the rate of return on the market portfolio, $\sigma_{am}$ is the covariance between the rate of return of EUA prices and $r_t^m$, and $\sigma_{mm}$ is the variance of $r_t^m$.

The variable $E_t$ represents rational expectations conditional on information available at time $t$. The first term on the right hand side $r_t^f p_t$ represents the Hotelling rule for cost-minimising intertemporal arbitrage in the EU ETS market, second term $\frac{\sigma_{am}}{\sigma_{mm}} (r_t^m - r_t^f)$ is the risk premium of holding EUAs as part of a diversified portfolio, while $(r_t^m - r_t^f)$ stands for the excess return on the market portfolio at time $t$. The risk premium for holding allowances is positive when $\sigma_{am}$ is greater than zero, i.e., allowances need to earn a positive premium when the covariance is positive – with risk averse investors, an asset return that varies positively with the market portfolio is a liability.

Because of uncertainty that may arise due to unexpected market movements, unanticipated regulatory actions or unforeseen technological developments, the expected value of $p_{t+1}$ is
known only with errors at time $t$, $p_{t+1}$ can be expressed as $E_{t+1} + e_{t+1}$. Substituting $E_{t+1}$ yields in equation (4.3):

$$p_{t+1} - p_t = r^f_t p_t + \frac{\sigma_{am}}{\sigma_{mm}} (r^m_t - r^f_t) p_t - \psi_t + e_{t+1}$$

(4.4)

with the dependent variable being the first log-differenced EUA price series and $e_t$ is error term.

Next, assuming that convenience yield is constant ($\psi_t = \psi$), and converting an equation (4.4) to an econometric model, we rewrite:

$$p_{t+1} - p_t = \alpha + \beta_1 r^f_t p_t + \beta_2 (r^m_t - r^f_t) p_t + \epsilon_{t+1}$$

(4.5)

where $\alpha = -\psi$, $\beta_2 = \frac{\sigma_{am}}{\sigma_{mm}}$, which is a standard practice for CAPM. This equation allows us to test the following hypothesis:

H1. $H_0: \beta_1 = 1$ – tests Hotelling rule.

The sign and significance of parameter $\beta_2$ provides information on the CAPM risk premium for EUAs. Empirically, the estimate of parameter $\alpha$ represents an average convenience yield over time.

In the case that Hotelling rule does not hold, prices do not meet equilibrium conditions and intertemporal arbitrage is possible. Alberola and Chevallier (2007) in their study conclude that Hotelling rule does not hold in EU ETS Phase I, and EUA spot prices do not adequately reflect abatement costs at the installation level. Hence, institutional setting during Phase I did not meet the necessary conditions for an efficient intertemporal price signal to emerge. Following the permit over-allocation in Phase I, Phase II allocations were tightened, and new investigation over efficiency of the new market setting is required. Section 6.5 of this paper will try to fill this gap.
4.4. Relating Spot Prices to Futures Prices

Futures are one of the most liquid and accessible instruments that might offer effective hedging of carbon risk. For example, power companies tend to sign long-term agreements with their industrial clients. In order to match their revenues with costs, they tend to start hedging production input factors two years in advance. Since they are heavily exposed to carbon price movements, they would be interested to hedge them along the basket of other input commodities. However, in order to employ effective risk management techniques, a long-run relationship must exist between spot and futures prices. In the absence of this relationship spot and futures prices would follow independent stochastic paths and long futures position would lead to taking undesirable additional risk. Therefore it is important to investigate if underlying asset prices follow the same trend or process as futures. Having specified the EUA spot price path under no-arbitrage condition, we will further study the efficiency of EU emissions cap-and-trade system and specify the equilibrium spot and futures prices relationship.

4.4.1. Cost-of-Carry and Convenience Yield Approach

As we have specified earlier, CO2 is an input, utilised in production process, and like any other consumption commodities, it is subject to supply/demand shocks. This makes industrial players interested in holding inventories of permits in order to avoid having to adjust their production levels due to the shortage of emission allowances. Holding EUA inventories grants industrials with additional flexibility, as production in any period no longer needs to be equal to consumption. As a result, the market-clearing price is driven not only by the current production and consumption, but also by the changes in inventory holdings (Pindyck, 2001).

Due to above described carbon parallels with a factor of production, cost-of-carry\(^\text{21}\) model is applied under no-arbitrage condition to analyse spot and futures price interdependence in the permits market: Borak et al. (2006), Milunovich and Joyeux (2007), Uhrig-Homburg and Wagner (2008), Chevallier (2008), Daskalakis et al. (2009) and Tiits (2009). In cost-of-carry model differences between spot and futures prices are explained by foregone interest due to

\(^{21}\) Cost-of-carry – a portion of the total cost of storing a commodity, namely the physical storage, insurance and transport costs, plus the forgone interest.
storing commodity and convenience yield (net of physical storage costs). Under this approach, the futures price is specified by

\[ F_t(T) = e^{(r_t^f - \psi_t)(T-t)}S_t \]

where \( S_t \) stands for the spot price at time \( t \), while \( F_t(T) \) is the futures price at time \( t \) of a contract with delivery in time \( T \), \( r_t^f \) – risk free interest rate, \( \psi_t \) – convenience yield net of storage costs and \( (T - t) \) – years to maturity. Uhrig-Homburg and Wagner (2008) point out, that by some researchers convenience yield is interpreted as correlation to some exogenously given variables. As an example for such a correlated variable they suggest the stock of inventory of EUA permits and note, that in this case the convenience yield itself may be stochastic and in this manner weaken the link between spot and futures prices.

Since carbon trading transactions are conducted electronically and there are no costs associated to its storage, discounted future price of allowance should differ from respective spot price only by gross convenience yield. However, as discussed previously, industrials have to surrender allowances, matching their actual emissions level just by April 30 of the next year. Then, as argued by Uhrig-Homburg and Wagner (2008), if futures mature before the end of the next compliance date, there is no benefit of holding spot EUAs compared to holding the corresponding long futures position and convenience yield one year before the expiration of the future should be equal to zero. In this case industrials could sell some of their carbon allocation and use money market to replicate a short forward position. In this case, the relationship between spot and forward prices should be described entirely by the cost-of-carry approach and riskless profit may be achieved if EUA spot prices fall below the discounted futures price. Investors could exploit arbitrage opportunity by buying the underlying asset and simultaneously entering into a short futures contract.

Mixed conclusions were drawn in the previous research carried on Phase I data by various authors. Milunovich and Joyeux (2007) in their study conclude that 2006 and 2007 futures

\[ (T - t) \] is calculated as the number of days to expiry date divided by 360.

\[ \psi_t = r_t^f - \frac{\ln(S_t)}{(T-t)} \]

Note, that the implied convenience yield with zero storage costs can be derived from (4.6) as:
contracts do not demonstrate cost-of-carry relationship with Phase I spot prices. In the contrast, Daskalakis et al. (2009) by simply calculating mean squared error deviation between theoretical and actual futures expiring in Phase I price, find the parity between spot and discounted future prices. Research carried by Uhrig-Homburg and Wagner (2008) confirm those findings, but add that this dependence no longer holds in Phase II. Chevallier (2008) reaches the same conclusions and specifies, that Phase I spot price fails to reflect Phase II future prices due to the ban of banking of Phase I allowances to Phase II. They explain this, as regulators intent to correct inefficiencies causes by the over-allocation of EUAs in Phase I and achieve an efficient price pattern, leading to effective abatement efforts in Phase II. Tiits (2009) is the first researcher investigating how phase II spot prices explain futures maturing in Phase II. However, author fails to confirm existence of the cost-of-carry relationship in Phase II due to the lack of evidence that a necessary cointegration condition between spot and futures prices exist. To our knowledge, this was the only intent to investigate if cost-of-carry relationship holds under Phase II market setting. Moreover, it should be noted that this analysis was carried in very early stage of Phase II market hence, updated investigation is needed and this paper will fill-in this gap by providing an updated empirical results in section 6.6.

4.4.2. Cointegration analysis
Our analysis of the futures and spot prices relationship will rely heavily on the cointegration methodology, which is widely used in research, investigating cost-of-carry model (Brenner and Kronner (1995), Milunovich and Joyeux (2007), Uhrig-Homburg and Wagner (2008), Tiits (2009). Using the simplified definition, variables are considered to be cointegrated, if they, despite being individually non-stationary, form a linear combination that is stationary\textsuperscript{24}. This means that there is a long-term equilibrium between them.

It is natural for the spot and future prices to demonstrate cointegration relationship, since futures prices converge to the spot prices at the date of maturity. Indeed, financial time series are found to be strongly cointegrated, however fewer evidences demonstrate cointegration of commodity spot and future prices. This is due stochastic and idiosyncratic costs-of-carry, related to physical storage, insurance and transportation of commodities. Since carbon permits are traded

\textsuperscript{24} The concept of stationarity will be explained in detail in section 5.1.
electronically and not in physical markets, taking natural logarithms of cost-of-carry model (derived under no arbitrage conditions) expressed in equation (4.6), it is easy to see the grounds of popularity of cointegration approach:

\[ f_t = s_t + r_t^f (T - t) - \psi_t (T - t) + \epsilon_t \]  

(4.7)

where \( f_t \) stands for \( \ln F_t \), \( s_t \) stands for \( \ln S_t \) and \( \epsilon_t \) is a stationary zero-mean error term whose variance is determined by the extent of various market imperfections. The above equation demonstrates that there must be a linear relationship between the natural logarithmic values of spot and futures prices.

Assuming that convenience yield is constant (\( \psi_t = \psi_t \)), we can test cost-of-carry relationship, by converting (3.4.2) equation into and econometrical model:

\[ f_t = \beta_0 + \beta_1 s_t + \beta_2 r_t^f (T - t) + \epsilon_t \]  

(4.8)

where \( \beta_0 = -\psi \). With the help of the above equation, following Milunovich and Joyeux (2007) approach, we test the following hypotheses:

H2. \( H_0: \epsilon_t \) is stationary and equation (4.8) forms a cointegrating relationship.

H3. \( H_0: \beta_1 = \beta_2 = 1 \) is a non-arbitrage restriction implied by the cost-of-carry model.

In the case of joint acceptance of H2 and H3 we are able to conclude, that Phase II EU ETS market setting is efficient (i.e. no risk free arbitrage opportunities are available) and that there exists a long-term cointegrating relationship between EUAs spot and futures prices as well as interest rate. While the acceptance of only H2 would imply that although EU ETS market is inefficient, and risk free arbitrage opportunities are present, a long-term cointegration relationship exists and futures are an adequate measure to hedge carbon risk. However, failure to

\[ \text{When prices are cointegrated, natural logarithms will also be cointegrated. Thus, following the common practice here and throughout our cointegration analysis we will use natural logarithms of prices.} \]

\[ \text{By removing an assumption, that convenience yield is constant, we are able to estimate the convenience yield parameter } \psi \text{ and test zero-convenience yield, using the following econometric specification: } f_t = -\psi T + \beta_1 s_t + \beta_2 r_t^f (T - t) + \psi_t + \epsilon_t \text{ (Milunovich and Joyeux, 2007, p.7)} \]
accept both: H2 and H3 would suggest that neither futures of the permits are suitable to hedge away the CO2 risk, nor that EU ETS market is efficient, hence arbitrage opportunities are present.

It is important to emphasise, that finding that variables under investigation are cointegrated, does not mean that they always move together in unison – short term distortions of an equilibrium relationship may exist. To model short-term dynamics of cointegrated variables we will utilise Vector Error Correction Model (VECM) that is based on linear regression analysis of returns. However, the presence of equilibrium relationship between variables also implies that statistical causality between the returns exists. Both, VECM and causality test will be discussed in more detail in the next chapter.
5. Econometric Methodology

In this chapter relevant econometrical and statistical theory will be presented, that will later be used in empirical analysis. We will start with stationarity concept, and then move to correlation discussion. We then explain a cointegration approach, where short term deviations are captured by Vector Error Correction Model (VECM). The chapter is concluded with an introduction to copula methodology.

5.1. The Concept of Stationarity

In order to know, whether price series are predictable, or wander in an unpredictable manner, we first need to know if our time series are stationary or follow a random walk. Another reason is that non-stationary variables may lead to unreliable results and cause a so called spurious regression.

To start from the very beginning, stochastic process is said to be stationary if its mean, variance and autocovariance (at various lags) remain constant no matter at what point of time we measure them, that is they are invariant (Gujarati, 2002, p.798). If the above conditions are not fulfilled – time series is said to follow a random walk or have a unit root.

There are three main ways to test if analysed time series is stationary: a) a graphical method, b) a correlogram test and c) a unit root test. Since the first two methods are rather straightforward to implement we will only discuss unit root test in a more detailed manner. Three most popular unit root methodologies that will be later adopted in our analysis are: augmented Dickey-Fuller (1979, 1981), Phillips-Perron (1988) and Kwiatkowski, Phillips, Schmidt and Shin (1992)

The **Augmented Dickey-Fuller (ADF)** test is an extension of the Dickey-Fuller (DF) test, where a unit root stochastic process is analysed. Consider the general equation:

\[
Y_t = \alpha + \beta t + \rho Y_{t-1} + \epsilon_t, \quad -1 \leq \rho \leq 1
\]

(5.1)

27 Very high \(R^2\) (always close to 1) and severely biased \(t\)-ratios.
28 Stochastic process – a collection of random variables ordered in time. (Gujarati, 2002, p.797)
where $\alpha$ is a drift term, $\beta$ is a deterministic $t$ trend parameter and $\epsilon_t$ is a white noise term$^{29}$. In case that $\rho = 1$, then above equation (5.1) reflects a non-stationary stochastic process$^{30}$. The DF test is carried on by fitting slightly transformed (5.1) equation by ordinary least squares (OLS):

$$\Delta Y_t = \alpha + \beta t + \delta Y_{t-1} + \epsilon_t$$

(5.2)

where $\delta = \rho - 1$ and $\Delta$ is the first-difference operator ($\Delta Y_t = Y_t - Y_{t-1}$). The null hypothesis $H_0$: $\delta = 0$ tests if there is a unit root (non-stationarity), rejection of $H_0$ leads to alternative hypothesis $H_1$: $\delta < 0$ – time series is stationary.

If under DF test it was assumed that error terms $\epsilon_t$ are i.i.d., the ADF test “augments” the standard test equation by adding the lagged values of the dependent variable and thus, accounting for autocorrelation of residuals:

$$\Delta Y_t = \alpha + \beta t + \delta Y_{t-1} + \sum_{i=1}^{k} \alpha_i \Delta Y_{t-i} + \epsilon_t$$

(5.3)

where $k$ is a number of lags which are added to the model to remove autocorrelations from residuals $\epsilon_t$ and $(\Delta Y_{t-1} = Y_{t-1} - Y_{t-2})$, $(\Delta Y_{t-2} = Y_{t-2} - Y_{t-3})$, and so on. The number of lags is estimated via a minimisation of the Bayesian Information Criterion (BIC). Like in DF test a null hypothesis $H_0$: $\delta = 0$ tests if there is a unit root.

**The Phillips-Perron (PP)** test can be viewed as DF statistic that has been adjusted by using Newey-West heteroskedasticity and autocorrelation consistent covariance matrix estimator, and thus is robust to serial correlation of error terms.

The above-mentioned popular ADF and PP tests specify the null hypothesis as data being non-stationary, but it has been noticed that due to this reason they frequently do not reject $H_0$ of a unit-root in the context of economic and financial time series. An alternative would be to use the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test, which unlike ADF and PP specifies its

$^{29}$ Strong white noise is independently and identically distributed (i.i.d.).

$^{30}$ In case $\alpha = 0$ and $\beta = 0$ we have a pure random walk process, if $\alpha = 1$ and $\beta = 0$ we have a random walk with drift, while $\alpha = 1$ and $\beta = 1$ defines a random walk with drift around stochastic trend.
null hypothesis $H_0$ that the data is trend stationary and an alternative hypothesis $H_1$ that it contains a unit root. To test stationarity hypothesis KPSS test adopts a Lagrange Multiplier methodology.

Although, it is common, that economical and financial data is not stationary, however differencing technique usually can be used to transform data into a stationary time series. If original data is stationary, it is said to be integrated at level 0 and denoted as I(0), if stationarity is reached after taking first-differences it is said that data is integrated of order 1 – I(1). In case analysed time series has a deterministic trend (i.e. I(0)+trend process), stationary transformation is reached by taking deviations from a fitted trend line (we need to fit a regression, where we take time trend as an independent variable and then use the residuals).

It should be added, that testing stationarity of commodity prices time series is rather complicated, as they are likely to contain jumps, which may lead to a rejection of unit root more difficult.

5.2. Correlation

Before proceeding with any other type of analysis it is important to understand a statistic, which is widely used in risk and portfolio management – correlation. In this section we will present both the advantages and the pitfalls of correlation analysis. It is important to be aware that correlation is just one of many measures and use it with care.

Correlation is a measure of the strength or degree of linear association between two variables. A population version of standard correlation metric, called Pearson’s product-moment correlation coefficient is defined by:

$$
\rho(X,Y) = \frac{\text{Cov}[X,Y]}{\sqrt{\sigma^2[X]\sigma^2[Y]}}
$$

(5.4)

where $\sigma^2[X]$, $\sigma^2[Y]$ denote the variances of X and Y and $\text{Cov}[X,Y]$ is the covariance between X and Y and is defined by:

$$
\text{Cov}[X,Y] := \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])] = \mathbb{E}[XY] - \mathbb{E}[X] \mathbb{E}[Y].
$$

Linear correlation coefficient may be interpreted as follows: if variable $X$ tends to increase, when variable $Y$ increases and tends to decrease, when variable $Y$ decreases, we can say that there is a
positive correlation between those variables. In case variable $X$ tends to decrease, when variable $Y$ increases and tends to increase, when variable $Y$ decreases, we can say that there is a negative correlation between those variables. In the case of perfect linear dependence $\rho(X,Y) = \pm 1$. If there is no association between movements of variables $X$ and $Y$ they have zero correlation. The generalisation of correlation to multivariate variables is straightforward.

It is easy to see, why correlation is so popular – it is easy to calculate and interpret as well as manipulate using linear operation. Nonetheless, there are important drawbacks of its application:

- Classical correlation metric can be applied only on variables that follow multivariate normal or elliptical distributions otherwise it gives misleading information of the real dependency between variables and tells us nothing about the degree of dependence in the tail of underlying distribution. Only in the case of multivariate normal distribution we can conclude that zero correlation (i.e. $\rho(X,Y) = 0$ implies independence of variables.
- Pearson’s correlation is only appropriate to use, if time series is stationary. In case correlation coefficient is calculated for non-stationary data, estimated correlation metric will be dominated by a trend and can lead to the appearance of high correlation where in truth there is none. Thus correlation of financial data is usually calculated on returns.
- Another important pitfall of linear correlation is that it is not invariant under nonlinear transformations (e.g. $\rho(X,Y) \neq \rho(lnX,lnY)$
- Linear correlation metric is sensitive to outliers.
- Correlation coefficient measures co-movements of returns that are subject to different shocks over time. And since returns are de-trended data – correlation is a short term measure, which requires frequent rebalancing.
- A standard formula of Pearson’s correlation coefficient gives equal weight to all data points. In dynamic world, it would make sense to give higher weights for recent observations – one of the ways to correct for that would be to use exponentially weighted moving average correlation (EMWA correlation)

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31 It is important to note, that existence of the linear relationship between variables does not imply an existence of any cause-and- effect relationship.
32 The so called spurious correlation.
Although standard linear correlation measure is very popular and widely applied in risk management, the above listed fallacies makes it appropriate metric only in one case – when returns follow a multivariate normal or Student-t distribution. When used with other distributions – it can lead to misleading results. It was demonstrated by various researchers, such as Luciano and Marena (2003), McNeil et al. (2005) and Junker et al. (2006) that since financial market returns do not follow normal distribution, Pearson’s correlation coefficient may be inadequate to measure the dependence structure between asset returns, and may provide with poor risk indication of extreme joint movements of the returns.

To correct for some of the above mentioned fallacies one can employ rank correlation techniques - non-parametric measures of dependence based on ranked data. Under this approach data is converted to ranked form by ranking the smallest value with the rank 1, the second smallest with the rank 2 and so on. Then, only the ranks of the observations are retained. Therefore rank correlations can be considered as measures of the degree of monotonic dependence between random variables \( X \) and \( Y \), while the linear correlation measures the degree of linear dependence. The transformation of \( X_1, \ldots, X_n \) to \( 1, \ldots, n \) has the consequence that the marginal distributions of \( X \) and \( Y \) will be ignored. This is why rank correlation is a distribution independent measure. The two most popular techniques are Spearman’s and Kendall’s rank correlations.

**Spearman’s rank correlation** – is an important substitute to Pearson’s linear correlation measure and is calculated by applying the Pearson correlation formula to the ranks of the data, rather than to the actual time series values. This method reduces many of the distortions, which we discussed above. Let \( X \) and \( Y \) be random variables with distribution functions \( F_1 \) and \( F_2 \) and joint distribution function \( F \). Then the population version of Spearman’s rank correlation \( \rho_S \) is:

\[
\rho_S(X,Y) = \rho(F_1(X),F_2(Y))
\]

where \( \rho \) is Pearson’s linear correlation. If \( \rho_S = +1 \) – we have a perfect agreement between the two sets of ranks, while \( \rho_S = -1 \) means, that there is a complete disagreement between the two data sets.
Kendall’s rank correlation – like Spearman’s \( \rho_S \) – Kendall’s \( \tau \) is a measure of correlation between two ordinal level variables. Consider a vector of continuous random variables \((X, Y)\), where we observe a random sample with \( n \) observations \( \{(X_i, Y_i), \ldots, (X_n, Y_n)\} \). Kendall’s \( \tau \) is calculated by comparing all possible pairs of observations \( \{(X_i, Y_i), (X_j, Y_j)\} \) for \( i \neq j \). There are \( \frac{n(n-1)}{2} \) possible pairs of observations in the sample and each pair is either concordant\(^{33}\) or discordant\(^{34}\).

\[
\tau = \frac{N_C - N_D}{n(n-1)}/2
\]

where \( N_C \) is a number of concordant pairs, while \( N_D \) is a number of discordant pairs. Like Spearman’s \( \rho_S \), Kendall’s \( \tau \) is in the range of -1 to +1. Although assumptions under Spearman’s \( \rho_S \), Kendall’s \( \tau \) are similar, however one-to-one comparison of those metrics is not possible. The relationship of the two metrics can be expressed in by: \(-1 \leq \{3\tau - 2\rho_S\} \leq +1\). Both Spearman’s \( \rho_S \) and Kendall’s \( \tau \) can be considered to be measures of the degree of monotonic dependence between \( X \) and \( Y \), while Pearson’s correlation coefficient is only a linear dependence measure. However, there are some differences in their interpretation. Spearman’s \( \rho_S \) is very similar to the standard correlation measure, in the way it accounts for proportion of variability, while Kendall’s \( \tau \) represents the difference between the probability that the observations are in the same order versus the probability that they are not in the same order (hence concordance and discordance measures).

The main advantage of the rank correlations is that, unlike Pearson’s correlation measure they are invariant to monotonic transformations. Another advantage is that rank correlations are not as sensitive to outliers like linear correlation. While the main disadvantage is, that they are more difficult to manipulate than Pearson’s correlation metric.

\(^{33}\) Concordance metric \( N_C(X, Y) \) is a numerical measure of association between two continuous random variables \( X \) and \( Y \). Observations \( (X_i, Y_i) \) and \( (X_j, Y_j) \) are concordant, if \( X_i < X_j \) and \( Y_i < Y_j \) or equally if \( X_i > X_j \) and \( Y_i > Y_j \).

\(^{34}\) Observations \( (X_i, Y_i) \) and \( (X_j, Y_j) \) are discordant, if \( X_i < X_j \), but \( Y_i > Y_j \) or equally if \( X_i > X_j \), but \( Y_i < Y_j \).
5.3. Cointegration
As we have already discussed in sections 4.5 and 5.1, most portfolio and risk management techniques are based on correlation metric, which can be subject to high volatility and thus fail to reflect a long-term relationship between the analysed data. What is more, financial asset correlation matrix is based on the asset returns, which means that any long-term trend is removed. Cointegration on the other hand, through exploring common driving factors, seeks to identify common long-term stochastic trends between variables and thus, analyse market structure.

Alexander (2008b, p.228) simplifies Engle and Granger (1987) definition and explains that “a set of integrated series are cointegrated if there is a linear combination of these series that is stationary”. For example, when two variables $X$ and $Y$ individually are integrated of at least order one (I(1)), but their linear combination:

$$Z = X - \delta Y$$

(5.7)

is stationary (i.e. integrated of order zero – I(0)), then they are cointegrated. Stationarity in this case implies, that the combination of the two variables $Z$ exhibits mean reverting pattern around long term equilibrium. As Alexander (2008b, p.228) defines a “cointegrating vector is the vector of constant coefficients in $Z$” and specifies that in the bivariate case there can exist just one cointegrating vector $(1,-\delta)$, and, in the case of a multivariate case there can be up to $(n-1)$ cointegrating vectors. It is important to note, that the more cointegrating vectors there are, the stronger is the long-term equilibrium between the analysed variables.

Although cointegration analysis does not attempt to predict where integrated asset prices will be in the long term, however it has a characteristic, which is very important for risk management. Since cointegrated prices have a common stochastic trend, by establishing a long-term equilibrium relationship we can predict the spread between spot and futures prices in our investigation of the cost-of-carry model, or spread between carbon and other commodities in carbon dependence on energy commodities and financial markets analysis. Due to finite variance of this spread, cointegrated asset prices tend not to float too far from each other and revert to

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35 $Z$ is also called disequilibrium because it shows the deviations from the expected equilibrium value.

36 Integrated process has an infinite unconditional variance, and thus, it can be at any level over a period of time.
long-term equilibrium relationship. This is very a desirable and valuable property in risk management, which may be utilised for hedging of carbon risk.

To structure our discussion of cointegration analysis further, we will divide it into the following stages:

- Long-term equilibrium analysis, where long-run relationships between variables are defined.
- Establishment and correction for short-term disequilibrium factors. Here a dynamic Vector Error Correction Model (VECM) that is based on linear regression analysis of returns is utilised.

While testing for cointegration it is very important to study a sufficiently long period of data, otherwise no common long-term relationship can be detected, thus the time span of the data under analysis is more important that the frequency of the observations. The most common way to test for cointegration is to perform an Engle and Granger (EG, 1987) test. According to EG methodology a simple OLS regression (which is called Engle and Granger regression\textsuperscript{37}) of one integrated variable on the other integrated variable has to be performed and then a unit root test has to be carried on the residuals of the regression. In the case of our previous example, if the unit root test indicates, that the error process of EG regression is stationary, then variables $X$ and $Y$ are cointegrated and disequilibrium parameter $Z$ the equation (5.7) is stationary linear combination of integrated variables, and represents long-term equilibrium.

Although EG test is easy to apply, there are several problems which lead for the search of other tests in order to confirm the existence of the long-term equilibrium relationship. First, in the case of testing for cointegration an equation of more than two variables, the results of the test will be influenced by the choice of the dependent variable. And second, but not less important, is that the EG regression allows us to find just one cointegrating vector, however, as explained earlier, in multivariate setting there might be up to $(n-1)$ stationary linear combinations representing the long-term equilibrium.

\textsuperscript{37} It should be noted that EG regression is the only situation, where OLS regression can be applied to non-stationary data. In the case, when cointegration does not hold, OLS estimators will be inconsistent.
As an alternative to EG test, a more sophisticated Johansen test may be proposed, that is able to consistently determine the number of cointegrating vectors. The purpose of Johansen test is to find a linear combination which is most stationary. Johansen test is based on VAR(1) process. Let \( y_t = (y_{1t}, \ldots, y_{kt})' \), \( t = 1, 2, \ldots \) denote a \( k \)-dimensional vector of random variables of interest. The VAR(1) is expressed:

\[
y_t = \delta + \Phi_1 y_{t-1} + \epsilon_t
\]

where the \( \epsilon_t \) vector is a vector white noise process with \( \epsilon_t = (\epsilon_{1t}, \ldots, \epsilon_{kt})' \) such that \( E(\epsilon_t) = 0 \), \( E(\epsilon_t \epsilon_t') = \Sigma \), and \( E(\epsilon_t \epsilon_s') = 0 \), for \( t \neq s \); \( \delta = (\delta_1, \ldots, \delta_k)' \) is a constant vector and \( \Phi_1 \) is a \( k \times k \) matrix. After subtracting \( y_{t-1} \) from both sides:

\[
\Delta y_t = \delta + \Pi y_{t-1} + \epsilon_t \quad \text{where } \Pi = \Phi - I \text{ and } I \text{ is } k \times k \text{ identity matrix.}
\]

To correct for potential autocorrelations of residuals we follow a methodology of ADF test (discussed in section 5.1.) and add to the test lagged dependent variables:

\[
\Delta y_t = \delta + \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_q \Delta y_{t-q} + \epsilon_t
\]

where \( q \) is the number of lags so the autocorrelation of residuals is removed. According to this equation \( \Pi y_{t-1} \) must be stationary, and when the rank of the matrix \( \Pi = r \), where \( r>0 \), there will be \( r \) independent linear relations between \( y_t \) vector of stationary variables. Consequently, Johansen cointegration test is a test on the rank of \( \Pi \) or equivalently the number of cointegrating vectors. The way the test is carried is by first testing null hypothesis \( H_0: r=0 \) versus alternative hypothesis \( H_1: r>0 \). In case \( H_0 \) is not rejected, we can conclude that there are no cointegrating relationships in our model, however if \( H_0 \) is rejected, we repeat the test with forming a new null hypothesis, where \( H_0: r=1 \) and alternative hypothesis \( H_1: r>1 \). If we do not reject \( H_0 \), we conclude that there is one stationary linear combination representing the long-term equilibrium, in case we are able to reject \( H_0 \) we repeat the testing routine again until we are not able to reject \( H_0 \).

Having discussed the two most common cointegration tests, it is important to note, that although EG test has significant limitations, however sometimes it might be preferred from the risk
management point of view. In line with risk optimisation objective, EG test is searching for a minimum variance, while Johansen test is looking for a solution for maximum stationarity.

Extending our discussion in section (4.4.2.) where we were debating the applicability of cointegration analysis under equilibrium cost-of-carry model, it is important to provide findings of Brenner and Kronner (1995). The authors argue that if “interest rates have a stochastic trend, then the spot and futures prices will not be cointegrated by themselves, and the differential should be included in the cointegrating vector. And that the “same results hold if convenience yields are stochastically trended, as long as interest rates and convenience yields are not cointegrated with cointegrating vector (1,-1)”. Thus, any studies of cointegration of spot and futures prices should include either trivariate study of cointegration between interest rates and spot and futures prices or multivariate analysis of long-term equilibrium of spot and futures prices, interest rates and some stochastically trending elements of the cost-of-carry model.

After a comprehensive discussion of the first stage of cointegration modelling, where we explained how long-term equilibrium relationship between asset prices is detected, we will now focus on the second stage that takes care of short-term deviations from the equilibrium relationship.

**Vector Error Correction Model.** Granger (1986) has shown that if two or more variables are cointegrated, then VAR model is miss-specified as disequilibrium term is missing, however there must exist an ECM under which lagged disequilibrium terms are included as explanatory variables. This is known as Granger representation theorem. In this situation VECM is well specified, as it introduces a dynamic self-regulation system due to which all the deviations from long-term equilibrium are automatically corrected. According to Boutaba (2008) VECM captures three channels of causality – a) lagged values of the differenced variables (weak (short-run) causality, b) error-correction as long run causality measure and c) joint significance of the first two channels of causation.
The connection between the two stages of cointegration analysis is the already defined disequilibrium term $Z$. Assuming there are $r$ cointegrating vectors the general form of ECM is expressed as:

$$\Delta y_t = \delta + \sum_{i=1}^{p} \Phi_i \Delta y_{t-i} + \sum_{j=1}^{q} \Gamma_j Z_{t-j} + \epsilon_t$$

where $Z_t = \begin{pmatrix} Z_{1t} \\ \vdots \\ Z_{rt} \end{pmatrix}$

(5.11)

ECM can be estimated by applying OLS methodology in estimation of each equation separately. Coming back to our description of cost-of-carry model in section 4.4.1 we can express a simple bivariate carbon spot and futures prices ECM as:

$$\Delta s_t = \alpha_0 + \sum_{i=1}^{m} \alpha_1 \Delta f_{t-1} + \sum_{i=1}^{n} \alpha_2 \Delta s_{t-1} + \alpha_3 u_{t-1} + \epsilon_t^s$$

$$\Delta f_t = \beta_0 + \sum_{i=1}^{k} \beta_1 \Delta f_{t-1} + \sum_{i=1}^{p} \beta_2 \Delta s_{t-1} + \beta_3 u_{t-1} + \epsilon_t^f$$

(5.12)

where $s_t=lnS_t$ and $f_t=lnF_t$, and $u_{t-1}$ is the lagged error term from the cointegrating vector, standing for deviations from the long-term equilibrium.

To wrap up our discussion of cointegration technique and highlight the advantages of a two-step equilibrium approach we will come back to the comparison with correlation metric with which we started this section. First of all, it is important to note that although both methods measure dependency, high correlation between asset returns does not automatically imply existence of cointegrating relationship of asset prices. The opposite is also true – the existence of cointegrating relationship does not imply the existence of a significant correlation between the variables. This leads to one important conclusion – correlation is not satisfactory instrument to measure long-term interdependence between analysed variables. Cointegration adequately corrects for this drawback on top of that integrating correlation based dynamic approach to correct for short-term market deviations. As a main drawback of cointegration application we may list a difficulty to give economical interpretation to multivariate cointegrating vectors in multivariate cointegration investigation (Maddala 2001). However, as argued by author, this should not be surprising, as “cointegration is purely statistical concept based on properties of the
time-series considered” and thus not need to have any economical interpretation – the most important is its ability to improve econometrical models.

**Granger Causality.** The existence of cointegration implies the existence of statistical causality between the asset returns. Alexander (2008b, p.246) explains that “X Granger causes Y if lagged values of X help to predict current and future values of Y better than just lagged values of Y alone”.

To carry Granger causality test, we may utilise ECM specification used for short-term correction of equilibrium relationship. If we consider our previous bivariate example of ECM of carbon spot and future prices expressed in equation (5.12) we would test spot price causality of futures price by testing a joint significance of all the variables containing lagged spot prices in first equation, while testing futures causality of spot prices we would test for the joint significance of all variables containing lagged futures prices in first equation. That is we would have the following hypotheses:

- Spot prices Granger cause future carbon prices, when \( H_0: \alpha_{2i} = 0 \)
- Futures prices Granger cause spot carbon prices, when \( H_0: \beta_{1i} = 0 \)

It should be noted, that Granger causality test is nothing more, than a lead-lag relationship between variables and that there is a possibility that movements in returns are actually caused by a third variable (Alexander 2008b).

**5.4. Copula analysis**

Copula is another useful tool to study relationships between the random variables and it is widely used in modern risk management theory. In Latin *copulae* means to link, to tie or connect. While in statistical terms, it refers to a function which allows to combine univariate distributions to create a joint distribution with a particular dependence structure. It is neither a short term, nor long term dependency measure, as it depends on which model it is fitted. For example, if we fit it to ARIMA, where autocorrelation exhibits a geometric rate of decay, we would be looking at a short term measure, while in case of ARFIMA (Autoregressive Functionality Integrated Moving
Average), where autocorrelation follows a hyperbolic decay rate we would analyse long term dependency structure.

The concept of copulas was initially introduced by Sklar (1959), and its first statistical applications were carried in mid-eighties. However, copula methodology was not in the horizon of empirical economic and financial practice until the end of the nineties, when Embrecht’s, McNeil’s and Straumann’s (2002) study was circulated as working paper in 1999. A year later Li (2000) applied copula concept for the default modeling in risk management and since then copula theory has gained popularity among modern financial, microeconomical, macroeconomical and risk management theoretics and practitioners.

The strenght of copula draws from the fact, that unlike classical risk metrics as volatility or correlation, it does not require normality assumption. Dowd (2004) emphasized that copula does not require any assumptions on the joint distributions among analysed assets – it is a powerfull technique that can separate the marginal distributions of considered assets returns from their joint dependence structure. This allows to apply copulas to any marginal distributions (i.e. normality assumption is no longer necessary) and, in addition, it does not require that those marginal follow the same distribution. It is important to note, that different copulas produce different joint distirbutions, when applied to the same marginals. This is because, copulas do not differ so much in the degree of dependence they provide, but rather in the dependence structure – in which part of the distribution this dependence is strongest. Another important property is that, most copulas allow to account for the extreme values in distribution tails.

5.4.1. Definition of Copula and Sklar’s theorem

Copula – is a multivariate distribution function (d.f.) $F$ of random variables $X_1, \ldots, X_n$ with standard uniform marginal distributions $F_1, \ldots, F_n$, i.e. $X_i \sim F_i$, $i=1, \ldots, n$. Alternatively, copula is any function $C: [0,1]^n \rightarrow [0,1]$ with the following properties:

38 Li’s (2000) relatively simple concept with the help of the Gaussian copula to model credit default correlation while using market data of credit default swap prices came into a wide use. Even an influential rating agency Moody’s has incorporated it into their rating methodology in 2004. Finally, the formula became so widely used (and mis-used!) that it was later even blamed by many for causing a 2008 credit crisis and later financial crisis. The problem was that too many people put faith in a formula that did not account for tail dependence and dramatic underestimation of risk occurred. (More on Gaussian copula and tail dependence in next sections)
\( C(x_1, \ldots, x_n) \) is increasing in each component \( x_i \).

(ii) \( C(1, \ldots, 1, x_i, 1, \ldots, 1) = x_i \) for all \( i \in \{1, \ldots, n\} \), \( x_i \in [0,1] \).

(iii) For all \( (a_1, \ldots, a_d), (b_1, \ldots, b_n) \in [0,1]^n \) with \( a_i \leq b_i \), we have:

\[
\sum_{i_1=1}^{2} \cdots \sum_{i_n=1}^{2} (-1)^{i_1+\cdots+i_n} C(x_{i_1}, \ldots, x_{i_n}) \geq 0, \quad \text{where } x_{j_1} = a_j \text{ and } x_{j_2} = b_j \text{ for all } j \in \{1, \ldots, n\}
\]

\( (5.13) \)

**Sklar’s Theorem.** Let \( F \) be a joint \( n \)-dimensional d.f. with univariate margins \( F_1, \ldots, F_n \) (not necessarily continuous). Let \( A_j \) denote the range of \( F_j \), \( A_j := F_j([0,1]) \) (\( j = 1, \ldots, n \)). Then there exists a copula \( C \) that for all \( (x_1, \ldots, x_n) \in \mathbb{R}^n \),

\[
F(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_n(x_n))
\]

\( (5.14) \)

If the margins are continuous then \( C \) is unique; otherwise \( C \) is uniquely determined on the range of marginals \( A_1 \times \cdots \times A_n \).


The above theorem, also permits for convenient converse implication: if \( C \) is a \( n \)-dimensional copula and \( F_1, \ldots, F_n \) are univariate d.f.’s, then the function \( F: \mathbb{R}^n \to [0,1] \) defined above in (5.14) is a \( n \)-dimensional joint d.f. with margins \( F_1, \ldots, F_n \). This is a very useful property, if one is considering separate univariate components of a random vector reflected by some copula.

Differentiating (5.14) with respect to \( x_1, \ldots, x_n \), we obtain the associated copula density function:

\[
c(F_1(x_1), \ldots, F_n(x_n)) = \frac{\partial^n C(F_1(x_1), \ldots, F_n(x_n))}{\partial F_1(x_1) \cdots \partial F_n(x_n)}, \quad \text{regarded as a function } u_n = F_n(x_n)
\]

\( (5.15) \)

If one has a copula density and marginal densities \( f_\nu(x) = F_\nu(x) \), then one can obtain the joint density of the original variables, using

\[
f(x_1, \ldots, x_n) = f_1(x_1) \cdots f_n(x_n) c(F_1(x_1), \ldots, F_n(x_n))
\]

\( (5.16) \)

\(^3\) The proof of Sklar’s theorem was not given in Sklar (1959), just a sketch was provided in Sklar (1973). Some “indirect” proofs of Sklar’s theorem were later discovered by various scholars.
As already mentioned, the values of the d.f.’s of the marginals are uniformly distributed. Thus, uniformly distributed variables \( u_n \in [0,1] \) can be used to denote copula d.f.’s \( F_n(x_n) \). In this manner, implicit formula can be expressed as:

\[
C(u_1, \ldots, u_n) = F(F_1^{-1}(u_1), \ldots, F_n^{-1}(u_n)),
\]

(5.17)

where \( u_n \) are the quantiles of the marginals and \( F_n^{-1} \) is a quantile function of \( F_n \) given by

\[
F_n^{-1}(\alpha) = \inf\{x|F_n(x) \geq \alpha\}, \alpha \in (0,1).
\]

Thus, copulas can be seen, as a way to transform a random vector \( (X_1, \ldots, X_n) \) into another random vector \( (U_1, \ldots, U_n) = (F_1(X_1), \ldots, F_n(X_n)) \), with uniform margins \([0,1]\) and with dependence among the components maintained. Equally, by utilising Sklar’s theorem, any copula can be combined with different univariate d.f.’s in order to obtain the n-variate d.f.

There is a wide range of different copulas, see e.g. Nelsen (1999) or Cherubini (2004), but most of the commonly used copulas may be attributed to one of the two families – elliptical and archimedean. As this paper, studies bivariate copulas, for the sake of simplicity bivariate copulas will be discussed going forward. However, the multivariate generalisation is straightforward.

5.4.2. Elliptical Copulas

Copula is called elliptic, if its d.f. is elliptic. Elliptical copulas are implicit, as they are directly derived from multivariate distributions by inverting Sklar’s (1959) theorem. The most famous examples of elliptical copula family are Gaussian (normal) and Student’s-t copulas. Elliptical copulas are symmetric copulas in the sense, that \( C(u_1, u_2) = C(u_2, u_1) \) and hence, their lower and upper tail dependence coefficients are the same. Since elliptical copulas have elliptically contoured distributions, their parameter is a simple linear correlation parameter \( \rho \), that shows linear dependence between variables.

**Gaussian (normal) copula** is defined as follows:

\[
C(u_1, u_2) = \Phi(\Phi^{-1}(u_1), \Phi^{-1}(u_2)),
\]

(5.18)

\footnote{In this case bivariate}
where $\Phi_p$ denotes the bivariate standard normal d.f. with correlation parameter $\rho \in (-1,1)$ and $\Phi^{-1}$ the inverse of the univariate standard normal d.f. Gaussian copula does not allow for tail dependence, i.e. it has zero tail dependence. Since it has just one parameter, it is relatively easy to calibrate. The bivariate density function of Gaussian copula is:

$$
c(u_1, u_2; \rho) = (1 - \rho^2)^{-\frac{1}{2}} \exp\left(-\frac{\rho^2 \lambda_1^2 - 2 \rho \lambda_1 \lambda_2 + \rho^2 \lambda_2^2}{-2(1 - \rho^2)}\right)
$$

(5.19)

where $\lambda_1 = \Phi^{-1}(u_1)$ and $\lambda_2 = \Phi^{-1}(u_2)$ are quantiles of standard normal variables.

**Student-t copula** is defined as follows:

$$
C(u_1, u_2) = t_{\nu, \rho}(t_{\nu}^{-1}(u_1), t_{\nu}^{-1}(u_2)),
$$

(5.20)

where $t_{\nu, \rho}$ is the bivariate Student-t d.f. with two parameters: correlation $\rho \in (-1,1)$ and $\nu > 1$ degrees of freedom and where $t_{\nu}^{-1}$ is the inverse univariate Student-t d.f. with $\nu$ degrees of freedom. Student-t copula can be used, where tail dependence should be modelled, however it allows just for symmetrical tail dependence, that is:

$$
2 \nu t_{\nu+1}(-\sqrt{\nu + 1}/\sqrt{1 - \rho}/(1 + \rho) \quad 
$$

and its density function is:

$$
c(u_1, u_2; \rho, \nu) = \rho^{-\frac{1}{2}} \Gamma\left(\frac{\nu + 2}{2}\right) \Gamma\left(\frac{\nu}{2}\right) \left(1 + \frac{\lambda_1^2 - 2 \rho \lambda_1 \lambda_2 + \lambda_2^2}{\nu(1 - \rho^2)}\right)^{-\frac{(\nu + 2)}{2}} \prod_{j=1}^{\nu} \left(1 + \frac{\lambda_j^2}{\nu}\right)^{-\frac{(\nu + 2)}{2}}
$$

(5.21)

where $\lambda_1 = \Phi^{-1}(u_1)$ and $\lambda_2 = \Phi^{-1}(u_2)$ are quantiles of standard normal variables and $\Gamma$ is the usual Euler function.

### 5.4.3. Archimedean Copulas

Unlike elliptical copulas that are inversely derived from d.f, Archimedean copulas are based on so called *generator functions* and are defined as:

$$
C(u_1, u_2) = \varphi^{-1}(\varphi(u_1) + \varphi(u_2)),
$$

(5.22)
where, \( \varphi \) is generation function, \( \varphi : [0,1] \rightarrow [0, \infty) \) is a continuous strictly decreasing convex function such that \( \varphi(1) = 0 \) and \( \varphi^{-1} \) is the pseudo-inverse.

Like in the case of elliptical copulas, where copula parameter was linear correlation coefficient \( \rho \), Archimedean copula parameter \( \theta \) might be interpreted as a measure of the degree of dependence between variables. The greater the \( \theta \), the stronger is the dependence between the analysed variables – especially in the tails.

Archimedean copula family has a very important property – they allow to capture asymmetric tail dependence, which is frequent in economic and financial data. With Archimedean copulas one can model e.g. heavy lower tail dependence with zero upper tail dependence or model both, but of different weights. This property is very important in risk management, where extreme dependence in the lower tail may lead to heavy loses.

**Clayton copula** introduced by Clayton (1978) has a d.f. as follows:

\[
C(u_1, u_2) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-\frac{1}{\theta}}
\]

(5.23)

Its generation function is \( \varphi(t) = \frac{1}{\theta}(t^{-\theta} - 1) \) and parameter \( \theta > 0 \). Clayton copula has strong lower tail dependence, that is expressed as: \( (2 - 2^{\frac{1}{\theta}}) \), while its upper tail dependence is zero.

**Gumbel copula** first mentioned in Gumbel (1960) is expressed in the following way:

\[
C(u_1, u_2) = \exp\{-[(-\log u_1)^{\theta} + (-\log u_2)^{\theta}]^{\frac{1}{\theta}}\}
\]

(5.24)

The generation function is \( \varphi(t) = -(\log t)^{\theta} \), parameter is \( \theta \geq 1 \), Gumbel copula has zero lower tail, however, it has strong upper tail dependence – \( (2 - \frac{1}{\theta}) \).

**Frank copula** described in Frank (1979) has the following distribution function:

\[
C(u_1, u_2) = -\theta^{-1} \log \left( \frac{(1 - e)^{-\theta} - (1 - e^{-\theta u_1})(1 - e^{-\theta u_2})}{(1 - e)^{-\theta}} \right)
\]
The generation function is \( \varphi(t) = -\log\left(\frac{e^{-\theta t} - 1}{\theta - 1}\right) \) and parameter range is \( 0 < \theta < \infty \). Frank copula has zero tail dependence.

**Joe copula**, first described by Joe (1993) is a generalised case of the Gumbel and Clayton copulas and has the following form: 

\[
C(u_1, u_2) = 1 - ((1 - u_1)^\theta + (1 - u_2)^\theta - (1 - u_1)\theta (1 - u_2)^\theta) \frac{1}{\theta}
\]

The generation function is: \( \varphi(t) = -\log(1 - (1 - t)^\theta) \), parameter is \( \theta \geq 1 \) and like Gumbel copula it has zero lower tail dependence, while its upper tail dependence is strong and is expressed as: \( 2 - 2\frac{1}{\theta} \).

### 5.4.4. Correlations and Copulas

Schweizer and Wolff (1981) have demonstrated that rank correlation measures Kendall’s \( \tau \) and Spearman’s \( \rho \) could be expressed in terms of copula:

\[
\rho_S(X, Y) = 12 \int_0^1 \int_0^1 [C(u_1, u_2) - u_1 u_2] du_1 du_2;
\]

\[
\tau = 4 \int_0^1 \int_0^1 C(u_1, u_2) dC(u_1, u_2) - 1.
\]

In case of one parameter copula, the above given equations (5.27) provide us with a method to calibrate copula, while using an estimate of rank correlation. It also can be shown, that in the case of Gaussian and Student-t copulas, with parameter that is linear correlation \( \rho \), the relationship with Kendall’s \( \tau \) is

\[
\tau = \frac{2}{\pi} \arcsin(\rho)
\]

and with Spearman’s \( \rho_S \):

\[
\rho_S = 2 \sin \left(\frac{\pi}{6} \rho\right).
\]
In the case of one parametrical Archimedean copulas\textsuperscript{41}, the concept is even more straightforward. Genest and MacKay (1986) showed that the population version of the Kendall’s τ can be directly expressed in copula generation function \( \varphi \):

\[
\tau = 1 + 4 \int_{0}^{1} \frac{\varphi(t)}{\varphi'(t)} dt.
\]

(5.30)

Authors also demonstrated that copulas can be ordered in the same way, as their association parameters. If we denote copulas generated by \( \varphi_1 \) and \( \varphi_2 \) respectively as \( \mathcal{C}_1 \) and \( \mathcal{C}_2 \), then \( \mathcal{C}_1 < \mathcal{C}_2 \iff \tau_{\mathcal{C}_1} \ll \tau_{\mathcal{C}_2} \). The same is true with Spearman’s \( \rho \): \( \mathcal{C}_1 < \mathcal{C}_2 \iff \rho_{\mathcal{C}_1} \ll \rho_{\mathcal{C}_2} \).

(5.31)

For the Archimedean copulas, to be analysed in this paper, the relationship between Kendall’s \( \tau \) and copula parameter \( \theta \) is as follows:

\begin{align*}
\text{Clayton:} & \quad \frac{\theta}{\theta + 2}; \\
\text{Gumbel:} & \quad 1 - \frac{1}{\theta}; \\
\text{Frank}\textsuperscript{42}: & \quad 1 - \frac{4}{\theta} + 4 \frac{D_1(\theta)}{\theta}; \\
\text{Joe}\textsuperscript{43}: & \quad 2\theta - 4 + 2\gamma + 2\log 2 + \Psi\left(\frac{1}{\theta}\right) - \Psi\left(\frac{2 + \theta}{2\theta}\right). 
\end{align*}

(5.32) \quad (5.33) \quad (5.34) \quad (5.35)

5.4.5. Fitting Copulas to Data

Boerger et al. (2007) applied copula approach to explore cross-commodity joint asset return distributions and their application to risk management of power utility. Gronwald et al. (2011) use dynamic Gaussian, Student-t, Clayton and Gumbel copulas to analyse the dependence structure between the 2008-2009 EUA future returns and those of financial assets and commodities. Authors advocate that copulas allow to model the dependence in a more general and flexible setting, than linear dependence approach, that is captured by correlation. In this

\textsuperscript{41} Please note, that there are Archimedean copulas, with more than one parameter, however, they are not in the scope of this paper.

\textsuperscript{42} \( D_1(\theta) = \int_0^\theta \frac{c^\theta}{\exp(c) - 1} dx \) (Debye function)

\textsuperscript{43} \( \gamma = \lim_{n \to \infty} (\sum_{i=1}^{n} \frac{1}{i} - \log n) \approx 0.57721 \) (Euler’s constant), \( \Psi(x) = \frac{d}{dx} \log(\Gamma(x)) \) (Diagamma function)

51
paper, however, focus on static copulas: Gaussian and Student-t both of which belong to Elliptical copulas family and Archimedean copulas: Clayton, Gumbel, Frank and Joe.

We will estimate copula parameters based on two methodologies: a non-parametric Kendall’s τ Moment Estimator method and Maximum Pseudo-Likelihood approach.

**Kendall’s τ Moment Estimator (KME)** is a straightforward one step nonparametric approach suggested by Genest and Rivest (1993) and is based on the relationship of copula parameters with Kendall’s τ that was discussed in the previous section. The advantage of the KME method is that one does not need to know marginal distributions of returns in order to estimate copula parameter. Because of this property, KME is generally preferred, when there are outliers in the data or in the case that the distributions of the marginals are heavy tailed.

On the other hand, if the number of observations is relatively large, then Maximum Pseudo-Likelihood (MPL) method, proposed by Genest et al. (1995), might be preferred, as it gives a more precise copula parameter estimate. MPL is basically a two-step approach: we use empirical cumulative distribution functions (ecdf) of marginal distributions to transform the observations \{(X_{i1}, X_{i2}), i=1, ..., n\} of a bivariate distribution into pseudo-observations with uniform margins \((U_{i1}, U_{i2})\). Then, copula parameter can be estimated by maximizing copula density:

\[
\hat{\theta} = \arg \max_{\theta} \sum_{i=1}^{2} \log c(u_{i1}, u_{i2}; \theta).
\]

(5.36)

In order to decide, which of the analysed copulas fits best the dependence structure of the variables we employ a method proposed by Genest et al. (2006). Based on Kendall’s τ process, introduced by Genest and Rivest (1993), authors compute p-values for Cramér-von Mises and Kolmogorov-Smirnov statistics: \(S_n\) and \(T_n\) respectively. Under this procedure, we test for \(H_0: C \in \mathcal{C}\), where \(\mathcal{C}\) is a parametric class of copulas that is being tested. In case the observed value of \(S_n\) or \(T_n\) is greater than the 100(1-\(\alpha\))th percentile of the distribution tested, the null hypothesis is rejected.
6. Empirical Analysis

In this chapter, we will present the main findings of our empirical study. We will first present data used in our analysis, then we will move to graphical and descriptive analysis. We then check the stationarity of time series and look for the evidence that current EU ETS market setting calls for internal abatement of CO2 emissions. In section 6.4, we start carbon prices interdependency analysis by looking for a correlation with energy commodities and financial assets. Then, in section 6.5, we continue studying the efficiency of the EU ETS market, by investigating if Hotelling rule holds in Phase II. In this section, we deepen carbon interdependency analysis, by providing OLS linear regression estimates. In section 6.6, we investigate if cost-of-carry relationship, between EUAs spot and futures prices, holds. We further deepen our understanding of carbon prices interdependency with other tradable assets with the help of cointegration technique. Granger causality estimates are also presented in section 6.7. Finally, we fit various copulas to AR(1) marginals residuals and give a new perspective on previous prices interdependency findings.

6.1. Data Discussion

The data analysed in this paper ranges from January 2\textsuperscript{nd}, 2007 to July 30\textsuperscript{th}, 2011 and consists of 1155 daily observations. The values are closing prices at the end of each trading day. All data, except EUAs spot and futures prices are obtained from Bloomberg (BBG) platform.

Since one of the major emitting sectors – power utilities – tends to start hedging their open positions\textsuperscript{44} two years in advance, we artificially create continuous future price series by concatenating year-ahead data. For simplicity, at the beginning of each year we take futures price quoted for the subsequent year and use until the end of the year (e.g. at the beginning of 2008 we incorporate 2009 futures prices and use until the end of 2008, then at the first trading day we introduce 2010 futures prices). Following this methodology, there is a data point, when corresponding returns can not be computed, since observations are not comparable.

\textsuperscript{44} As explained in section 4.1.
**Carbon Price.** The EUA spot (denoted as $CO2S$) and futures ($CO2Y1$) prices, analysed in this paper, are data presented in Point Carbon\textsuperscript{45} (Thomson Reuters) database. We use daily closing prices, expressed in EUR/ per ton of CO2. Since 2008 spot EUAs started trading just after the compliance event for Phase I\textsuperscript{46}, the time series of CO2 spot prices range from April 4\textsuperscript{th}, 2008 to July 30\textsuperscript{th}, 2011. This affects all the models (namely Hotelling and cost-of-carry), and approaches were spot price dynamics is analysed.

**Rates of Return.** The risk free rate of return ($rf$) (BBG ticker: EUR003M) is the 3 months Euribor presented as daily rates\textsuperscript{47}. We choose Dow Jones EuroSTOXX 50 Index ($EuroS50$) (BBG: SX5E) as market rate of return and present it as daily rates. We also consider a variable from fixed income market – 10Y German Bund (BBG: GTDEM10Y).

**Energy Prices.** Since Germany’s power sector is a single biggest emitter of CO2 gases, in our analysis we choose to use German electricity prices ($PowerY1$) (BBG: ELGB) in EUR/MWh. We also choose Brent ($BrentY1$) (BBG: CFWDCO) futures traded on InterContinental Exchange (ICE), the European leader on energy futures. The price of Brent is converted to EUR/bbl in order to avoid Foreign Exchange effects. The natural gas price ($GasY1$) is the daily natural gas prices negotiated on TTF (Title Transfer Facility) Hub (BBG:TTFG) in Netherlands, presented in EUR/MWh. Coal price is API2 (CoalY1) (BBG: API2), a benchmark coal price for coal imported to northwest Europe, converted to EUR/t.

**Foreign Exchange Rate** is used in order to avoid currency bias. EUR/ USD exchange rates are also taken from BBG platform (BBG:EURUSD)

**Switching Price**, expressed in €/MWh, can be viewed as a proxy of carbon abatement costs among power generation companies. We include it in our analysis, because, as already discussed,

\textsuperscript{45}http://www.pointcarbon.com

\textsuperscript{46}As discussed, Phase I was characterised by the overallocation of the credits that caused CO2 prices to approach levels close to zero.

\textsuperscript{47}Daily observations of annual interest rate were transformed to daily interest rate by using the following formula: $= \exp \left( \frac{\ln(1+i)}{n} \right) - 1$, where $rf$ is daily 3M interest rate, $i$ – annual interest rate and $n$ – number of trading days.
power generation segment is the single biggest carbon emitter in the EU ETS. We calculate switching price, following Point Carbon methodology (Appendix 7):

\[
\text{switch price} = \frac{\text{Gas cost} - \text{Coal cost}}{\text{Coal Emissions (t/MWh) - Gas Emissions (t/MWh)}}
\]  

(6.1)

6.2. Descriptive and Graphical Analysis

The EUAs spot price time series, analysed in this paper ranges from 3\textsuperscript{rd} of April, 2007 to 31\textsuperscript{st} of October, 2011. The minimum spot price, during this period stood at 8.00€/t (12/02/09) as a consequence of global financial and economical crisis, while the maximum price was 29.38€/t (01/07/08), couple of months before the global turmoil, when fuel prices were still booming. Similarly the lowest value of EUAs future price during the period under analysis (01/01/2007-31/10/2011)\textsuperscript{48} was 8.45€/t (12/02/09) and maximum 30.60€/t (12/02/09). The average EUA spot price was 15.62 €/t, while average future price stood at 17.31. Both price series are characterised by very high variance: 18.00€/t and 19.47€/t respectively.

A graphical analysis of EUAs spot and futures prices, presented in Figure 7-1, exhibits a fair degree of co-movement, supporting cointegrating approach, chosen to study relationship between carbon spot and future prices. Furthermore, we observe that analysed time series presents various spikes, with a significant sharp decline after the collapse of Lehman Brothers (15/09/2008). This encourages us to implement a CHOW test for structural break, in order to check if we are dealing with several time series, with different characteristics and to be able to present comparable and robust results. Chow test rejects $H_0$: no structural break hypothesis at 15/09/2008 at 10\% significance level, and at 21/11/2008 at 5\% significance level. We consider those dates as the beginning and the end of the structural break of the analysed series. We also checked 02/03/2009, 14/06/2011 and 23/06/2011, however $H_0$ can not be rejected at any significant level.

\textsuperscript{48} As explained in Chaper 6, two datasets are analysed. Analysis, related to spot market investigates CO2 prices (from 03/04/2008 to 31/10/2011) since the day Phase II EUAs started trading. Since Phase II EUAs futures started trading already in Phase I, we analyse longer time series, namely from 02/01/2007 to 31/10/2011.
6.3. Stationarity tests

In order to check, if current EU ETS market rules are stringent enough to create a scarcity of EUAs, we investigate if European carbon credits prices follow a mean reverting path around an upward trend. We perform ADF and PP tests with H₀: that time series is not stationary. We find, that all prices are characterised by unit root. First differences of the prices are found to be stationary and price series are found to be integrated of level one – I(1). Since ADF and PP are prone to accept unit root, when in fact there is none, we also implement KPSS test, where H₀ is that time series is stationary. When tested on prices, test statistics is not significant at any acceptable level, while carried on first differences, test statistic is significant at 1%, leading to confirm ADF and PP tests results, that price series are I(1). *This finding, however, allows us to conclude that EU ETS legal setting is not strict enough to generate internal abatement of carbon emissions, since carbon prices present a unit root.*
6.4. Correlations

A simple linear correlation analysis\textsuperscript{49} concludes that there is a strong and significant, almost unity, relationship between EUAs spot and futures prices. It has also demonstrated that a significant and relatively strong positive relationship between EUAs and power returns. What is more, the relationship is even stronger during the structural-break period. We are able to confirm Mansanet-Bataller (2007) and Hintermann (2010) results, that a significant positive relationship between carbon and Brent exists. However, this relationship is rather weak, although a little bit stronger in time series during structural-break. In line with Mansanet-Bataller (2007) and Hintermann (2010) findings we find that coal returns are insignificant in relation to EUAs. We are also able to confirm Rickets \textit{et al.} (2007) and Alberola \textit{et al.} (2008) results that coal is negatively related to carbon. This is true for our time series, except period during structural-break. Gas was found to have an insignificant and weak relationship with European CO2 allowances. Interestingly, the existing relationship is found to be negative before the structural-break and positive afterwards. Surprisingly, switch price was found to be weak and insignificant “predictor” of EUAs.

In relation to financial markets, we find a significant relationship between carbon and stock market index, as well as fixed income market instrument-German Bund. The discovered relationship is weak, although strong during the period of structural-break. Interestingly, as commented above, in general the returns seem to be correlated stronger during the period of structural-break. This confirms general findings of researchers, that markets tend to be correlated stronger during turbulent times.

6.5. Hotelling Model

Following theoretical model, presented in section 4.3, we check if EUAs Phase II spot market is efficient. Similar to the stochastic trend framework, Hotelling rule serves as an indicator of scarcity of exhaustible resources. Since in section 6.2 we demonstrated that structural break is present in analysed time series, we calculate four separate models for different subperiods: full

\textsuperscript{49} Selected results presented in Appendix 2, while more detailed results may be found in CD/ROM, included in this paper.
period, pre-structural break, structural break and post-structural break models. The selected results of our calculations are presented in Appendix 3.\textsuperscript{50} In order to present robust results, we also extend the base model and calculate three models:

- Base model is as given by equation (4.5). In order to capture potential impact from outliers, we include two dummy variables \( p_{\text{min}} \) and \( p_{\text{max}} \) that capture biggest swings in EUAs returns. Both dummy variables are significant at 5% during all sub-periods, thus helping to improve the quality of analysed model.
- Shock model – to check the robustness of base model estimates, we add shock energy variables. Following Helfand et al. (2006) methodology, we calculate step-ahead values\textsuperscript{51} of energy commodities. Just unexpected new information on power prices seem to be significant (at 5% level) in respect to carbon price movements.
- Arbitrage Pricing Theory model – we introduce step-ahead values of 10Y Bund and EuroStoxx50, however none of them are significant according to obtained results.

What regards to the testing of Hotelling rule – results are twofold. Given the confidence limits, presented in Figure 0-5, we do not reject \( H_1:H_0 \) that Hotelling rule holds in both pre-structural break and structural-break subperiods, however we are able to reject \( H_0 \) in full period, as well as post-structural break analysis. This would suggest that although Phase II market was efficient enough to achieve competitive intertemporal prices equilibrium, however due to over-allocation, triggered by financial and economical crisis, this equilibrium relationship no longer hold and EUAs prices are subject to intertemporal arbitrage.

Though it is interesting to note, that the standard errors of the estimates during periods, when \( H_1:H_0 \) can not be rejected, are large, hence the confidence intervals are wide. This could be due to the fact, that the analysed sub-periods are relatively short, and could indicate that we do not have enough information to reject \( H_1:H_0 \). \textit{We thus conclude that Hotelling rule does not hold in Phase II, and confirm that Alberola and Chevallier (2007) findings can be extended to Phase II as well.}

\textsuperscript{50} Interested reader may find a separate EXCEL file “Hotelling Output” in CD/ROM, where more detailed results are presented.

\textsuperscript{51} For each value at \( t \) we calculate forecast \( t+1 \). Each forecast uses data for all prior days in order to produce a forecast for \( t+1 \). We then compute a forecast error as a difference between the actual value and the forecast and interpret this variable as new or so called shock information. (See Helfand et al. (2006) for details)
6.6. Cointegration Analysis and Cost-of-Carry Model

To further study interdependence between EUAs spot and futures prices, as well as deepen our understanding of emission allowances relationship with energy commodities and financial assets, we employ cointegration technique. As in previous studies, we divide the analysed time series to four sub-periods.

From stationarity analysis, carried in section 6.3 we already know that all price series are I(1), thus, the necessary condition, that variables have to be integrated at the same level is fulfilled. We first employ Johansen test (results are presented in Appendix 5), to examine if a bivariate cointegration relationship exists between considered assets. We fail to confirm correlation results, as we discover no long-term relationship between carbon and power or Brent prices. This is an example, when no structural relationship exists between correlating assets. However, we find that cointegrating relationships exist between EUAs futures and Switch price in full period and that carbon futures prices are related to EuroSTOXX50 index in post-structural break sub-period. This finding suggests, that energy commodities could not be used to hedge away carbon risk (e.g. by the means of pair-trading).

In respect to EUAs spot and futures prices, we find that a long-term cointegrating relationship exists in full period – this is in line with our correlation analysis results. However, we fail to confirm cointegration between carbon spot and futures prices during the three subperiods. We attribute it to the fact, that data series for pre-break and break sub-periods is too short. It can also be explained by the argument of Brenner and Kronner (1995) presented previously – in the case that stochastic trend is present in interest rates, no cointegrating relationship between the spot and futures prices will be present, and a trivariate cointegration study needs to be performed – this supports our cost-of-carry approach. We hence focus on the verification of cost-and-carry model.
In order to estimate a long-term cointegrating vector and its error correction model, we implement Engle and Granger (EG) OLS regression and find that the long-run cointegrating relationship is:

\[
\hat{\pi}_t = 0.0963 + 0.97714 s_t + 0.1748 r_{fm} + u_t
\]

\[se \quad (0.0058) \quad (0.0022) \quad (0.0009)\]

\[p-val \quad (<.0001) \quad (<.0001) \quad (<.0001)\]

where \(\hat{\pi}_t\) is futures value and \(s_t\) is spot value of EUAs in natural logarithms, \(r_{fm}\) is interest rate times time to maturity and \(u_t\) is regression error term. The low standard errors of the estimates and small \(p\)-values of the \(t\)-statistic, show that all estimates are significant. Having estimated the long-run cointegration relationship, we are now able to obtain a short-term ECM model by regressing the differenced values of spot prices, interest rate and lagged error term on differenced value of futures price. The short-term ECM are as follows:

\[
\Delta s_t \quad 0.96384 \\
se \quad (0.0068) \\
p-val \quad (<.0001)
\]

\[
\Delta r_{fm} \quad 0.00021 \\
se \quad (0.0016) \\
p-val \quad (<.0001)
\]

\[
u_{t-1} \quad 0.39753 \\
se \quad (0.0108) \\
p-val \quad (<.0001)
\]

Once again, the standard errors are small (except interest rate, where standard error is relatively big compared to the estimate itself), and \(p\)-values of \(t\)-statistic show that all variables are significant. The estimated coefficients of ECM can be interpreted as long-term equilibrium speed of adjustment to short-term fluctuations, where a value of “1” indicates that long-run cointegrating relationship is restored in one day, and “0”, that this relationship is not restored at all. From the above results we see, that structural relationship between EUAs spot and future prices is restored during very short period of time, confirming that carbon futures is an appropriate instrument to hedge carbon risk.

From the equation (6.1) we get \(\beta_1\) and \(\beta_2\) estimates, required to test \(H_3\), defined in section 4.4.2, where \(H_0: \beta_1 = \beta_2 = 1\). Since \(r_{fm}\) estimate is very far from 1 and it’s standard error is very low, we can conclude without the formal test, that cost-of-carry relationship does not hold in Phase II. This finding can be also supported by joint test, where the estimated \(p\)-value is less than 0.0001. Hence, we conclude that Phase II carbon futures prices are not characterised by cost-of-carry relationship with spot prices and risk free arbitrage opportunities are present. It can be partially explained by the fact, noted by Uhrig-Homburg and Wagner (2008) that during the freeze of credit markets many carbon markets participants chose to sell their emission allowances on the
spot market, and buy futures contracts. This has led to a super-contango situation, where futures prices are much higher, than they should be under the equilibrium cost-of-carry relationship.

**Granger Causality.** Since our cointegration analysis indicate that just CO2 spot prices, EuroSTOXX50 index and switch price are cointegrated with carbon futures, we will further only on those relationships. Granger causality $H_0$ is that variable is influenced by its lagged values and not by other variables.

$$\chi^2 \quad \text{Pr}>\chi^2$$

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\chi^2$</th>
<th>Pr $&gt;\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO2Y1 by Switch</td>
<td>1.88</td>
<td>0.391</td>
</tr>
<tr>
<td>Switch by CO2Y1</td>
<td>4.42</td>
<td>0.110</td>
</tr>
<tr>
<td><strong>CO2Y1 by EuroStoxx50</strong></td>
<td><strong>7.47</strong></td>
<td><strong>0.024</strong></td>
</tr>
<tr>
<td><strong>EuroStoxx50 by CO2Y1</strong></td>
<td><strong>6.16</strong></td>
<td><strong>0.046</strong></td>
</tr>
<tr>
<td>CO2Y1 by CO2s</td>
<td>3.16</td>
<td>0.206</td>
</tr>
<tr>
<td>CO2s by CO2Y1</td>
<td>0.91</td>
<td>0.633</td>
</tr>
<tr>
<td>CO2Y1 by CO2s and rf</td>
<td>1.29</td>
<td>0.863</td>
</tr>
<tr>
<td>CO2s by CO2Y1 and rf</td>
<td>4.71</td>
<td>0.319</td>
</tr>
</tbody>
</table>

Table 6-1. Granger Causality Test Results

*Source: Author Calculations*

From the Table 6-2 presented below, it can be seen that we reject $H_0$ hypothesis that EUAs futures price is influenced by itself and not by EuroStoxx50 index changes, at the same time we are able to reject $H_0$ for EuroStoxx index and conclude that it is Granger caused by carbon futures prices. Other price series, although cointegrated, do not seem to Granger cause each other. As already noted, in section 5.3, since Granger causality test is nothing more, than a lead-lag relationship between variables, there is a possibility that movements are actually caused by a third (or fourth in case of spot, futures and interest rate) variable.

6.7. Copula Analysis

In order to proceed with copula analysis, we need to calculate standardised residuals for the analysed time series. First, we choose an appropriate ARMA-GARCH model – based on the lowest Akaike Information Critetia (AIC), ARMA (1,0) model looks the best fit for all variables.
We then standardise residuals by using their standard deviations and finally, we transform all data series to uniform \([0,1]\) domain, as the space of copula functions is a unit-hypercube.

In copula analysis, we do not devide data into the sub-periods, as it does not diminish the quality of our estimates of the copula parameters. On the contrary, it improves our calculations, since copulas permit us to capture tail dependencies of marginal distributions (as explained in section 5.4). Thus, the presence of structural break actually allows us to account for tail dependencies of marginal distributions in case of the extreme joint movements of returns. As in previous analysis, in the study of EUAs spot and futures prices we work with the data ranging from 03/04/2008 to 31/10/2011, while in case of dependence analysis of European carbon futures with energy commodities and selected financial assets data from 01/01/2007 to 31/10/2011 in analysed.

We first estimate Kendall’s \(\tau\) correlation coefficients for bivariate time series of marginals – results are presented in Table 6-2. In line with results presented in correlation analysis (section 6.4), we find that significant relationship exists between the marginals of CO2 spot and futures prices as well as between CO2 futures prices and Brent, power, EuroStoxx50 index and 10Y German Bund. The strength and direction of the relationships between innovations is also in line with the ones found in correlation analysis of asset returns.

Following the methodology discussed in section 5.4.5, we calculate copula parameters based on non-parametric Kendall’s \(\tau\) Moments Estimator (KME) and semi-parametric Maximum Pseudo Likelihood (MPL) approaches (results presented in Table 6-2). Since Gaussian and Student-t copula parameters, respectively \(\rho_G\) and \(\rho_t\), are correlation coefficients, we can do a one-to-one comparisons with correlation analysis. The obtained results are consistent with previous findings – we find a strong positive dependence relationship between EUAs spot and futures prices, as well as carbon futures and power prices.

In order to examine, which of the proposed copulas describes best the dependence structure between analysed variables we calculate Cramér-von Mises and Kolmogorov-Smirnov tests statistics \(p\)-values, where low \(p\)-value rejects \(H_0\): that examined copula provides an appropriate fit to generate the joint marginals distribution, at 5% confidence level. For each of the copula, we
### Calculation of copula parameters

We do not calculate inverse relationship for a two parameter Student-t copula, as it is only well-defined for one parameter copulas. 

### Table 6-2. Copula Parameter Estimates

<table>
<thead>
<tr>
<th></th>
<th>Kendall p*</th>
<th>Gaussian ρp**</th>
<th>Student-t ρt**</th>
<th>Clayton θ</th>
<th>Gumbel θ</th>
<th>Frank θ</th>
<th>Joe θ</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>MPL</td>
<td>KME</td>
<td>MPL</td>
<td>MLP</td>
<td>KME</td>
<td>MPL</td>
<td>KME</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CO2S</td>
<td>0.8851</td>
<td>0.976</td>
<td>0.984</td>
<td>10.648</td>
<td>15.400</td>
<td>8.057</td>
<td>8.700</td>
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<td></td>
<td>(&lt;.0001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.233)</td>
<td></td>
<td>(1.027)</td>
<td></td>
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<tr>
<td>CO2Y1</td>
<td>0.1971</td>
<td>0.196</td>
<td>0.214</td>
<td>0.257</td>
<td>0.305</td>
<td>1.121</td>
<td>1.245</td>
</tr>
<tr>
<td></td>
<td>(&lt;.0001)</td>
<td>(0.026)</td>
<td>(0.028)</td>
<td>(0.040)</td>
<td></td>
<td>(0.022)</td>
<td></td>
</tr>
<tr>
<td>Y10Bund</td>
<td>-0.0420</td>
<td>-0.034</td>
<td>-0.033</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>CoaY1</td>
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<td>(0.029)</td>
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<td>(0.172)</td>
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<td>0.737</td>
<td>0.663</td>
<td>1.379</td>
<td>1.856</td>
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<td>1.000</td>
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<td></td>
<td>(0.2531)</td>
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<td>(0.029)</td>
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<td>(0.033)</td>
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<tr>
<td>CO2Y1</td>
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<td>(0.033)</td>
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<td>PowerY1</td>
<td>0.2253</td>
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<td>0.253</td>
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<td>0.347</td>
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<td>EuroS50</td>
<td>0.1832</td>
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<td>0.198</td>
<td>0.260</td>
<td>0.284</td>
<td>1.103</td>
<td>1.224</td>
</tr>
<tr>
<td></td>
<td>(&lt;.0001)</td>
<td>(0.026)</td>
<td>(0.028)</td>
<td>(13.106)</td>
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<td>(0.021)</td>
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<tr>
<td>CO2Y1</td>
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<td>-0.001</td>
<td>-0.002</td>
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<td>N/A</td>
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<td>(0.4841)</td>
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<td>(0.030)</td>
<td>(10.861)</td>
<td></td>
<td>(0.173)</td>
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</table>

Source: Authors’ calculations

* Brackets under correlation are p-values.
** Brackets under copulas are standard errors.
*** We do not calculate inverse relationship for a two parameter Student-t copula, as it is only well-defined for one parameter copulas.
**** Calculation of copula parameters is not available, as those copolas do not support negative asset dependence.

### Table 6-3. Goodness-of-Fit

<table>
<thead>
<tr>
<th>Goodness-of-fit p-values</th>
<th>Gaussian p-values</th>
<th>Student-t p-values</th>
<th>Clayton p-values</th>
<th>Gumbel p-values</th>
<th>Frank p-values</th>
<th>Joe p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CvM</td>
<td>KS</td>
<td>CvM</td>
<td>KS</td>
<td>CvM</td>
<td>KS</td>
</tr>
<tr>
<td>CO2S</td>
<td>0.11</td>
<td>0.50</td>
<td>0.95</td>
<td>0.87</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>CO2Y1 BrentY1</td>
<td>0.21</td>
<td>0.27</td>
<td>0.45</td>
<td>0.45</td>
<td>0.49</td>
<td>0.45</td>
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<tr>
<td>CO2Y1 CoalY1</td>
<td>0.25</td>
<td>0.41</td>
<td>0.41</td>
<td>0.31</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>CO2Y1 PowerY1</td>
<td>0.08</td>
<td>0.02</td>
<td>0.37</td>
<td>0.16</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td>CO2Y1 GasY1</td>
<td>0.03</td>
<td>0.04</td>
<td>0.12</td>
<td>0.27</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>CO2Y1 Switch</td>
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<td>0.82</td>
<td>0.20</td>
<td>0.37</td>
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<td>N/A</td>
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<td>CO2Y1 EuroS50</td>
<td>0.01</td>
<td>0.01</td>
<td>0.04</td>
<td>0.04</td>
<td>0.33</td>
<td>0.09</td>
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<tr>
<td>Y10Bund</td>
<td>0.16</td>
<td>0.10</td>
<td>0.54</td>
<td>0.57</td>
<td>0.96</td>
<td>0.89</td>
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<td>CO2Y1 rf</td>
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<td>0.05</td>
<td>0.14</td>
<td>0.08</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations
create 100 bootstrap samples and determine the distance between the empirical and estimated copula.

Gronwald et al. (2011) in their study find that the best fit to data is provided by Gaussian and Student-t copulas. Our results partially confirm these conclusions, although we find that Gaussian copula appropriately describes just the dependency between EUAs futures and switch price, Student-t copula is found to be appropriate to model joint distribution of European carbon spot and futures prices, emission allowances and Brent and coal year-ahead futures, as well as carbon futures and 10Y German Bund.

The second copula family that is found to present most of the fits to model joint distributions is Joe. It is found to be appropriate in the case of carbon spot and futures price, as well as to model relationship between EUAs and Brent, power, EuroStoxx futures and 10Y German Bund interest rates. Clayton copula seems to be appropriate to model European carbon futures and 10Y German Bund joint distribution of innovations. We conclude that neither Gumbel, nor Frank copulas are appropriate to model dependence between carbon and energy commodities as well as selected financial assets. One of the reasons is, that they do not allow to model negative interdependence, which is discovered between carbon and coal, gas and switch prices.

It should be noted, that the implemented tests of goodness-of-fit do not empower us to conclude, which copula family fits the joint distribution of the marginals. Some more advanced techniques are presented in Chen and Fan (2006) and could be investigated by further research.
7. Application to Risk Management

In this chapter, we will present our recommendations for carbon risk management practices.

The establishment of the EU ETS market and the introduction of a new tradable asset has created a new type of risk – carbon risk, to which more and more companies and investors are becoming exposed in all over the globe. Risk management is comprised of three steps: a) identification b) analysis and c) mitigation or acceptance of the risk.

**Identification.** For a successful carbon risk management, a profound understanding of the potential market drivers is essential. To facilitate the identification process of CO2 risk factors, we have already presented the potential drivers of carbon market in section 3.3.5., where we categorised them into two main groups: a) policy and regulatory issues and b) market fundamentals that are directly related to the production of emissions. As defined in section 2.2. it is not within the scope of the paper to focus on policy and regulatory issues and hence, we will concentrate on the second group.

**Analysis.** Identification and modelling of the dependencies are at the very heart of risk analysis. Until recently, linear correlation was the main metric computed by risk practitioners. In the financial theory both CAPM and arbitrage pricing theory (APT) use correlation as a measure of the dependence between financial instruments under consideration. However, as discussed in section 5.2., robust practical application of correlation is limited to the data that follows a normal distribution. If applied to other distributions – it may lead to misleading results. Relatively recently introduced risk exposure measure – Value-at-Risk\(^{52}\) (VaR), also depends on correlation (and volatility), however, VaR approach does not specify, how these values should be estimated.

VaR as a metric\(^{53}\) is a simple quantile of possible losses distribution. In a univariate case it is defined by asset returns distribution and the variance of returns. However, in a multivariate case,

---

52 In 1994 J.P.Morgan has published first VaR methodology, while in 1999 a worldwide adoption of the concept took place, after the Basel II Accord.
53 VaR can be also referred to as a measure. In that case it is considered as a certain risk aversion threshold level of an investor, so that the probability of mark-to-market losses of the portfolio over a certain period of time, do not exceed this value.
we need an asset interdependence factor. Usually, a normal distribution of asset returns is assumed, thus correlation matrix is used to model asset interdependence. However, as argued above, financial assets rarely follow a normal distribution and correlation matrix is inappropriate metric to model VaR. This claim is also supported by our empirical finding, that Gaussian copula is not adequate distribution to model carbon and analysed assets interdependency. Hence, we suggest non-Gaussian copula approach that overcomes the limitations of the correlation and allows to model non-normal distributions with different tail dependencies.

Mitigation. There are various ways to mitigate risks and the two most common approaches in financial theory are: diversification and hedging. As discussed by Grinblatt and Titman (2002), if the expected value of bad outcome exceeds the expected value of good outcome – a company or investor should consider hedging their risk. Based on the results of empirical analysis, we conclude, that EUAs futures is a suitable instrument to hedge spot prices movements. One of the approaches would be to calculate the so called optimal hedge ratio (OHR).

Hatemi-J and Roca (2006) define OHR as the quantities of the spot and hedging instruments, that ensure that the total value of the hedged portfolio does not change (Fan et al. (2010, pp.6). Authors express it as: $V_h = Q_s S - Q_f F$, where $V_h$ is the value of the hedged portfolio, while $Q_s$ and $Q_f$ are respectively spot and future instruments quantities and $S$ is the price of spot instrument, $F$ is the price of futures instrument. If $\Delta V_h = 0$, then: $\frac{Q_f}{Q_s} = \frac{\Delta S}{\Delta F}$, and if we define $h$ as the optimal hedge ratio, such that $h = \frac{Q_f}{Q_s}$, then $h = \frac{\Delta S}{\Delta F}$.              (7.1)

Fan et al. (2010) analyses different OHRs, based on OLS, two-step and simultaneous VECM, as well as GARCH-VECM models. Authors conclude that in-line with other markets carbon hedge ratio is in the range of 0.5 to 1.0 and tat best results can be achieved by the OHR based on a simple linear regression model. However, due to the fact, that carbon may not follow elliptical distribution, we would suggest that it would be to implement optimal hedge ratio, that would be based on non-quantile regression (that is explained in Alexander, 2008b), where linear quantile model would be replaced by the q quantile curve of copula. Based on the results reported in
Table 6-3., we propose to calculate non-linear quantile regression with Joe and Student-t copulas quantile curves.
8. Summary of Conclusions and Perspectives

The EU ETS is the first and the biggest international carbon trading scheme and is a cornerstone of the EU’s effort to reduce GHGs emissions. It was launched in January 2005 and already from the start was subject to significant EUAs price fluctuations and jumps. Each day more and more companies and investors are becoming exposed to a new type of risk – carbon risk and although there is handful lot of papers, investigating Phase I of the EU ETS, however the amount of the empirical research on Phase II remains scarce.

The aim of this paper was to apply correlation, cointegration and copula approaches in order to analyse the EU ETS asset prices development as well as provide some insights for sound investment and risk management decisions.

In order to answer the research question, we first investigate the efficiency of the European carbon market setting and establish three different conceptual frameworks: stochastic trend, Hotelling rule and cost-of-carry model and discover that:

- Analysed period is characterised by structural break. Hence, we divide our study period into three sub-periods: pre-structural break, structural break and post-structural break periods.
- Current EU ETS legal setting is not strict enough to generate internal abatement of carbon emissions, since carbon prices present a unit root, and do not exhibit a mean reversion around an upward trend, as would be expected if scarcity of allowances existed.
- Hotelling rule does not hold in Phase II, and carbon prices do not meet the equilibrium conditions of exhaustible resources, presenting intertemporal arbitrage opportunities. This finding suggests that EU ETS institutional market setting did not meet the necessary conditions for an efficient intertemporal price signal to emerge. While carrying sub-period Hotelling analysis – interesting results emerged. We found, that Hotelling rule held during pre-structural break and structural break subperiods. Hence, we conclude that inefficiency of Phase II market was caused by financial and economical crisis, rather than ineffective institutional setting.
- Based on cointegration analysis we determine that cost-of-carry model does not hold. We partially explain this result by the fact, that because of the freeze of the credit markets,
many carbon market participants chose to finance themselves by selling their emission allowances on the spot market by buying futures contracts instead. This behaviour triggered a big demand of futures contracts and caused a situation, where futures prices were much higher, than they should have been under the equilibrium cost-and-carry relationship. Since we find a long-term cointegrating relationship between EUAs spot and futures prices, but fail to confirm the existence of cost-of-carry relationship, we conclude that risk-free arbitrage opportunities are present and futures are an adequate measure to hedge carbon risk.

Hence, we conclude that Phase II of European carbon market failed to create an efficient environment, needed to trigger internal abatement.

By investigating carbon prices interdependency with other energy commodities and selected tradable financial assets we find, that:

- Carbon spot prices seem to have significant short-term relationship with Brent and coal prices. Interestingly it was found not to correlate with gas and switching price. In respect to financial assets, a significant relationship between EUAs and EuroSTOXX50 and 10Y German Bund was discovered. What is more, as expected, carbon returns seem to present a stronger correlation with other markets during the period of structural break.

- Contrary to results of short-term correlation analysis, European carbon credits seem no to have a structural relationship with energy commodities. Selected financial assets also do not seem to present a long-term relationship with EUAs. This implies that carbon risk could not be hedged away by the use of energy commodities or analysed financial assets.

- Copula analysis in general supports our conclusions, drawn from the correlation analysis. It should be noted, that just Student-t and Joe copula families seem to be fit to model joint distribution of the marginals. However, our implemented goodness-of-fit tests do not empower us to conclude, which copula family fits the joint distributions of carbon and energy commodities and selected financial markets, and some more advanced techniques are needed in order to investigate it.

In this paper we explain the drawbacks of traditional correlation analysis based risk management strategies. Based on our empirical findings we suggest:
• To model carbon portfolio VaR by the use of non-normal copulas, that allow to capture different tail dependencies.
• To implement non-linear quantile regression technique, based on the q quantile curve of Student-t and Joe copulas.
References


Appendices
Appendix 1

Unit Root TESTS

### Level values

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<th></th>
<th>ADF</th>
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### First Difference

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Table 0-1. Unit Root Tests

Source: Authors’ calculations

The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) test the $H_0$: time series is non-stationary, while Kwiatkowski, Phillips, Schmidt and Shin (KPSS) specifies $H_0$: as stationary. Specification based on lowest reported $p$-value.

The above tests specify, that level values of analysed time series are non-stationary, however their first differences are stationary.

For ADF $p$-values, see Said and Dickey (1984), KPSS – Kwiatkowski et al. (1992).
Appendix 2

Selected Pearson Correlation Coefficients

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<thead>
<tr>
<th></th>
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<th>Post-Break</th>
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<td>RCO2S*</td>
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Table 0-2: Selected Pearson Correlation Coefficients

p-values are provided in the brackets.
All price series are natural logarithms.
* Time series starting from 03/04/2007

Source: Author’s calculations
### Appendix 3

#### Selected Hotelling Rule Model Output

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**Table 0-3. Selected Hoteling Model Output – Part 1 of 2.**

Note: - heteroscedasticity consistent standard errors in the brackets.
** significant at 5% level

Source: Author’s Calculations
## Selected Hotelling Rule Model Output continued

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<td>18,323 19,938 21,492 19,613 **</td>
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Table 0-4. Selected Hotelling Model Output – Part 2 of 2.

Heteroscedasticity consistent standard errors in the brackets.
** significant at 5% level

Source: Author’s Calculations
**Appendix 4**

Confidence Intervals to test Hotelling Rule

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</tr>
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<td>Regression (7)</td>
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<td>[-4.30341; 6.84688] *</td>
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<td>[-2.78662; 3.04643] *</td>
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Table 0-5. Confidence Intervals for the Test of Hotelling Rule

Source: Author’s Calculations

H₀ is β₁=1.

* Do not reject H₀.
Appendix 5

Johansen Test for Cointegration

<table>
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Table 0-6. Johansen Test for Cointegration

All variables are natural logarithms
*Additional calculations were carried.

Source: Author’s Calculations
## Main Characteristics of the EU ETS Market

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<tr>
<th>Category</th>
<th>Phase I</th>
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<th>Phase III</th>
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<td><strong>Member Countries</strong></td>
<td>EU-25</td>
<td>EU-27, Norway and Lichtenstein</td>
<td>EU-27, Norway and Lichtenstein</td>
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<tr>
<td><strong>Economy-wide reduction target</strong></td>
<td>None</td>
<td>Corresponds EU-15 (8% below 1990) and member states targets under Kyoto</td>
<td>20% below 1990 by 2020. Could be increased up to 30%</td>
</tr>
<tr>
<td><strong>Reduction target of ETS</strong></td>
<td>None</td>
<td>Corresponds to member states Kyoto targets</td>
<td>21% below 2005 by 2020</td>
</tr>
<tr>
<td><strong>Total allocation, Mt CO2e/y</strong></td>
<td>Average 2,197</td>
<td>Average of 2,150</td>
<td>2,127 in 2016</td>
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<tr>
<td><strong>Auctioning, Mt CO2e/y</strong></td>
<td>&lt;1%</td>
<td>Average of 68</td>
<td>1,075 in 2016</td>
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<td><strong>Free allocations, Mt CO2e/y</strong></td>
<td>Average of 1,967</td>
<td></td>
<td>1,992 in 2016</td>
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<td><strong>New Entrants, Mt CO2e/y</strong></td>
<td>Average of 115</td>
<td></td>
<td>59 in 2016</td>
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<td><strong>Regional Emissions, Mt CO2e/y</strong></td>
<td>5,039 in 2007</td>
<td>4,940 in 2008</td>
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<td><strong>Emissions covered by ETS, Mt CO2e/y</strong></td>
<td>2,072 in 2007</td>
<td>2,100 in 2008</td>
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<td><strong>Share of Emission Covered</strong></td>
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<td>Power, heavy industry and aviation (from 2012)</td>
<td>Power, heavy industry and aviation</td>
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<td><strong>Regulatory Approach</strong></td>
<td>Point of emission</td>
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<td><strong>Regulatory threshold</strong></td>
<td>20 MW(power), for other sectors consult Annex-I Directive 2003/87/EC</td>
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<td><strong>Number of covered entities</strong></td>
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<td>10,888 in 2009</td>
<td>To be determined</td>
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<td><strong>Gases covered</strong></td>
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<td>CO2, some N2O</td>
<td>CO2, some N2O</td>
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Table 0-7. Main Characteristics of the EU ETS Market

Source: Point Carbon
Appendix 7
Dark and Spark Spreads

The Dark spread is the theoretical gross margin earned by a coal-fired power station selling 1MWh of electricity having bought the required amount of coal to produce this unit of electricity. The spark spread is the equivalent for a gas-fired power plant.

The dark and spark spreads tell us the relative profitability of coal and gas fired power plants, thus indicating what the preferred mode of production is, if there is spare capacity.

Dark spread (€/MWh) = Power price (€/MWh) - "coal cost" (€/MWh)
Spark spread (€/MWh) = Power price (€/MWh) - "gas cost" (€/MWh)

The coal and gas costs can then be calculated as:

Coal cost (€/MWh) = Coal price ($/t) * Heat rate (GJ/MWh)/Energy content (GJ/t) * FX (€/$)
Gas cost (€/MWh) = Gas price (pence/therm) * Conversion (therm/GJ)* Heat rate (GJ/MWh) / Energy content (GJ/t) * FX (€/£)/100

Coal prices are normally nominated in $/t, while gas prices in the UK are nominated in pence/therm. To convert these prices to €/MWh, heat rates and energy content for coal and gas have to be taken into account in addition to currency exchange (FX) rates, see the table below.

Clean dark and spark spreads (CDS and CSS)

CDS and CSS take into account the cost of carbon allowances required to cover the amount of CO2 emitted when producing 1 MWh of power.

CDS (€/MWh) = Dark spread (€/MWh) – "coal carbon cost" (€/MWh)
CSS (€/MWh) = Dark spread (€/MWh) – "gas carbon cost" (€/MWh)
Coal (gas) carbon cost (€/MWh) = Carbon cost (€/tCO2) * coal (gas) emissions factor (tCO2/GJ)* Heat rate (GJ/MWh)

Implied fuel switching price

The implied fuel switching price is the CO2 price that would put the CDS and CSS into equilibrium, thus encourages power producers to switch from coal fired production to gas fired production if capacity permits.

"Coal carbon cost" – "Gas carbon cost" = "Gas cost" – "Coal cost"

Implied switching price = ("Gas cost" – “Coal cost”)/(Coal (t/MWh) – Gas (t/MWh))

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<th>Gas</th>
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<tr>
<td>Average plant efficiency (%)</td>
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<td>Heat rate (GJ/MWh)</td>
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<td>GJ/therm conversion</td>
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</tr>
<tr>
<td>Stationary combustion emission factor (tCO2/GJ)</td>
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Source: Point Carbon

http://www.pointcarbon.com/trading/cmeu/resources/methodologies/modeldescriptions/darkandsparkspreads
Appendix 8

CD/ ROM with time series and additional empirical findings